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ALTERNATING CURRENTS

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ALTERNATING CURRENTS

THEIR THEORY, GENERATION, AND TRANSFORMATION

BY

ALFRED HAY, D.Sc., M.I.E.E.

LONDON AND NEW YORK
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PREFACE

In the following pages an attempt is made to furnish the reader with a general account of the principles, construction, and use of alternate-current measuring instruments, generators, motors, and transforming machinery. Special attention is devoted to methods of testing. The first three chapters contain a sketch of alternate-current theory. The author has tried, as far as possible, to exclude everything of purely academic or historical interest; on the other hand, he has not hesitated to devote a good deal of space to matters which are either not generally understood, or which are of too recent origin to have found their way into many text-books.

The author desires to express his indebtedness to the various firms who supplied him with working drawings and placed other information at his disposal; especially would he like to mention in this connection The Electric Construction Co., Ltd.; Messrs. Everett, Edgcumbe and Co.; Messrs. Kelvin and James White, Ltd.; and Messrs. Nalder Bros. and Thompson, Ltd.

The bulk of the illustrations have been specially prepared for this book.

A. H.

London, N.W.,
October, 1905.

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ALTERNATING CURRENTS

THEIR THEORY, GENERATION, AND TRANSFORMATION

CHAPTER I

§ 1. Alternating currents. Frequency and wave-shape. Form factor and amplitude factor—§ 2. Simple sine waves. Vector diagrams—§ 3. Relations connecting amplitude, r.m.s., and arithmetic mean values of simple sine wave—§ 4. Impressed and induced e.m.f.'s. Self-inductance—§ 5. Fundamental equation for a circuit in which the current is variable—§ 6. Sine waves in circuits containing resistance, self-inductance, and capacity—§ 7. Power in alternating-current circuit.

§ 1. Alternating Currents. Frequency and Wave-shape. Form Factor and Amplitude Factor

An alternating electric current is a current which periodically passes through a definite cycle of changes, the cycle consisting of two half-cycles, during one of which the current is positive, and during the other negative; the negative half-cycle being, however, an exact reproduction of the positive half-cycle, and the only difference being one of algebraic sign.

The graph of an alternating current would therefore consist of a curve such as that shown in Fig. 1. We may speak of the complete cycle of changes as a complete *wave* of current; and of the two half-cycles as the positive and negative half-waves.

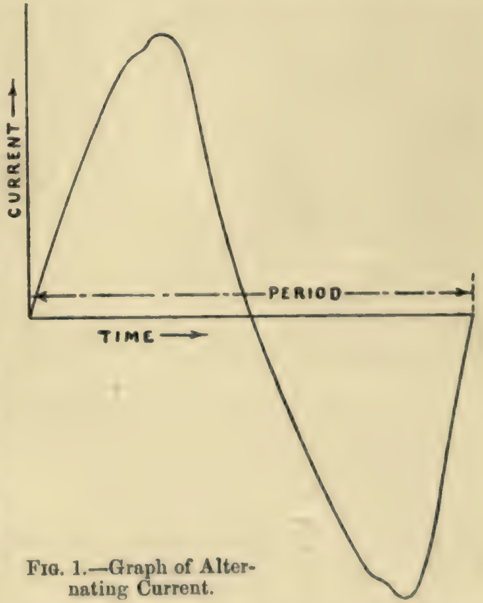

Fig. 1.—Graph of Alternating Current.

The time taken by the current to run through a complete cycle of changes is spoken of as the *period* of the alternating current; and the number of cycles per second is the *frequency* (sometimes also termed *periodicity*) of the current.

In the practical applications of alternating currents, we have to deal with currents whose graphs differ widely from one another. The shape of the graph is spoken of as the *wave-shape* or *wave-form* of the current, and is in many problems a matter of considerable importance. In a later chapter, we shall explain how a record of the wave-form or graph of an alternating current may be obtained experimentally.

Since an alternating current varies from instant to instant, it is obvious that in deciding on a system of measurement, we must define exactly what we mean by the *numerical value* of an alternating current.

Now in connection with the most important applications of alternating currents—electric lighting, electric power transmission and distribution, electric furnaces—the useful effect produced by the current at any instant depends on the value of the *square* of the current at that instant. Hence, in order that an alternating current may be equivalent (as regards its effect) to a given continuous current, the mean value of the square of the alternating current over a period * must be equal to the square of the continuous current. Otherwise, the two currents are equivalent if the square root of the mean square value of the alternating current is equal to the continuous current. This root-mean-square value is generally termed the r.m.s. value of the current, and when we speak of the numerical value of an alternating current, we mean its r.m.s. value.

In some few cases, we have to consider the *arithmetic mean value* of a current; in others, we have to take into account its *maximum* value.

The relations connecting the r.m.s., the arithmetic mean, and the maximum values of an alternating current depend on its wave-form. It is usual to consider the ratios of the r.m.s. value to the other two, and in this connection two terms introduced by Dr. Fleming, and known as the *form factor* and *amplitude factor*, are convenient.

$$\text{The } \textit{form factor} \text{ of a given wave} = \frac{\text{r.m.s. value}}{\text{mean value}}$$

$$\text{The } \textit{amplitude factor} \quad ,, \quad ,, \quad ,, \quad = \frac{\text{r.m.s. value}}{\text{maximum value}}$$

Hitherto, we have spoken of alternating *currents;* but all that has been said applies equally to alternating p.d.'s or e.m.f.'s.

---

* Or half-period, since the numerical values during a negative half-wave are identical with those during a positive half-wave.

§ 2. Simple Sine Waves. Vector Diagrams

In some cases (as, *e.g.*, in incandescent lighting, or electric furnace work) the results obtained by using an alternating current of given r.m.s. value are entirely independent of its *wave-form;* so long as the r.m.s. value is unaltered, we may use a current of any wave-form we please without in any way affecting the results. In other cases, however, the effects produced will, for a given r.m.s. value, depend, to a greater or less extent, on the *wave-form* (as, *e.g.*, in connection with motors, transformers, arc lights, and *especially* in cases where *capacity* is present, as in concentric cables).

Now in order to simplify the theoretical treatment of the subject as much as possible, it is an obvious advantage to select a wave-form which shall lend itself readily to mathematical treatment; and of all possible wave-forms, the simplest and easiest to deal with is that known as a *simple harmonic* or *sine wave.* The equation to such a wave is—

$$y = \mathrm{Y} \sin pt \, ^*$$

and its graph is shown in Fig. 2.

In the classical theory of alternating currents, it is usual to assume that the waves dealt with are simple sine waves. Since there are, as mentioned above, cases in which the results obtained are independent of the wave-form, the assumption of the simplest wave-form in such cases is perfectly justifiable, and leads to correct results while considerably simplifying the treatment. Unfortunately, there are other cases in which the assumption of sine waves is no longer admissible, and leads to results more or less erroneous, and at times entirely misleading.

For this reason, the use of sine waves has been severely criticized by some writers. But there is nothing else that can be usefully substituted for them, and since in a large number of problems they yield results sufficiently accurate for practical purposes, their use is certainly justifiable. It must be remembered, however, that results deduced on the sine-wave hypothesis must be used with due caution when applied to certain problems.

There is, however, a still further justification for the assumption of sine waves in alternating-current theory, and this is due to the gradual recognition of the fact that for most practical purposes also the sine wave is a *desideratum.* Especially is this the case in connection with long-distance power transmission. Successful attempts have recently been made to construct alternators capable of giving pure sine waves of e.m.f.; and although many of the alternators at present in use give somewhat irregular wave-forms, it seems highly probable that in the practice of the future the sine wave will play as prominent a part as it does in the theory of the present.

---

* The angle pt, where t is the time, is expressed in *radians.*

A sine wave may be represented either analytically, by means of an equation of the form—

$$y = \mathrm{Y} \sin pt,$$

or graphically as in Fig. 2, t being plotted horizontally and y vertically. There is still another mode of representation, however, very generally used, and known as the *vector* or *clock-face* diagram method. In this, we suppose a straight line of constant length OP (Fig. 3) = Y to revolve with constant angular velocity p (radians per sec.) about one of its extremities O as centre. Such a rotating line of constant length may be termed a *rotating vector*. Suppose that at the time $t = 0$ the line is in the horizontal position (shown dotted in Fig. 3). After t secs., it will have swept out an angle pt, so that its projection on the vertical will be $\mathrm{OP} \cos\left(\frac{\pi}{2} - pt\right) = \mathrm{Y} \sin pt = y$. As, therefore, the line OP rotates, its projection on the vertical at any instant gives us the magnitude and sign of the alternating current at that instant. The extremity of this projection moves with a simple harmonic motion, and the projection itself is spoken of as an *alternating vector.*

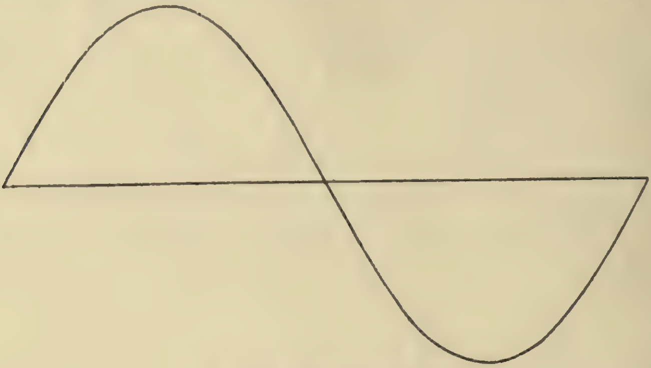

FIG. 2.—Sine Wave.

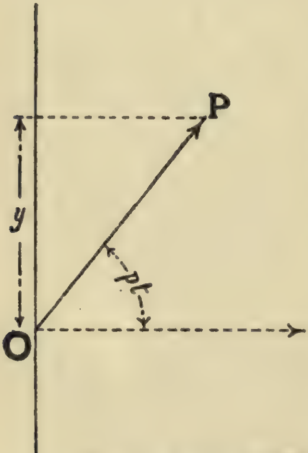

FIG. 3.—Vector, or Clock-face Diagram.

The maximum value Y of the sine wave is termed its *amplitude*. A complete cycle of changes clearly corresponds to a complete revolution of OP, or to the time T taken by OP to sweep out an angle of 2π radians. Since the angular velocity is p, we have $p\mathrm{T} = 2\pi$, or $p = \frac{2\pi}{\mathrm{T}}$. We have already (§ 1) termed T the *period*. Now if the *frequency* or number of complete cycles per second be denoted by n, we have $\mathrm{T} = \frac{1}{n}$, so that $p = 2\pi n$; i.e. *the angular velocity of the rotating vector in the vector diagram is equal to 2π times the frequency.*

If the equation of the sine wave is given in the form—

$$y_1 = \mathrm{Y}_1 \sin (pt + \theta)$$

then y_1 may as before be represented by the projection on the vertical of a rotating vector of length Y_1, the only difference now being that at the time $t = 0$ this rotating vector is not horizontal, but makes an angle $+\theta$ with the horizontal.

Since the rotating vectors from which y and y_1 are derived have the same angular velocity p, the angle between them must remain constant and equal to θ, which is the angle they make with each other at the time $t = 0$. This constant angle θ is the *phase difference* between the two sine waves y and y_1. The wave y is said to *lag* behind y_1, and y_1 is said to *lead* with respect to y, the angle θ being

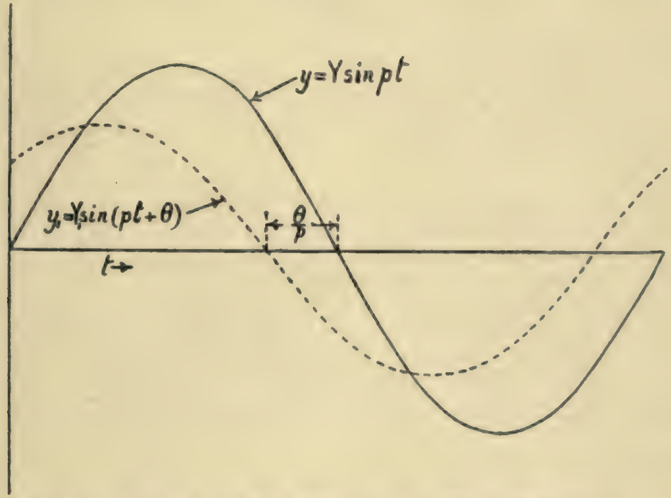

FIG. 4.—Graphs of Two Sine Waves, with a Phase Difference θ.

referred to as the *angle of lag* and *lead* respectively in the two cases.

The ordinary graphs of the two waves y and y_1 are shown in Fig. 4, the full-line curve representing y and the dotted one y_1.

It is, of course, obvious that two sine waves whose equations are $y_3 = Y_3 \sin(pt + \theta_3)$ and $y_4 = Y_4 \sin(pt + \theta_4)$ have a phase difference $\theta_3 - \theta_4$; this being the fixed angle between the two corresponding rotating vectors (of lengths Y_3 and Y_4) in the clock diagram.

Instead of assuming an axis fixed in space, and considering the projection on it of a rotating vector, the direction of rotation of which is counter-clockwise, we may suppose the vector to remain fixed, and consider its projection on a *rotating axis*, the angular velocity being *clockwise* and numerically equal to p. It is evident that both these methods yield identical results.

What we are concerned with in practice when dealing with a number of sine waves of the same frequency is to know (1) the r.m.s. value of each simple sine wave; and (2) their phase differences. Hence in constructing a vector diagram it is generally convenient to make the length of each vector correspond to the *r.m.s. value* of the sine wave, instead of, as in the original mode of representation, to its maximum value.

§ 3. Relations connecting Amplitude, r.m.s., and Arithmetic Mean Values of Simple Sine Wave

In order to enable us to pass from the maximum value or amplitude of a sine wave to its r.m.s. value, we have to find the relation connecting these two quantities. The r.m.s. value is the square root of the mean value of $y^2 = Y^2 \sin^2 pt$ over a period. Now the mean value of $y^2 = Y^2 \sin^2 pt$ may be found by determining the area of the graph of this function over a period, and dividing this area by the length of the base-line, *i.e.* by T. The area is, however, represented by the definite integral $\int_0^T Y^2 \sin^2 pt \,.\, dt$, the value of which may be obtained as follows :—

$$\begin{aligned}\int_0^T Y^2 \sin^2 pt \,.\, dt &= \tfrac{1}{2}Y^2\int_0^T (1 - \cos 2pt)dt \\ &= \tfrac{1}{2}Y^2\left\{\int_0^T dt - \int_0^T \cos 2pt \,.\, dt\right\} \\ &= \tfrac{1}{2}Y^2\left\{\left[t\right]_0^T - \left[\tfrac{1}{2}\sin 2pt\right]_0^T\right\} \\ &= \tfrac{1}{2}Y^2T\end{aligned}$$

since $\sin 2p\mathrm{T} = \sin 4\pi = 0$. Dividing the area by T, we find that *the mean value of the square of a sine function over a period equals* $\frac{1}{2}\mathrm{Y}^2$, i.e. *equals half the square of the amplitude.* Hence the r.m.s. value is $\frac{\mathrm{Y}}{\sqrt{2}} = 0{\cdot}707$ times the amplitude. The *amplitude factor* (§ 1) for a simple sine wave is thus $\frac{1}{\sqrt{2}} = 0{\cdot}707$.

By a similar method, we can find the arithmetic mean value and the form factor of a sine wave. To find the arithmetic mean value, we may determine the area of a half-wave, *i.e.* the value of the integral $\int_0^{\frac{\mathrm{T}}{2}} \mathrm{Y} \sin pt \,.\, dt$, and divide this by $\frac{\mathrm{T}}{2}$. Now—

$$\int_0^{\frac{\mathrm{T}}{2}} \mathrm{Y} \sin pt \,.\, dt = \mathrm{Y}\Big[-\frac{1}{p}\cos pt\Big]_0^{\frac{\mathrm{T}}{2}} = \frac{2}{p}\mathrm{Y}$$

Hence the mean value is $\frac{4}{p\mathrm{T}}\mathrm{Y}$, or, since $p\mathrm{T} = 2\pi$, the mean value is $\frac{2}{\pi}\mathrm{Y}$, and the *form factor* (§ 1) of a sine wave is thus—

$$\frac{\frac{\mathrm{Y}}{\sqrt{2}}}{\frac{2\mathrm{Y}}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$$

§ 4. Impressed and Induced e.m.f.'s. Self-inductance

In order to maintain an alternating current in a circuit, it becomes necessary to introduce into the circuit a source of alternating e.m.f. The e.m.f. provided by such a source is spoken of as the *impressed e.m.f.*, in order to distinguish it from other e.m.f.'s which are generally called into play as soon as the alternating current begins to flow.

A current flowing in a circuit gives rise to a definite number of lines of magnetic induction, which become linked with the circuit. By the great principle discovered by Faraday in 1831, any change in the magnetic flux linked with a circuit is accompanied by the induction of an e.m.f. around the circuit; *the direction of the induced e.m.f. being always such as to oppose the change which gives rise to it* (Lenz's law).

Now, since an alternating current is changing from instant to instant, the magnetic flux to which it gives rise will similarly change, and will thus give rise to an *induced e.m.f.*, which becomes superposed on the *impressed e.m.f.* The resultant e.m.f., which is instrumental in maintaining the current through the resistance of the circuit, is at any instant equal to the algebraical sum of the impressed and induced e.m.f.'s.

It is therefore obvious that, for given values of the resistance and the impressed e.m.f., the magnitude of the current will be determined by the *magnetic flux* to which the current gives rise. In other words, *resistance* is, in the case of an alternating current circuit, not the only factor determining the value of the current, which also depends on the *total flux linked with the circuit when conveying a unit current.*

This latter quantity—the flux linked with the circuit when conveying unit current—is defined to be the *self-inductance*,* or *inductance* simply, of the circuit.

§ 5. Fundamental Equation for a Circuit in which the Current is Variable

Let us suppose, in the first instance, that all the quantities are expressed in C.G.S. units. If L = self-inductance of circuit, and i = value of current at time t, then the total flux linked with the circuit at time t is $\mathrm{L}i$. The induced e.m.f. is numerically equal to the rate of change of the magnetic flux, but since it always opposes the changes which give rise to it, it must be taken with a negative sign. Thus the induced e.m.f. at time t is given by—

$$-\frac{d}{dt}(\mathrm{L}i)$$

Now, although $\mathrm{L} = \frac{\text{magnetic flux}}{\text{current}}$ is constant for coreless coils, it is no longer so in the case of coils provided with iron cores. It may, however, be assumed to be approximately constant even in this latter case, so long as the magnetization is well below the knee of the B − H curve. Assuming, then, L to be constant, we have for the induced e.m.f. the value—

$$-\mathrm{L}\frac{di}{dt}$$

If e = impressed e.m.f. at time t, then the resultant e.m.f. at the same instant is given by—

$$e - \mathrm{L}\frac{di}{dt}$$

---

* The older term is *self-induction*, or *coefficient of self-induction.*

and it is this resultant e.m.f. which maintains the current against the resistance r of the circuit. Hence—

$$i = \frac{e - L\frac{di}{dt}}{r}$$

This equation may be written in the form—

$$e = ri + L\frac{di}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the physical interpretation of which is, that the impressed e.m.f. at any given instant may be regarded as employed in two ways: (1) in balancing the drop of potential ri due to the resistance of the circuit; and (2) in balancing the opposing e.m.f. of self-inductance.

Equation (1) is of great importance, and we shall frequently have to make use of it.

Let us now suppose that the practical units—the volt, the ohm, and the ampere—are substituted for the C.G.S. units, and let us choose the practical unit of self-inductance so that equation (1) will also hold for the practical units, without being complicated by the introduction of any constants. Let the practical unit of self-inductance contain 10^x C.G.S. units. In order to avoid unnecessary constants, we must choose x so that—

$$L \times 10^x \times 10^{-1}\frac{di}{dt} = 10^8 . L\frac{di}{dt}$$

or $x = 9$, so that the corresponding practical unit of self-inductance, termed the *henry*, is equal to 10^9 C.G.S. units.

The corresponding practical unit of magnetic flux is equal to 10^8 C.G.S. units. [It has been proposed to call this unit of magnetic flux a *weber*.]

In some cases, the self-inductance of a circuit is so small as to be practically negligible in comparison with its resistance. As an example of such a circuit, we may consider an insulated wire which has been doubled on itself and then wound into a coil—as in the usual method of winding standard resistance coils. Such a circuit is termed a *non-inductive* one. An incandescent lamp—whose resistance is very high and self-inductance very low—is another example of a non-inductive circuit.

§ 6. Sine Waves in Circuits containing Resistance, Self-inductance, and Capacity

Consider a non-inductive circuit of resistance r in which there is an alternating current represented by—

$$i = I \sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since L is negligible, the impressed e.m.f. is, by equation (1), equal to—

$$e = ri = r\text{I} \sin pt$$

The impressed e.m.f. is thus *in phase* with the current, and the vectors corresponding to the impressed e.m.f. and the current in a vector diagram lie along the same straight line, as shown in Fig. 5. Further, considering r.m.s. values of e.m.f. and current, we see that—

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

Let us next consider another extreme case—that, namely, in which the self-inductance of a circuit is so high that its resistance may be neglected in comparison. An example of such a circuit is furnished by a coil consisting of a very large number of turns of fairly thick wire. Let L stand for the inductance of the circuit, the current being as before given by (2). Then since r is by supposition negligible, (1) gives—

$$e = \text{L}\frac{di}{dt} = \text{L}\frac{d}{dt}(\text{I} \sin pt)$$
$$= p\text{LI} \cos pt$$

Let OP = I in Fig. 6 represent the current vector. Draw OR at right angles to OP, making OR = pLI. At any instant t, the projection of OP on the vertical, gives the value of the current, while at the same instant the projection of OR, which is given by OR cos $pt = p\text{LI} \cos pt = e$, represents the value of the impressed e.m.f. Thus OR is the e.m.f. vector, and we see that there is a phase difference of $\frac{\pi}{2}$ or 90° between the impressed e.m.f. and the current, the current *lagging behind the impressed e.m.f.* When two sine waves differ in phase by $\frac{\pi}{2}$, they are said to be *in quadrature* with each

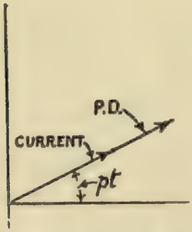

Fig. 5.—Vector Diagram for Pure Resistance.

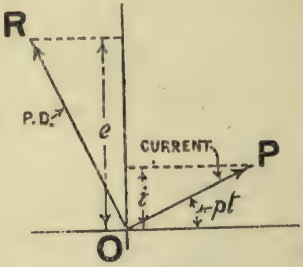

Fig. 6.—Vector Diagram for Pure Inductance.

other, and we see that in the case considered the impressed e.m.f. and the current are in quadrature with each other. Further, again considering r.m.s. values, we see that—

$$\text{current} = \frac{\text{e.m.f.}}{p\text{L}}$$

The quantity $p\text{L}$ is termed the *reactance* of the circuit.

Consider a condenser of capacity C, across whose terminals there is an impressed p.d. given by—

$$v = \text{V} \sin pt$$

Let q = instantaneous quantity or charge in the condenser. Then—

$$q = \text{C}v = \text{CV} \sin pt$$

If i = instantaneous current, then clearly $i = \frac{dq}{dt}$, or—

$$i = p\text{CV} \cos pt$$

In a vector diagram, therefore, the current would be represented by a vector which is $\frac{\pi}{2}$ or 90° *ahead* of the p.d. vector, as in Fig. 7. The p.d. and current are *in quadrature* with each other, and the p.d. *lags behind the current.*

The r.m.s. values of the p.d. and current are connected by the relation—

$$\text{current} = \text{p.d.} \times \text{C}p = \frac{\text{p.d.}}{\frac{1}{\text{C}p}}$$

and $\frac{1}{\text{C}p}$ is spoken of as the *reactance* of the condenser.

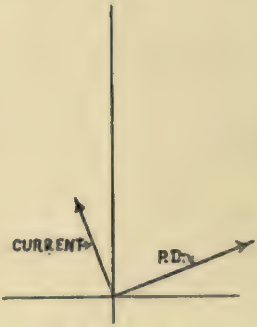

Fig. 7.—Vector Diagram for Pure Capacity.

Let the resistance of a circuit be r, and its self-inductance L. In order to maintain a current—

$$i = \text{I} \sin pt$$

in the circuit, we must, by equation (1), provide an impressed e.m.f. of amount—

$$\begin{aligned} e &= ri + \text{L}\frac{di}{dt} \\ &= r\text{I} \sin pt + p\text{LI} \cos pt \end{aligned}$$

The first component of the impressed e.m.f., viz. $r\text{I} \sin pt$, may be represented, in a vector diagram, by the projection of a rotating vector OA, Fig. 8, of length rI, on the vertical axis; while the second component, pLI cos pt, may be represented by the projection on the same axis of a rotating vector $\text{OB} = \text{L}p\text{I}$. The impressed e.m.f. at any instant is the algebraical sum of these two projections; but this is clearly the same as the projection of the diagonal OE of the rectangle constructed on OA and OB as sides. Hence OE will be the rotating vector corresponding to the impressed e.m.f. But since—

$$\text{OE} = \sqrt{\text{OA}^2 + \text{OB}^2} = \sqrt{r^2\text{I}^2 + p^2\text{L}^2\text{I}^2} = \text{I}\sqrt{r^2 + p^2\text{L}^2}$$

we see that—

$$\text{current} = \frac{\text{e.m.f.}}{\sqrt{r^2 + p^2\text{L}^2}}$$ [*]

and that the current *lags* behind the e.m.f. by an angle AOE such that—

$$\tan \text{AOE} = \frac{\text{L}p}{r}$$

The quantity $\sqrt{r^2 + p^2\text{L}^2}$ is termed the *impedance* of the circuit, and the angle $\text{AOE} = \tan^{-1} \frac{\text{L}p}{r}$ is the *angle of lag* of the current behind the impressed e.m.f.

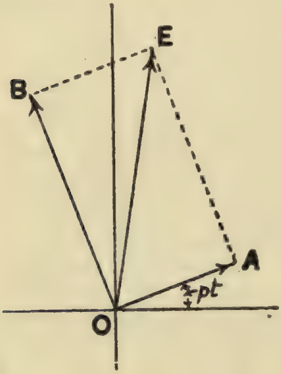

FIG. 8.—Vector Diagram for Inductive Resistance.

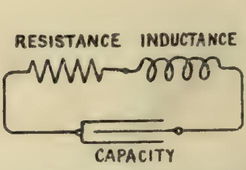

FIG. 9.—Arrangement of Resistance, Inductance, and Capacity in Series.

We may now consider a circuit containing resistance, inductance, and capacity. Such a circuit is diagrammatically represented in Fig. 9.[*] The impressed e.m.f. may be regarded as made up of the following three components, represented by the vectors OA, OB, and OC in Fig. 10:—

---

[*] This equation holds good whether maximum or r.m.s. values be considered.

[†] We shall, in the diagrammatic representation of a circuit, use a coiled line to indicate an inductive resistance, and a zig-zag line to indicate non-inductive resistance.

(1) The component $OA = rI$, in phase with the current.
(2) The component $OB = LpI$, 90° in advance of the current.
(3) The component $OC = \frac{1}{Cp}I$, 90° behind the current.

The lengths of the vectors representing the amplitudes of the components, their projections at any instant correspond to the instantaneous values of the components, and the algebraical sum of the projections gives the instantaneous value of the impressed e.m.f.

Now the vectors OB and OC lying in the same straight line and being oppositely directed, the algebraical sum of their projections is equal to the projection of a single vector OD of length equal to $OB - OC = I\left(Lp - \frac{1}{Cp}\right)$. Finally, the sum of the projections of OA and OD is equal to the projection of the single vector OE, which is the diagonal of the rectangle constructed on OA and OD as sides, and which represents the impressed e.m.f. Now—

$$OE = \sqrt{\overline{OA}^2 + \overline{OD}^2} = \sqrt{r^2I^2 + \left(Lp - \frac{1}{Cp}\right)^2 I^2} = I\sqrt{r^2 + \left(Lp - \frac{1}{Cp}\right)^2}$$

Hence we see that—

$$\text{current} = \frac{\text{impressed e.m.f.}}{\sqrt{r^2 + \left(Lp - \frac{1}{Cp}\right)^2}}$$

and the current lags behind the impressed e.m.f. by an angle $AOE = \theta$ such that—

$$\tan\theta = \frac{Lp - \frac{1}{Cp}}{r}$$

The quantity $Lp - \frac{1}{Cp}$ is the *reactance* of the circuit, while $\sqrt{r^2 + \left(Lp - \frac{1}{Cp}\right)^2}$ is termed the *impedance* of the circuit.

It is easy to see that the impedance, resistance, and reactance of a circuit are capable of being represented by the three sides of a right-angled triangle, one of whose angles corresponds to $\theta = \tan^{-1}\frac{Lp - \frac{1}{Cp}}{r}$ (the triangle OAE, in Fig. 10, may, to a suitable scale, be taken to represent these three quantities).

The diagram of Fig. 10 has been drawn on the supposition that the reactance Lp due to inductance exceeds the reactance $\frac{1}{Cp}$ due to capacity. In this case, θ is positive, *i.e.* the current lags behind the impressed e.m.f.; θ is frequently spoken of as the *angle of lag*. It may, however, happen that $\frac{1}{Cp} > Lp$, θ then becoming negative, *i.e.* the current *leading* instead of lagging.

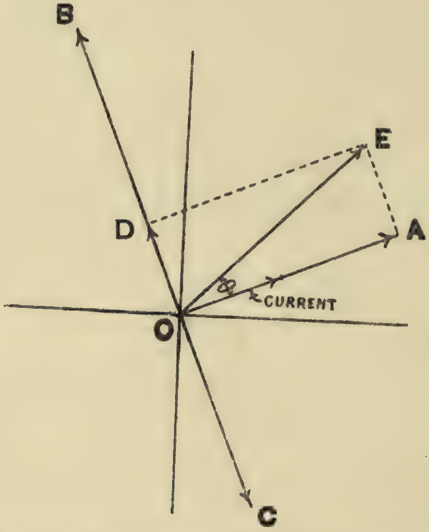

FIG. 10.—Vector Diagram for Inductive Resistance in Series with Capacity.

We may, further, consider the special case in which $Lp = \frac{1}{Cp}$; θ now vanishes, *i.e.* the current comes into phase with the impressed e.m.f., and the impedance becomes simply equal to the resistance. The current is, therefore, the same as that which would be obtained with the same impressed e.m.f. in a non-inductive circuit of resistance r. Under these very special circumstances, the circuit is said to exhibit *electrical resonance* or *syntony*; the effect due to self-inductance being completely neutralized by that due to capacity.

When electrical resonance occurs in a highly inductive circuit, *i.e.* one whose self-inductance is very large in comparison with its resistance, we have the very remarkable result that the p.d.'s across the inductive resistance and the condenser may very largely exceed the impressed e.m.f. This is at once evident from the vector diagram of Fig. 10, since in this case each of the vectors OB and OC becomes very large in comparison with OA. The impressed e.m.f. and the p.d.'s across the two portions of the circuit are given by—

$$\begin{aligned}\text{impressed e.m.f.} &= \text{current} \times \text{resistance}\\ \text{p.d. across inductive resistance} &= \text{current} \times \sqrt{r^2 + p^2L^2}\\ \text{p.d. across condenser terminals} &= \frac{\text{current}}{Cp}\end{aligned}$$

§ 7. Power in Alternating Current Circuit

Let the current in a circuit be given by—

$$i = \mathrm{I} \sin pt$$

and the impressed e.m.f. by—

$$e = \mathrm{E} \sin (pt + \theta)$$

The power w at any instant is—

$$\begin{aligned}w = ei &= \mathrm{EI} \,.\, \sin (pt + \theta) \,.\, \sin pt\\ &= \tfrac{1}{2}\mathrm{EI} \,.\, 2 \sin (pt + \theta) \,.\, \sin pt\\ &= \tfrac{1}{2}\mathrm{EI} \{\cos \theta - \cos (2pt + \theta)\}\\ &= \tfrac{1}{2}\mathrm{EI} \cos \theta - \tfrac{1}{2}\mathrm{EI} \,.\, \cos (2pt + \theta)\end{aligned}$$

We therefore see that the expression for the instantaneous power consists of two terms, one of which, $\frac{1}{2}\mathrm{EI} \cos \theta$, is a constant, while the other is a cosine (or sine) wave of frequency equal to *double* the frequency of the e.m.f. and current waves.

Now when we speak, without in any way qualifying the expression, of the *power* in an alternating current circuit, we understand by this term the *mean value* of the power over a complete period. The mean value of the second term in the expression for the *instantaneous power* w is, however, zero over any whole number of periods. Thus the mean value of the power becomes equal to the first or constant term $\frac{1}{2}\mathrm{EI} \cos \theta$. This may be written in the form $\frac{\mathrm{E}}{\sqrt{2}} \cdot \frac{\mathrm{I}}{\sqrt{2}} \cdot \cos \theta$, and since $\frac{\mathrm{E}}{\sqrt{2}}$ = r.m.s. value of e.m.f., and $\frac{\mathrm{I}}{\sqrt{2}}$ = r.m.s. value of current, we see that

$$\text{mean power} = \text{e.m.f.} \times \text{current} \times \cos \theta$$

r.m.s. values of e.m.f. and current being understood. The power, therefore, is not simply equal to the product of the e.m.f. and current, but is equal to this product multiplied by $\cos \theta$. The multiplier which converts *volt-amperes* or *apparent power* into watts or *true power* is termed the *power factor* of the circuit. In the case considered, the power-factor is cos θ, but the term "power factor" is used generally, even in connection with circuits in which the waves are no longer of the simple sine form, to express the ratio $\frac{\text{true power}}{\text{volt-amperes}}$.

The expression for the mean power may be written in the form $\frac{E}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \cos \theta$. Now I cos θ is the projection of the current vector on the impressed e.m.f. vector, and since the power depends on the magnitude of this projection, we speak of $\frac{I}{\sqrt{2}} \cos \theta$ (*i.e.* r.m.s. current × cos θ) as the *power component* or *load component* of the current, while $\frac{I}{\sqrt{2}} \sin \theta$ is termed the *idle* or *wattless* component of the current. It is frequently convenient to regard the current in a circuit as split up into these two components, as shown in Fig. 11, where the vectors are taken to represent r.m.s. values of the current.

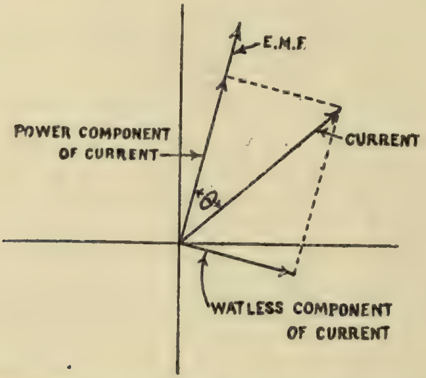

Fig. 11.—Decomposition of Current into Power and Wattless Components.

CHAPTER II

§ 8. Series arrangement of impedances—§ 9. Parallel arrangement of impedances—§ 10. Numerical example—§ 11. Mutual inductance and its effects—§ 12. Electromagnetic repulsion motor—§ 13. Skin effect.

§ 8. Series Arrangement of Impedances

Let two impedances, AB and BC, be connected in series as shown in Fig. 12 (*a*), and let it be required to find their combined impedance.

An impedance is not completely specified by its numerical value. For with the same numerical value of the impedance we may have widely differing values of the ratio $\frac{\text{reactance}}{\text{resistance}}$, which determines the phase difference between e.m.f. and current. The impedance is, however, uniquely determined if in addition to the ratio $\frac{\text{p.d.}}{\text{current}}$ we know the angle of phase difference θ between p.d. and current.

Since the two impedances in Fig. 12 (*a*) are arranged in series with each other, it follows that the current must have the same value in each of them at every instant. For this reason, we choose the current vector as our vector of reference in the vector diagram, and

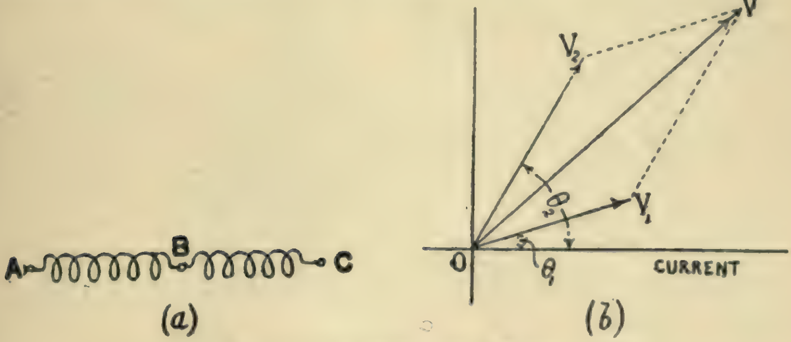

Fig. 12.—Two Impedances in Series.

we may conveniently lay off this current vector along the horizontal axis, as in Fig. 12 (*b*).

Let θ_1, θ_2 be the phase differences corresponding to the two given impedances, and let Z_1, Z_2 be their numerical values.

In the vector diagram, assume the current to have a value of *unity*. Then the p.d. across AB is given by a vector OV_1, Fig. 12 (*b*), of length Z_1, making an angle θ_1 with the current vector. Similarly, the p.d. across BC is given by a vector OV_2 of length Z_2, making an angle θ_2 with the current vector. From this it follows that the p.d. across AC is given by the diagonal OV of the parallelogram constructed on OV_1 and OV_2 as sides, and the angle made by this diagonal with the current vector is the angle by which the current lags behind the p.d. across AC.

Since, however, we have assumed the current to be unity, it follows that the length of OV will correspond to the numerical value of the total impedance, and the angle which OV makes with the current vector will correspond to the angle of phase difference for the total impedance. Thus the required impedance is completely determined.

It is now evident that the rule for the composition of two impedances in series with each other is identical with the rule for the composition of two forces acting at a point—we have simply to apply the *parallelogram law*.

The same rule may be extended to any number of impedances connected in series, and by applying the *polygon law* we can easily find, by a purely graphical method, both the magnitude of, and the angle of phase difference corresponding to, the total impedance.

Although a purely graphical method enables us to deal with this problem, yet where accuracy is required it may be preferable to have recourse to calculation. The most convenient method is then as follows.

Let Z_1, Z_2, Z_3 . . . be the given impedances, and θ_1, θ_2, θ_3 . . . the angles of phase difference, or the *phase angles*,* as we may briefly term them, between the current and the p.d. across each impedance. Resolve each impedance into two components, one of which lies along the horizontal, and the other along the vertical axis. Find the sum of all the horizontal and the sum of all the vertical components, square the two sums, add them, and extract the square root; this gives the total impedance. Thus—

total impedance =

$$\sqrt{(Z_1\cos\theta_1 + Z_2\cos\theta_2 + Z_3\cos\theta_3 + \ldots)^2 + (Z_1\sin\theta_1 + Z_2\sin\theta_2 + Z_3\sin\theta_3 + \ldots)^2}$$

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* The phase angle is to be reckoned *positive* if the current *lags* behind the p.d., and *negative* if it *leads*.

and the phase angle of the total impedance is given by—

$$\tan^{-1}\frac{Z_1 \sin \theta_1 + Z_2 \sin \theta_2 + Z_3 \sin \theta_3 + \ldots}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2 + Z_3 \cos \theta_3 + \ldots}$$

If, instead of being given the impedances Z_1, Z_2, etc., and their phase angles θ_1, θ_2 . . . , we are given the values of the resistances r_1, r_2, r_3 . . . , and of the reactances V_1, V_2, V_3 . . . , then clearly—

$$\text{total impedance} = \sqrt{(r_1 + r_2 + r_3 + \ldots)^2 + (V_1 + V_2 + V_3 + \ldots)^2}$$

With a series arrangement of impedances, it is, therefore, permissible to add the resistances arithmetically in order to obtain the total resistance; and to add the reactances algebraically in order to obtain the total reactance. But it is not permissible to add the impedances arithmetically; this latter addition must be carried out *vectorially*, *i.e.* in accordance with the polygon law.

As a result, we find that in the case of a series circuit such as the one shown in Fig. 13, if V_1, V_2, V_3 denote the r.m.s. values of the p.d.'s across AB, BC, and CD respectively, and V the r.m.s. value of the p.d. across the entire circuit AD, then in general $V_1 + V_2 + V_3 > V$, since the sum of the sides of any open polygon is in general greater than the closing side of the polygon. In the special case in which $\theta_1 = \theta_2 = \theta_3$, we have $V_1 + V_2 + V_3 = V$, the polygon in this case degenerating into a straight line.

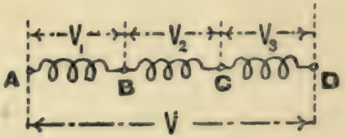

FIG. 13.—Series Arrangement of Impedances.

It may be well to point out that although the sum of the r.m.s. values of the p.d.'s in the case considered is in general different from the r.m.s. value of the total p.d., yet at every instant the sum of the *instantaneous values* of the p.d.'s must necessarily equal the *instantaneous* value of the total p.d. (the sum of the projections of the sides of an open polygon being always equal to the projection of the closing side of the polygon).

§ 9. Parallel Arrangement of Impedances.

Let a number of impedances, Z_1, Z_2, Z_3 . . . , having phase angles θ_1, θ_2, θ_3 . . . , be connected in parallel between two points, and let it be required to find their joint or parallel impedance.

The quantity which is common to all the branch circuits is the p.d. It is, therefore, suggested to take the p.d. vector as the vector of reference in our vector diagram; let this p.d. vector be laid off horizontally as in Fig. 14.

The value of the joint impedance being the same for all values of the p.d., we may, for the sake of convenience, assume the p.d. to have a value of *unity*. Since—

$$\text{current} = \frac{\text{p.d.}}{\text{impedance}} = \text{p.d.} \times \frac{1}{\text{impedance}}$$

we see that, on the assumption of unit p.d., the currents in the various branches are given by the *reciprocals of the impedances.* The reciprocal of the impedance of a circuit is termed its *admittance.* Let A_1, A_2, A_3 . . . be the admittances corresponding to the various branches, so that $A_1 = \frac{1}{Z_1}$, $A_2 = \frac{1}{Z_2}$, $A_3 = \frac{1}{Z_3}$ . . .; then, with unit p.d. across the terminals, the currents in the various branches are equal to A_1, A_2, A_3. . . .

An admittance, like an impedance, is a directed or vector quantity, so that it is not completely determined unless in addition to its magnitude we are also given its phase angle.

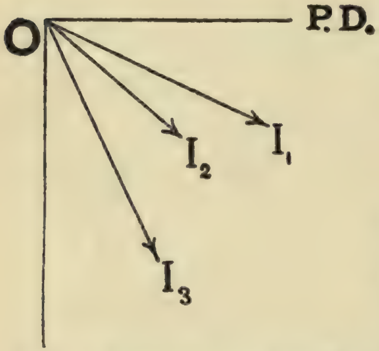

FIG. 14.—Vector Diagram of Currents in Branched Circuit.

The phase angles of the admittances A_1, A_2, A_3 . . . are equal in magnitude to θ_1, θ_2, θ_3 . . .

Let us now, in our vector diagram, lay off vectors OI_1, OI_2, OI_3 . . . (Fig. 14) of lengths A_1, A_2, A_3 . . . making angles θ_1, θ_2, θ_3 . . . with the p.d. vector. These vectors will be the current vectors for the various branches, and since the total current between the two points is *at any instant* equal to the sum of the instantaneous currents in the various branches, it is clear that the vector of total current is obtained by compounding the vectors according to the polygon law, as in Fig. 15, the closing side OI of the polygon giving the vector of total current. Since, however, the p.d. was assumed to have a value of unity, it follows that OI represents the *joint admittance* of the various branches, and its phase angle is the angle which it makes with the p.d. vector.

The *joint impedance* is at once obtained by taking the reciprocal of the joint admittance.

With a parallel arrangement of impedances, therefore, the admittances of the various branches are compounded according to the same *rule as that which governs the composition of impedances in a series circuit.*

It will be noticed that the sum of the r.m.s. values of the branch currents is in general greater than the r.m.s. value of the total current; but may become equal to it in the special case in which all the phase angles θ_1, θ_2, θ_3 . . . are equal.

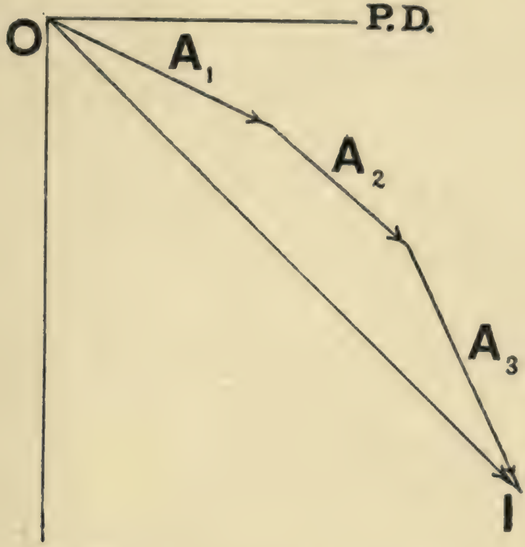

FIG. 15.—Vectorial Addition of Admittances.

In order to secure accuracy, we may as before *calculate* the joint impedance. The joint admittance is given by—

$$\sqrt{(A_1\cos\theta_1+A_2\cos\theta_2+A_3\cos\theta_3+\ldots)^2+(A_1\sin\theta_1+A_2\sin\theta_2+A_3\sin\theta_3+\ldots)^2}$$

and the reciprocal of this is the joint impedance. The phase angle θ corresponding to the joint impedance is such that—

$$\tan\theta=\frac{A_1\sin\theta_1+A_2\sin\theta_2+A_3\sin\theta_3+\ldots}{A_1\cos\theta_1+A_2\cos\theta_2+A_3\cos\theta_3+\ldots}$$

§ 10. Numerical Example.

In order to illustrate the application of the above principles, we shall work out in detail a fairly complicated numerical example.

In Fig. 16 is shown an arrangement of a number of resistances, inductive and non-inductive, and condensers. The magnitudes of these are marked in the diagram. An alternating p.d. of 500 volts, having a frequency of 50, is applied across the extreme terminals L and N. The problem is to find the currents in the various branches.

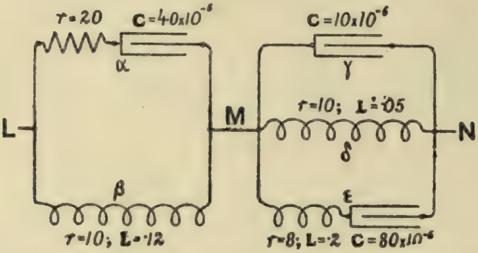

Fig. 16.—Circuit containing Resistances, Inductances, and Capacities.

We begin by finding the joint impedance of the branches α and β between L and M, and the joint impedance of the branches γ, δ, and ε between M and N.

The frequency being 50, we have $p = 2\pi \times 50 = 314$, say. Hence the capacity reactance in the branch α is $\dfrac{1}{p\mathrm{C}} = \dfrac{1}{314 . 40 . 10^{-6}}$ $= 80{\cdot}4$ ohms; and since the resistance is 20, the impedance of the branch α is $\sqrt{20^2 + 80{\cdot}4^2} = 82{\cdot}8$ ohms, and its phase angle is $\tan^{-1} \dfrac{-\,82{\cdot}8}{20} = -\,76^\circ\, 25'$. The *admittance* of this branch is $\dfrac{1}{82{\cdot}8}$ $= 0{\cdot}0121$, say.

Considering next the branch β, we have for its inductive reactance $p\mathrm{L} = 314 \times 0{\cdot}12 = 37{\cdot}7$. The impedance is $\sqrt{10^2 + 37{\cdot}7^2} = 39$, the admittance $\dfrac{1}{39} = 0{\cdot}0256$, and the phase angle $\tan^{-1} \dfrac{37{\cdot}7}{10}$ $= +\,75^\circ\, 8'$.

Applying the rule for the composition of parallel admittances, we have for the joint admittance of α and β—

$$\sqrt{(0{\cdot}0121 \cos 76^\circ 25' + 0{\cdot}0256 \cos 75^\circ 8')^2 + (-\,0{\cdot}0121 \sin 76^\circ 25' + 0{\cdot}0256 \sin 75^\circ 8')^2} = \sqrt{0{\cdot}00945^2 + 0{\cdot}013^2} = 0{\cdot}01605$$

Hence the joint impedance between L and M is 62·3 ohms, and the joint phase angle is $\tan^{-1} \dfrac{0{\cdot}013}{0{\cdot}00945} = +\,54^\circ$.

Taking next the branches γ, δ, and ε between M and N, and proceeding as before, we find—

		Impedance.	Admittance.	Phase Angle.
γ	...	319	0·00314	− 90°
δ	...	18·6	0·0537	+ 57° 30′
ε	...	24·35	0·0411	+ 70° 49′

Applying, as before, the rule for the composition of parallel admittances, we have—

$$0{\cdot}00314 \cos 90^\circ + 0{\cdot}0537 \cos 57^\circ\,30' + 0{\cdot}0411 \cos 70^\circ\,49' = 0{\cdot}0424$$

and—

$$-\,0{\cdot}00314 \sin 90^\circ + 0{\cdot}0537 \sin 57^\circ\,30' + 0{\cdot}0411 \sin 70^\circ\,49' = 0{\cdot}081$$

Thus the joint admittance is $\sqrt{0{\cdot}0424^2 + 0{\cdot}081^2} = 0{\cdot}0914$, the joint impedance 10·94, and the joint phase angle $\tan^{-1}\dfrac{0{\cdot}081}{0{\cdot}0424} = +\,62^\circ\,22'$.

Having obtained the joint impedance between L and M, and also that between M and N, and the corresponding joint phase angles, we next proceed to compound these two impedances, according to the law for the composition of series impedances. We thus find—

Joint resistance between L and M = 62·3 cos 54° = 36·62
„ „ „ M „ N = 10·94 cos 62° 22′ = 5·08

Total resistance between L and N = 41·70

Similarly—

Joint reactance between L and M = 62·3 sin 54° = 50·41
„ „ „ M „ N = 10·94 sin 62° 22′ = 9·69

Total reactance between L and N = 60·10

We now find for the total impedance between L and N the value $\sqrt{41{\cdot}7^2 + 60{\cdot}1^2} = 73{\cdot}15$ ohms, and for the total phase angle $\tan^{-1}\dfrac{60{\cdot}1}{41{\cdot}7} = +\,55^\circ\,15'$.

The remaining part of the problem presents but little difficulty. The total current is $\dfrac{500}{73{\cdot}15} = 6{\cdot}84$ amperes. The p.d. across LM is obtained by multiplying the total current by the impedance between L and M, and similarly the p.d. across MN is found by multiplying the total current by the impedance between M and N.

P.d. across LM = 6·84 × 62·3 = 426 volts
P.d. across MN = 6·84 × 10·94 = 75 volts

We may note, in passing, that the sum of the r.m.s. voltages across LM and MN, namely, 426 + 75 = 501, is very nearly equal to the voltage of 500 across LN. This close coincidence is purely accidental, and is due to the fact that the joint phase angle between L and M (54°) is not very different from that between M and N (62° 22′).

Having determined the voltages across LM and MN, we at once obtain the branch currents by multiplying the admittance of each branch by the voltage across it. We thus find—

Current in α = 426 × 0·0121 = 5·15
„ „ β = 426 × 0·0256 = 10·9
„ „ γ = 75 × 0·00314 = 0·235
„ „ δ = 75 × 0·0537 = 4·03
„ „ ϵ = 75 × 0·0411 = 3·08

It will be noticed that the current in the branch β is considerably greater than the total current.

§ 11. Mutual Inductance and its Effects

When two circuits are placed near each other, a current sent through one of them will in general produce an appreciable magnetic flux through the other, some of the lines produced by the first circuit becoming linked with the second. The circuits are said to possess *mutual inductance*, and their mutual inductance is defined to be the magnetic flux linked with either circuit when unit current flows round the other.*

By taking into account the principle known as Lenz's law, it is easy to arrive at the general nature of the effects produced by mutual inductance when the first circuit is supplied with an alternating current, and the second circuit, which contains no impressed e.m.f., is simply closed on itself. According to Lenz's law, the current induced in any circuit by a varying flux always opposes the changes of flux which give rise to it; and the circuit in which the current is induced is subject to mechanical forces tending to move the circuit so as to reduce the extent of the flux variations.

If both circuits are rigidly fixed, the currents induced in the second circuit when an alternating current is allowed to flow in the first circuit will tend to reduce the amplitude of the flux fluctuations. Hence one effect due to the presence of mutual inductance is equivalent to a *lowering of the self-inductance* of the first circuit. Again, the currents induced in the second circuit will produce heat in it, and since the power corresponding to this rate of heat generation comes from the first or inducing circuit, it follows that the total power supplied to the first circuit will be greater than the power taken by it at the same p.d. when the second circuit is absent. A second effect due to mutual inductance is therefore equivalent to an *increase in the resistance* of the first circuit.

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* The flux linked with the second circuit when unit current flows round the first is equal to the flux linked with the first circuit when unit current flows round the second. For a simple proof of this relation, the reader is referred to the author's "Principles of Alternate-Current Working."

Let us next suppose that the first circuit is fixed, and the second movable. If we assume the two circuits to be parallel to each other, the second circuit will be *repelled* by the first, since the result of such motion would be to reduce the amplitude of the flux fluctuations. Thus, a ring of copper or aluminium slipped over the pole of an alternate-current electromagnet will be projected upwards as soon as a sufficiently strong current is sent through the coil of the electromagnet, and if provided with suitable guides the ring may even be kept floating above the electromagnet, gravity being neutralized by electromagnetic repulsion. Striking experiments of this nature have been carried out by Prof. Elihu Thomson and Prof. Fleming.

If the second circuit is prevented from having a motion of translation, but is free to rotate about an axis, rotation will take place until the plane of the second circuit is parallel to the inducing field; for this is the position in which the flux fluctuations are completely suppressed. Since action and reaction are always equal and opposite, an equal and opposite couple will be experienced by the first circuit. A coil of wire, for example, conveying an alternating current will, when pivoted or suspended in front of a sheet of conducting material, experience a couple tending to turn it into a position at right angles to the conducting sheet.

§ 12. Electromagnetic Repulsion Motor

The principles just explained find a practical application in the construction of a certain class of alternate-current motor known as the *repulsion motor*. There are various types of this class of motor, but we shall here select for description one of the types originally devised by Elihu Thomson, and termed by him the *shaded-pole* motor.

In this form of motor, an alternating-current electromagnet is made to act on *two* secondary short-circuited circuits. Let us suppose that two rings of copper of the same size are suspended parallel to, and nearly in contact with, each other in the field of such a magnet. The currents induced in the rings by the alternating magnetic field will be nearly in phase with each other (the rings being nearly in the same region of the field), so that there will be attraction between them (since conductors conveying currents flowing in the same direction attract each other). Let us now suppose that the rings are displaced relatively to each other as in Fig. 17 (*a*), in a direction parallel to their planes. In Fig. 17 (*a*), the shaded portion represents the pole of the alternating-current electromagnet, which for the sake of simplicity is shown of circular shape. The attraction between the rings will tend to pull them into coincidence, and there will be a component of stress in a direction parallel to the planes of the rings. Next, suppose one of the rings to be replaced by a conducting sheet of metal, as in Fig. 17 (*b*)—in which the dotted circle shows the position of the pole—and let the ring be fixed while the conducting sheet is free to move. If the ring were removed, then by symmetry it is clear that the currents induced in the conducting sheet would (assuming the sheet to be of large extent, so as to project well beyond the polar edges) follow circular paths having their centres on the axis of the magnet. But with the ring in place, it is evident that the portion of the polar surface covered by the ring is subjected to a greater demagnetizing action than the portion lying outside the ring, and the result of this is to cause a shifting of the magnetic flux (whose distribution with the ring removed would be uniform) towards the right-hand "unshaded" crescent-shaped portion of the polar surface. But with this shifting of the flux the currents induced in the conducting sheet will also be shifted to the right, following paths similar to that roughly indicated by the chain-dotted line. Now, the portion of the conducting sheet forming the closed circuit indicated by the chain-dotted line and the ring will behave relatively to each other in the manner of the two rings of Fig. 17 (*a*), and since the

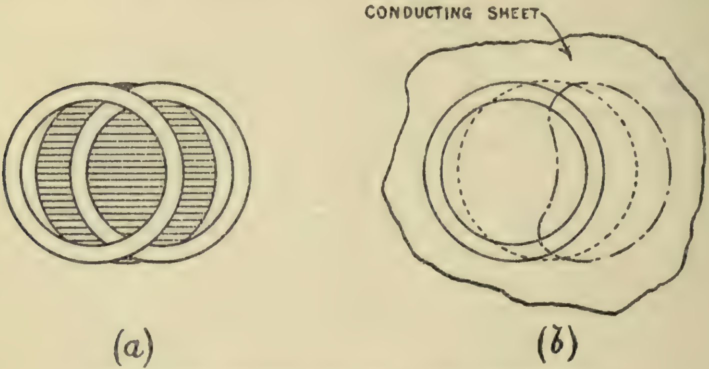

Fig. 17.—To illustrate Principle of Shaded-pole Motor.

ring is fixed, the sheet will move from right to left—*i.e.* from the "unshaded" to the "shaded" portion of the magnetic pole. Since, however, the conducting sheet is continuous, as it moves successive portions of it come into the position of the chain-dotted line, and so the pull is maintained during the motion.

A small motor constructed on this principle is shown in Fig. 18. The core of the magnet consists of a number of steel or soft-iron stampings of rectangular shape with an opening in the middle, and around one limb of the core is wound the exciting coil. The stampings are cut across the middle of the opposite limb of the core, so as to form a narrow air-gap. Around one half of one of the poles is placed the "shading" coil, which consists of a simple band of thick copper encircling half the pole-piece. A suitably mounted aluminium disc is free to rotate between the poles, and corresponds to the "conducting sheet" of Fig. 17 (*b*). From what has been said, it is evident that the direction of rotation will be that indicated by the arrow.*

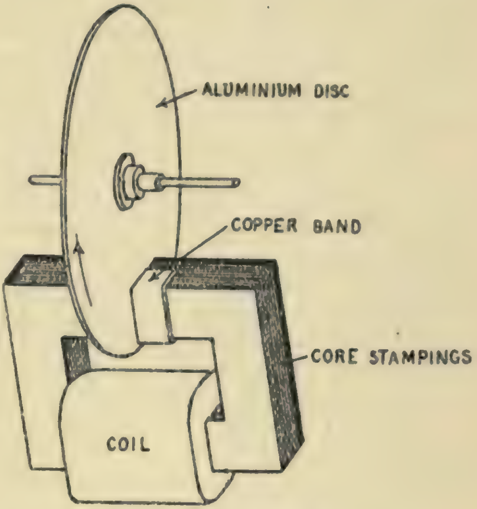

Fig. 18.—Shaded-pole Motor.

§ 13. Skin Effect

In the case of a conductor of large cross-section conveying a current, there will be an appreciable difference between the magnetic flux linked with the surface layers of the conductor and the flux linked with its central portion. For if we imagine the conductor split up into a large number of small parallel filaments, then the current along any filament at a considerable depth below the surface will be linked not only with the lines external to the conductor, but also with an appreciable number of lines in the substance of the conductor; whereas a surface filament is only linked with the external lines. As a consequence, the e.m.f. induced along a surface filament when an alternating current is sent through the conductor is less than that induced along a central filament, and since the various filaments may be regarded as different branches of a parallel circuit, it is obvious that, for a given cross-section of filament, the impedance will be greater, and hence the current less, in the case of a filament near the surface than in one near the centre of the conductor. Thus the current density will be greater in the surface layers, the current being *unevenly distributed* over the cross-section of the conductor. This uneven distribution of current is equivalent to a reduction of cross-section, or to an increase of resistance, of the conductor, and is frequently spoken of as the *skin effect.*

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* The form of motor sketched in Fig. 18 is used by Messrs. Brown, Boverie & Co., of Switzerland, in the relays employed by them in connection with their automatic maximum-current switches.

The skin effect with cylindrical *copper* conductors is, at ordinary frequencies, inappreciable until a diameter of about ½ inch is reached; it then increases very rapidly with the diameter. But with conductors constructed of magnetic materials, such as iron or steel, a very marked effect occurs even with conductors of small cross-section. This effect is of considerable practical interest in cases where an alternating-current system is used for working electric railways or tramways, and where the ordinary track rails are used as one of the conductors of the system. In the case of a steel conductor of large cross-section, the effective area of cross-section over which the current is distributed is confined to a comparatively thin surface layer—not exceeding ⅛ inch at all ordinary frequencies. Hence the loss occurring with a given current in such a conductor is very much greater for an alternating than for a continuous current.

CHAPTER III

§ 14. Polyphase currents—§ 15. Rotating field produced by polyphase currents—§ 16. Connections of polyphase systems—§ 17. Comparison of single-, two-, and three-phase systems—§ 18. Simple alternating wave of magnetic flux—§ 19. Analysis of alternating into two rotating waves, and *vice versâ*—§ 20. Production of rotating waves of magnetic flux by means of polyphase currents.

§ 14. Polyphase Currents

In connection with many important practical applications, the alternating currents employed consist not of simple or *single-phase* currents, but of a system of several currents of the *same frequency* but differing in phase. Such a system is termed a *polyphase system.*

The only two polyphase systems of practical importance are those known as the *two-phase* and the *three-phase* systems.*

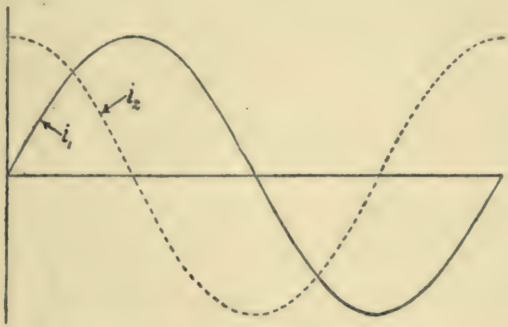

Fig. 19.—Graphs of Two-phase Currents.

A two-phase system consists of two currents of the same frequency differing in phase by 90°.

A three-phase system consists of three currents of the same frequency differing 120° in phase.

Fig. 19 shows the graphs of the currents (sine waves being assumed) forming a two-phase, and Fig. 20 the graphs of the currents forming a three-phase system.

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* A six-phase system is frequently used in the armature windings of rotary converters. But the transmission of power to the converter takes place by means of three-phase currents.

The vector diagram for a two-phase system consists, as shown in Fig. 21 (a), of two vectors at right angles to each other; while that for a three-phase system, Fig. 21 (b), consists of three vectors making angles of 120° with each other.

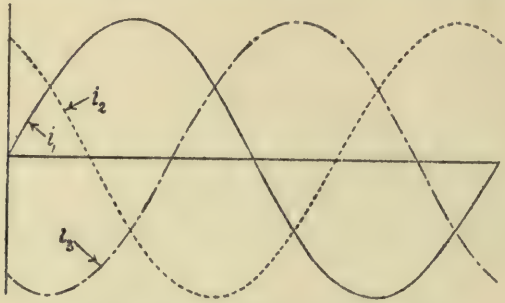

FIG. 20.—Graphs of Three-phase Currents.

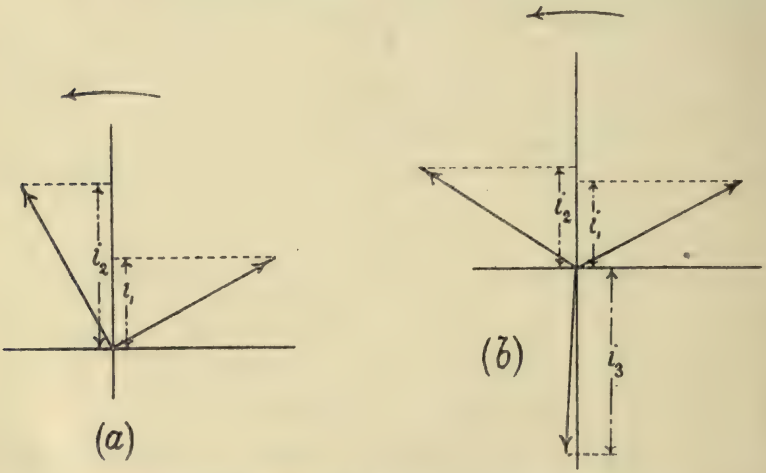

FIG. 21.—Vector Diagrams of Two- and Three-phase Systems.

Algebraically, the expressions for the currents forming a two-phase system may be written—

$$i_1 = I_1 \sin pt$$

$$i_2 = I_2 \sin\left(pt + \frac{\pi}{2}\right) = I_2 \cos pt$$

and those for the currents forming a three-phase system—

$$i_1 = I_1 \sin pt$$
$$i_2 = I_2 \sin\left(pt + \frac{2\pi}{3}\right)$$
$$i_3 = I_3 \sin\left(pt + \frac{4\pi}{3}\right)$$

It may be pointed out that the amplitudes of the currents forming a polyphase system need not necessarily be equal; if they are, the system is said to be *balanced.*

§ 15. Rotating Field produced by Polyphase Currents

Polyphase systems present several advantages* over the single-phase system, one of the most important being the possibility of producing a *rotating magnetic field* without the aid of any mechanical rotation.

In order to explain the production of such a field by means of two-phase currents, we may consider two similar coils placed at right angles to each other, as in Fig. 22; at any given point of space each coil will produce, when conveying a current, a field proportional to the current. Hence, if we suppose that the coils are traversed by the two currents of equal amplitude—

$$i_1 = I \sin pt$$

and—

$$i_2 = I \cos pt$$

respectively, the magnetic fields at the common centre O of the two coils due to the currents may be written (Fig. 22)—

$$x = M \sin pt, \text{ along the horizontal axis,}$$

and—

$$y = M \cos pt, \text{ along the vertical axis,}$$

and since the fields are at right angles to each other, the resultant field OR is given by the square root of the sum of their squares, *i.e.* the magnitude of the resultant field is—

$$OR = \sqrt{x^2 + y^2} = M$$

The magnitude of the resultant field is thus *constant.* In order to find its *direction* at the time t, we notice that if θ be the angle which it makes with the vertical axis—

---

* These are considered in § 17.

$$\tan \theta = \frac{x}{y} = \tan pt$$

so that—

$$\theta = pt$$

The angle therefore changes at a constant rate, *i.e.* the field rotates with the constant angular velocity p.

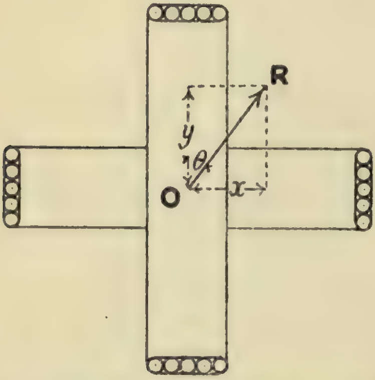

FIG. 22.—Production of Rotating Field by Two-phase Currents.

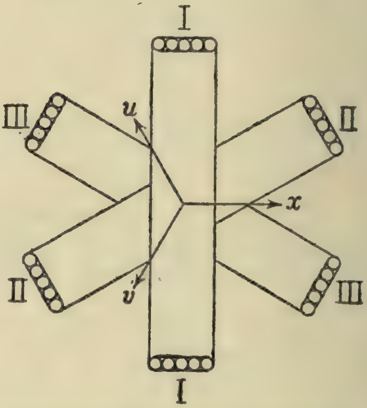

FIG. 23.—Rotating Field produced by Three-phase Currents.

Consider next three coils, I, II, and III, arranged at 120° to each other, as shown in Fig. 23, and conveying the three currents of equal amplitude—

$$i_1 = \mathrm{I} \sin pt$$

$$i_2 = \mathrm{I} \sin\left(pt + \frac{2\pi}{3}\right) = -\tfrac{1}{2}\mathrm{I} \sin pt + \frac{\sqrt{3}}{2}\mathrm{I} \cos pt$$

$$i_3 = \mathrm{I} \sin\left(pt + \frac{4\pi}{3}\right) = -\tfrac{1}{2}\mathrm{I} \sin pt - \frac{\sqrt{3}}{2}\mathrm{I} \cos pt$$

The directions of the magnetic fields produced by the three coils at their common centre are indicated by x, u, and v in Fig. 23. The values of these magnetic fields are given by—

$$x = \mathrm{M} \sin pt$$

$$u = -\tfrac{1}{2}\mathrm{M} \sin pt + \frac{\sqrt{3}}{2}\mathrm{M} \cos pt$$

$$v = -\tfrac{1}{2}\mathrm{M} \sin pt - \frac{\sqrt{3}}{2}\mathrm{M} \cos pt$$

In order to find the magnitude of the resultant field, we may determine the sum of all the horizontal components, then the sum of all the vertical components, and finally take the square root of the sum of the squares of the total horizontal and vertical components.

Now, the total horizontal component at time t is given by (Fig. 23)—

$$X = x - (u + v) \cos 60^\circ = M \sin pt + \tfrac{1}{2}M \sin pt = \tfrac{3}{2}M \sin pt$$

and the total vertical component by—

$$Y = (u - v) \sin 60^\circ = \tfrac{3}{2}M \cos pt$$

so that the magnitude of the resultant field is—

$$\sqrt{X^2 + Y^2} = \tfrac{3}{2}M$$

i.e. the resultant field is of constant magnitude. If ϕ be the angle which it makes with the vertical axis at time t, then—

$$\tan \phi = \frac{X}{Y} = \tan pt$$

so that the resultant field rotates with constant angular velocity p.

In this possibility of producing a rotating magnetic field without any mechanical rotation lies, as already mentioned, one of the main advantages of polyphase currents.

For the production of two-phase currents, two independent sources of e.m.f. are required, having the same frequency but a phase displacement of 90°. These two sources of e.m.f. are represented by two independent windings in the armature of a two-phase generator.

Similarly, the three sources of e.m.f. required for the production of three-phase currents are represented by three independent armature windings in the generator.

§ 16. Connections of Polyphase Systems

In the case of two-phase systems, the two phases or circuits of the generator, motor, or other receiving apparatus forming the system are, as a rule, kept entirely separate, as there is no advantage in electrically linking them. Sometimes, however, such linkage is resorted to, the arrangement adopted being that shown in Fig. 24.

If we suppose the system balanced, then the current in the common wire will be represented, in the vector diagram of Fig. 21 (a), by the diagonal of the square constructed on the two current vectors there shown as sides. Hence the current in the common wire is $\sqrt{2}$, or 1·414 times the phase current (*i.e.* the current in either phase), and if the current density in all the wires be the same, the cross-section of the common wire will have to be $\sqrt{2}$ times the cross-section of one of the outer wires.

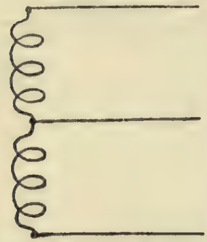

Fig. 24.—Interconnected Two-phase System.

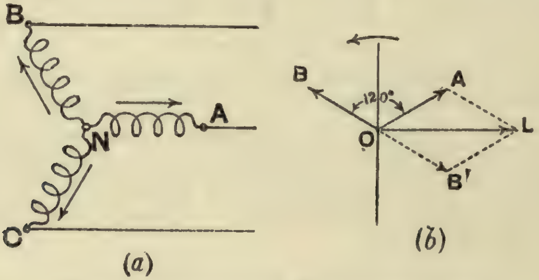

Fig. 25.—Star-connected Three-phase System.

Three-phase circuits are *invariably* linked together, as this effects a saving in the amount of copper required in the mains. There are two methods of coupling such circuits. One of these, shown in Fig. 25, is known as the *star* or Y coupling. In this, it will be noticed, the *line* current is the same as the *phase* current, and there is a point—known as the *neutral point*, marked N in Fig. 25 (*a*)—common to the three phases. The possibility of so coupling the circuits is due to the fact, at once evident from Fig. 21 (*b*), or from an inspection of the equations of § 14, that the algebraical sum of the three currents vanishes at every instant. With the star method of connection, the line currents are equal to the phase currents, but the line p.d. is much higher than the phase p.d. If we assume for the positive directions of the three p.d.'s the directions *away from* the neutral point N, as indicated by the arrows in the figure, then since in proceeding from B to N we move in the negative direction, and in proceeding from N to A in the positive direction, it is evident that the *line* p.d., *i.e.* the p.d. across BA, will at every instant be equal to the *difference* of the instantaneous p.d.'s across NA and NB. Now, if the instantaneous p.d. across NA be represented by the projection of OA in the vector diagram of Fig. 25 (*b*), and that across NB by the projection of OB, then the difference of these two projections, which gives the instantaneous value of the p.d. across BA, is equal to the projection of OA, *plus* the projection of OB *reversed*. In Fig. 25 (*b*), OB' is OB reversed, so that the p.d. across BA is equal to the sum of the projections of

OA and OB′, *i.e.* to the projection of OL. But since OL = 2OA . cos 30° = $\sqrt{3}$. OA, we see that, with a balanced system having a star coupling and supplied with sine waves of e.m.f., *the line p.d. is equal to* $\sqrt{3}$, *or* 1·732 *times the phase p.d.*

Another method of coupling three-phase circuits is that shown in Fig. 26, and known as the *mesh* or *triangle* or *delta* method. It is immediately obvious that the line p.d. is now equal to the phase p.d. The line *currents,* however, are not equal to the phase currents, each line current being at any given instant equal to the difference of the adjacent phase currents—as is at once evident from Fig. 26. Hence the vector diagram of Fig. 25 (*b*) is now applicable to the *currents,* and we see that with a mesh grouping and simple sine waves *the line current is equal to* $\sqrt{3}$, *or* 1·732 *times the phase current.*

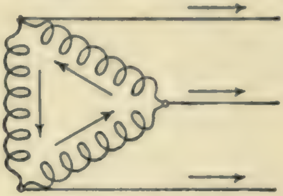

FIG. 26.—Delta-connected Three-phase System.

The advantage of coupling the circuits will now be readily understood. For if in Fig. 26 the three circuits had been kept separate, six line wires, each, we shall suppose, of cross-section *a,* would have been required (two wires for each phase or circuit). By coupling the circuits delta fashion, so as to maintain the original p.d. across each pair of mains unaltered, the number of wires is reduced to three, and the cross-section of each is, for the same current density as before, $\sqrt{3} . a$. Thus the ratio of the amount of copper in the coupled circuits to that in the uncoupled ones is $\frac{3 . \sqrt{3} . a}{6a} = \frac{\sqrt{3}}{2} = \frac{1{\cdot}732}{2} = 0{\cdot}866$, representing a saving of about 13 per cent., and this saving represents a clear gain, the coupling of the circuits not being attended with any disadvantages such as result, for example, from the coupling of two-phase circuits. In the latter case, the maximum p.d. between the two outer line wires is $\sqrt{2}$, or 1·414 times the phase p.d., which either increases the risk of a breakdown or else necessitates better and hence more expensive insulation for the line than would be necessary if the circuits were kept separate.

As regards the practical use of the two methods of coupling three-phase circuits, it may be said that the coils of generators and motors are generally coupled star fashion, while transformers are, for reasons to be explained later, in most cases delta-connected. Rotary converters are, from the nature of the case, also delta-connected; and so is an ordinary lamp load.

§ 17. Comparison of Single-, Two-, and Three-phase Systems

The relative advantages and disadvantages of the single-, two-, and three-phase systems of power generation and transmission may be briefly summarized as follows. The single-phase system is the simplest, since it requires only two conductors, with a correspondingly simple arrangement of switch-gear. Where the number of conductors is a matter of importance—as in connection with electric tramways and railways—the single-phase system possesses an important advantage over its polyphase rivals. On the other hand, polyphase generators are, for a given speed and power, lighter and cheaper than single-phase ones; induction motors and rotary converters of the polyphase type are also superior to single-phase machines.* The cost of the conductors to transmit a given amount of power with a given strain on the insulation is least in the case of a three-phase system, and it is for this reason that the three-phase system has been much more generally adopted than the two-phase one. The latter has been mostly used to replace existing old single-phase systems, in which case the single-phase concentric mains could be utilized; a three-phase system, requiring three-core or triple concentric mains, would have involved the scrapping of the existing system of single-phase mains.

§ 18. Simple Alternating Wave of Magnetic Flux

In § 15 we explained how a rotating field at a given point of space—the common centre of suitably arranged coils—may be produced by supplying the coils with polyphase currents. We now proceed to the study of the waves of magnetic flux which are produced in alternating-current machines.

Numerous types of alternating-current machinery consist essentially of two coaxial iron cylinders separated from each other by a narrow gap—the air-gap—as shown in Fig. 27. One of the cylindrical cores—generally the outer—is stationary, and forms the *stator* of the machine; while the other is maintained in rotation, and forms the *rotor*. The surfaces of the cores may be either continuous, as in Fig. 27, or more or less discontinuous. For the sake of simplicity, we shall assume that each surface is continuous. By means of suitable windings, embedded in the cores, a system of magnetomotive forces is made to act across the gap, producing in it a multipolar field, so that the direction of the flux is alternately from stator to rotor and rotor to stator. The distribution of this flux is generally more or less irregular, and depends on the particular arrangement of the windings, the discontinuities in the polar surfaces of the cores, etc. Now, just as in considering alternating e.m.f.'s and currents we made no attempt to deal with the numerous wave-shapes which occur in practice, but confined our attention to the simplest possible wave—the pure sine wave—so in the present instance, instead of dealing with the more or less irregular distribution of the magnetic flux which occurs in practice, we shall select for special treatment the simplest case of all—that, namely, in which the magnetic flux is distributed in the air-gap according to the simple sine law. If the distance x be measured along the circumference of the rotor (Fig. 28), and if we select for our origin a point at which the magnetic field vanishes or changes sign, then according to the above supposition the value of the magnetic induction y at any point of the gap distant x from the origin may be represented by—

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* The performance of a single-phase rotary converter is so poor that this type of machine is never used in modern practice.

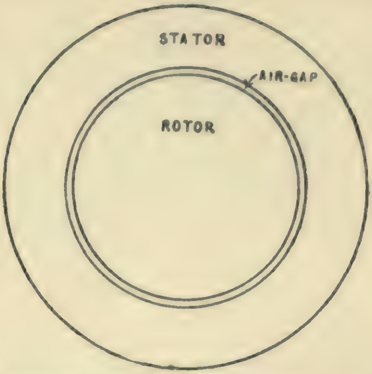

FIG. 27.—Type of Alternating-current Machine.

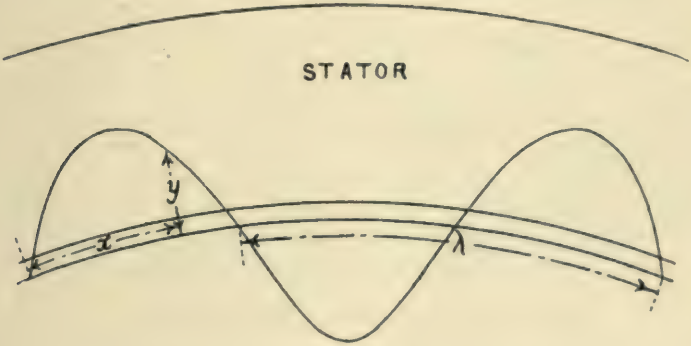

FIG. 28.—Simple Sine Wave of Flux.

$$y = b \sin qx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The distribution of the flux may be graphically represented as in Fig. 28, where the values of y are laid off, to a convenient scale, along lines perpendicular to the rotor circumference.

We may speak of the distance λ which separates two corresponding points on the curve of induction, and within which are included all possible values of the induction, as the magnetic *wave-length.* The wave-length clearly corresponds to the distance between the middle points of two pole-pieces of the same name, or to twice the pole-pitch. Since the substitution of $x + \lambda$ for x leaves y unaltered as regards both magnitude and sign, we must have—

$$\sin qx = \sin (qx + q\lambda)$$

or—

$$q\lambda = 2\pi, \text{ i.e. } q = \frac{2\pi}{\lambda}$$

Equation (1) and the corresponding curve of Fig. 28 give the distribution of the flux at a given instant, with a given current circulating around the coils which produce the m.m.f.'s.* Now, let us suppose that the current is alternating, so that the magnetic p.d. across the gap at any *given* point on the rotor circumference varies according to the simple harmonic law with the *time.* Then b in equation (1) will no longer be a constant, but will be represented by—

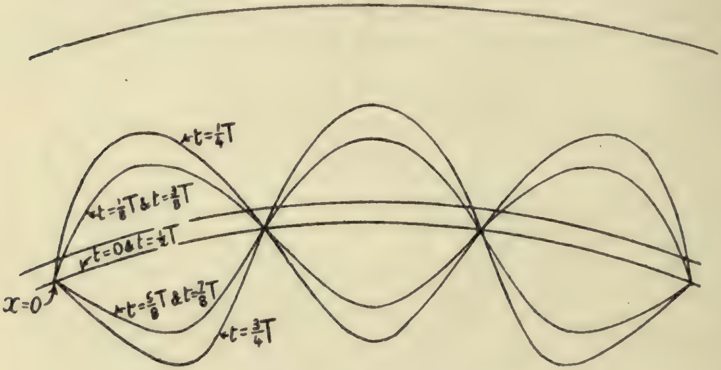
$t=\frac{1}{4}T$
$t=\frac{1}{8}T$ & $t=\frac{3}{8}T$
$t=0$ & $t=\frac{1}{2}T$
$x=0$
$t=\frac{5}{8}T$ & $t=\frac{7}{8}T$
$t=\frac{3}{4}T$

FIG. 29.—Oscillations of Simple Alternating Wave.

$$b = \text{B} \sin pt$$

where $p = \frac{2\pi}{\text{T}}$, T being the period.

---

* M.m.f. = magnetomotive force.

We now have—

$$y = \mathrm{B} \sin pt \,.\, \sin qx$$

From this equation we see that while at every instant (*i.e.* for every value of t) the magnetic flux is distributed in space according to the sine law, and while the points of zero flux remain fixed, the actual value of the flux at every point undergoes simple oscillations according to the sine law. This means that the ordinates of the curve in Fig. 28 oscillate about the base line. In Fig. 29 are indicated the positions of the curve corresponding to equal time intervals, each of which $= \frac{1}{8}\mathrm{T}$. Thus our wave of magnetic flux is an oscillating wave whose zero points or *nodes* remain fixed; such a wave is frequently spoken of as a *stationary wave.* We may also speak of it as a *simple alternating wave* of magnetic flux.

§ 19. Analysis of Alternating into Two Rotating Waves, and vice versâ

The expression for y may be thrown into a slightly different form. We have—

$$\begin{aligned} y &= \mathrm{B} \sin pt \,.\, \sin qx \\ &= \tfrac{1}{2}\mathrm{B} \,.\, 2 \sin pt \,.\, \sin qx \\ &= \tfrac{1}{2}\mathrm{B}\{\cos (pt - qx) - \cos (pt + qx)\} \\ &= \tfrac{1}{2}\mathrm{B} \cos (pt - qx) - \tfrac{1}{2}\mathrm{B} \cos (pt + qx) \end{aligned}$$

From this we see that y may be split up into—

$$y_1 = \tfrac{1}{2}\mathrm{B} \cos (pt - qx)$$

and—

$$y_2 = -\tfrac{1}{2}\mathrm{B} \cos (pt + qx)$$

Let us consider the meanings of y_1 and y_2. If we assign to t any definite value, then clearly both y_1 and y_2 will represent a magnetic flux distributed in the air-gap according to the simple sine (or cosine) law. Taking the particular instant $t = 0$, we find—

$$[y_1]_{t=0} = \tfrac{1}{2}\mathrm{B} \cos qx$$

and—

$$[y_2]_{t=0} = -\tfrac{1}{2}\mathrm{B} \cos qx$$

where the symbol $[y_1]_{t=0}$ denotes the value of y_1 at the instant $t = 0$. The values of $[y_1]_{t=0}$ and $[y_2]_{t=0}$ have been plotted in Fig. 30, and are shown by the full-line curves. When t has increased to $\frac{1}{8}\mathrm{T}$, we find—

$$[y_1]_{t=\frac{1}{8}T} = \tfrac{1}{2}B\cos\left(\frac{\pi}{4} - qx\right)$$

and—

$$[y_2]_{t=\frac{1}{8}T} = -\tfrac{1}{2}B\cos\left(\frac{\pi}{4} + qx\right)$$

These new values of y_1 and y_2 are also shown in Fig. 30, the dotted curve representing $[y_1]_{t=\frac{1}{8}T}$, and the chain-dotted curve $[y_2]_{t=\frac{1}{8}T}$.

From this we see that during a time interval $\frac{1}{8}T$ each curve has, without altering its size or shape, been displaced through a distance $= \frac{1}{8}\lambda$, and that the displacement for y_1 has been a *forward* one, and that for y_2 a *backward* one. Hence y_1 and y_2 represent two waves of magnetic flux *travelling* without change of size or shape in opposite directions around the rotor periphery. We speak of such a travelling or moving wave of magnetic flux as a rotating wave of flux or a *rotating magnetic field*, and the result just established shows that *a simple alternating or stationary wave is equivalent to, or may be replaced by, two rotating waves.*

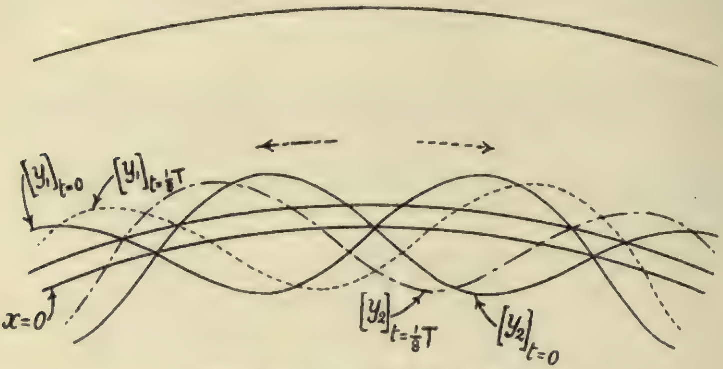

FIG. 30.—Rotating Waves of Flux.

The directions of rotation corresponding to y_1 and y_2 are indicated by the dotted and chain-dotted arrows respectively in Fig. 30. By adding the ordinates of $[y_1]_{t=\frac{1}{8}T}$ and $[y_2]_{t=\frac{1}{8}T}$ we, of course, obtain the ordinates of the curve corresponding to $[y]_{t=\frac{1}{8}T}$ in Fig. 29.

It will be noticed that the amplitude of each of the two component rotating waves is equal to *half* the maximum amplitude of the resultant alternating or stationary wave.

The relation just established between an alternating wave and two oppositely rotating waves of half the amplitude is a *reciprocal* one; *i.e.* any rotating wave may be analyzed into two simple alternating waves of half the amplitude. The proof of this proposition is extremely simple. For, taking the rotating wave—

$$u = \mathrm{U} \cos (pt - qx)$$

we have—

$$u = \tfrac{1}{2}\mathrm{U} \cos pt \cos qx + \tfrac{1}{2}\mathrm{U} \sin pt \sin qx$$

But each of the terms on the right-hand side represents an alternating wave, and the maximum amplitude of each alternating wave is half the amplitude of the given rotating wave. Thus the proposition is established.

The substitution of two rotating waves for an alternating one, or *vice versâ*, is a device frequently employed in alternating-current theory, as it in many cases leads to a considerable simplification in the treatment of certain classes of problems.

§ 20. Production of Rotating Waves of Magnetic Flux by Means of Polyphase Currents

We shall now show that a rotating wave of magnetic flux may be produced by using suitably arranged windings supplied with polyphase currents.

Let us suppose that a two-phase system of mains is available. Imagine a winding embedded in the stator (or rotor) which, when traversed by a simple alternating current—obtained from one phase of the two-phase system of mains—generates a simple alternating wave of magnetic flux represented by—

$$y_1 = \mathrm{B} \sin pt \sin qx$$

where, as before, $p = \frac{2\pi}{\mathrm{T}}$ and $q = \frac{2\pi}{\lambda}$, T and λ denoting the period and the wave-length respectively of the wave of magnetic flux.

Let another winding, precisely similar to the first, but displaced relatively to it by $\frac{1}{4}\lambda$ (or half a pole-pitch), be arranged on the stator, and let a current be sent through it from the remaining phase of the two-phase system. This winding will generate a simple alternating magnetic flux wave precisely similar to that generated by the first winding, but displaced relatively to it as regards both time and space. For sin pt we must now, by reason of the phase difference of $\frac{\pi}{2}$ which exists between the two currents, write $\sin\left(pt + \frac{\pi}{2}\right)$, and—

assuming that the origin from which x is measured remains unaltered, and that the second winding is displaced *backwards*, or in the negative direction of x, relatively to the first—for qx we must write $q\left(x + \frac{\lambda}{4}\right) = qx + \frac{\pi}{2}$. Hence, denoting by y_2 the value of the magnetic induction at any point x due to the current in the second winding, we have—

$$y_2 = \text{B} \sin\left(pt + \frac{\pi}{2}\right) \sin\left(qx + \frac{\pi}{2}\right)$$

Each of these simple alternating waves may, as already explained, be replaced by two rotating waves, so that—

$$y_1 = \tfrac{1}{2}\text{B}\{\cos(pt - qx) - \cos(pt + qx)\}$$

and—

$$\begin{aligned} y_2 &= \tfrac{1}{2}\text{B}\{\cos(pt - qx) - \cos(pt + qx + \pi)\} \\ &= \tfrac{1}{2}\text{B}\{\cos(pt - qx) + \cos(pt + qx)\} \end{aligned}$$

The resultant induction y at any point is given by—

$$y = y_1 + y_2 = \text{B} \cos(pt - qx)$$

and we see that the result of the superposition of the two alternating waves is a pure rotating wave. Thus a two-phase system of currents may be made to give rise to a rotating wave of magnetic flux.

Similarly, a three-phase system of currents may be used for producing a rotating wave. Imagine three similar windings on the stator or rotor, displaced relatively to each other by amounts $\frac{1}{3}\lambda$, to be traversed by three-phase alternating currents. If y_1, y_2, and y_3 denote the three component alternating flux waves, we may write—

$$y_1 = \text{B} \sin pt \sin qx$$

$$y_2 = \text{B} \sin\left(pt + \frac{2\pi}{3}\right) \sin\left(qx + \frac{2\pi}{3}\right)$$

and—

$$y_3 = \text{B} \sin\left(pt + \frac{4\pi}{3}\right) \sin\left(qx + \frac{4\pi}{3}\right)$$

or, using the simple transformation previously employed—

$$y_1 = \tfrac{1}{2}\text{B}\{\cos(pt - qx) - \cos(pt + qx)\}$$

$$y_2 = \tfrac{1}{2}\text{B}\left\{\cos(pt - qx) - \cos\left(pt + qx + \frac{4\pi}{3}\right)\right\}$$

$$y_3 = \tfrac{1}{2}\text{B}\left\{\cos(pt - qx) - \cos\left(pt + qx + \frac{8\pi}{3}\right)\right\}$$

Adding, we get for the resultant the rotating wave—

$$y = y_1 + y_2 + y_3 = \tfrac{3}{2}\mathrm{B} \cos (pt - qx)$$

Special attention may be drawn to the following points. In each case, when $t = 0$, the induction at $x = 0$ is at its maximum. This means that the crest of the resultant rotating wave reaches the origin at the instant when the amplitude of the first component alternating wave is at its maximum. In other words, at the instant when any one of the component alternating waves reaches its maximum amplitude, it is coincident in position with the resultant rotating wave.

Secondly, the amplitude of the rotating wave is, in the case of the two-phase system, equal to that of either component alternating wave, while in the case of the three-phase system the amplitude of the resultant is 1·5 times that of the component waves.

From the above it is at once evident that, while in the case of the two-phase system the e.m.f. induced in each phase by the resultant rotating wave is the same as that which would be induced by the alternating wave due to that phase if acting alone, the e.m.f. induced in each phase of the three-phase winding is increased 50 per cent. by the presence of the other two phases. This result may also be expressed by saying that there is no mutual inductance between the two circuits of a two-phase system, but that in a three-phase system the effect of mutual inductance is equivalent to a 50 per cent. increase in the self-inductance.

CHAPTER IV

§ **21**. Theory of the wattmeter. Effect of self-inductance—§ **22**. Effect of capacity —§ **23**. Effect of eddy currents—§ **24**. Effect of mutual inductance—§ **25**. Correction for loss in wattmeter—§ **26**. Power measurement in three-phase circuits—§ **27**. Relations of line p.d.'s and currents—§ **28**. Case of balanced load. Tangent formula—§ **29**. Power measurement by ammeters and voltmeters—§ **30**. Measurement of energy.

§ 21. Theory of the Wattmeter. Effect of Self-inductance

One of the most important measurements in alternating-current circuits is the measurement of *power*. By power is here meant, as explained in § 7, the mean value of the power over a complete period. Although numerous methods, some very ingenious, have been devised for the measurement of power, it is nowadays generally admitted that by far the most satisfactory method is that involving the use of a *wattmeter*.

The oldest type of this instrument is the electrodynamometer wattmeter, which resembles in general construction the well-known Siemens electrodynamometer for the measurement of currents. It consists of two coils, a fixed and a movable one, arranged with their planes at right angles to each other, the movable coil being provided with a torsion head, by means of which it can always be restored to its original position when deflected by an electrodynamic couple. The fixed coil is placed in the main circuit, while the movable coil is connected in series with a high non-inductive resistance, and is joined across the two points between which power is being measured.

The connections are indicated in Fig. 31, where F denotes the fixed or main or "current" coil of the instrument, M the movable or shunt or "pressure" coil, r_1 the high non-inductive resistance in series with the pressure coil, and A the portion of the circuit in which power is being measured. The current coil represents the ammeter part of the instrument, and the pressure coil the voltmeter part. The dotted line indicates an alternative mode of connection for the pressure coil.

Consider in the first place the *ideal* wattmeter, in which the self-inductance of the pressure or fine-wire circuit is negligible in comparison with the resistance, and in which the fine-wire circuit is also utterly devoid of capacity. The current in this circuit will then be in phase with, and proportional to, the p.d. Hence the instantaneous couple acting on the instrument will be proportional to the product p.d. × current, *i.e.* to the power, and the mean couple to the mean power. The angle of torsion will therefore be proportional to the mean power.

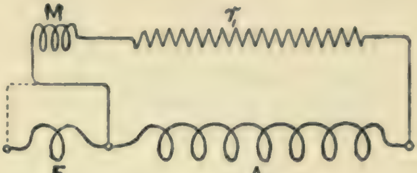

Fig. 31.—Connections of Wattmeter.

In the actual instrument, the self-inductance of the fine-wire circuit is made as small as possible by using a coil of the smallest possible number of turns. The capacity of this circuit is reduced to a practically negligible amount by winding the non-inductive resistance bifilarly in sections, which are then connected in series with each other.

It is more difficult in practice to reduce the self-inductance to a negligible amount than the capacity, and it is important to investigate what effect the presence of a small amount of self-inductance may have on the reading of the instrument.

Let R, L denote the resistance and self-inductance of the load respectively, and let r, l stand for the corresponding quantities in the fine-wire circuit of the wattmeter. If V = p.d., I = current in main circuit, then, denoting $\tan^{-1}\frac{\mathrm{L}p}{\mathrm{R}}$ by ϕ, and $\tan^{-1}\frac{lp}{r}$ by θ, we have—

$$\text{true power } w = \mathrm{VI}\cos\phi \; (\S\ 7)$$

and if l were negligible the wattmeter reading would correspond to VI $\cos\phi$. The effect of l is a twofold one: it reduces the current in the fine-wire circuit in the ratio $\frac{r}{\sqrt{r^2+p^2l^2}}$, and it further causes the current to lag behind the p.d. by an angle θ. The first effect reduces the couple in the given ratio, while the second is obviously equivalent to a change in the power factor of the load from $\cos\phi$ to $\cos(\phi-\theta)$. Thus the reading of the wattmeter will be—

$$w' = \frac{\mathrm{VI}r}{\sqrt{r^2+p^2l^2}} \cdot \cos(\phi-\theta) = \mathrm{VI}\cos\theta\cos(\phi-\theta)$$

We thus have, dividing the former equation by the latter—

$$w = w' \frac{\cos \phi}{\cos \theta . \cos (\phi - \theta)}$$

If, therefore, θ is not negligible, then the only case in which the wattmeter reading will be correct is that corresponding to $\phi = \theta$. For values of $\phi < \theta$ the reading of the wattmeter will be too low, and for values of $\phi > \theta$ too high.

The multiplier $K = \frac{\cos \phi}{\cos \theta . \cos (\phi - \theta)}$, which converts the wattmeter reading into true power, is spoken of as the *correcting factor*. It may be thrown into various forms. Thus, since—

$$\begin{aligned} \cos \theta . \cos (\phi - \theta) &= \cos \theta(\cos \phi \cos \theta + \sin \phi \sin \theta) \\ &= \cos^2 \theta \cos \phi(1 + \tan \phi \tan \theta) \end{aligned}$$

we have—

$$K = \frac{\sec^2 \theta}{1 + \tan \phi \tan \theta} = \frac{1 + \tan^2 \theta}{1 + \tan \phi \tan \theta}$$

Now, since θ is always very small, we may write approximately—

$$K = \frac{1}{1 + \tan \phi \tan \theta} = 1 - \tan \phi \tan \theta$$

a form which shows very clearly the *rapid increase in the error with decreasing power factor* $\cos \phi$.

Another simple approximate expression for the true power, which may be used when the power factor is very low, has been suggested by Dr. Drysdale.* Using the last form of approximate value for K, we have, since the wattmeter reading is roughly correct, *i.e.* since roughly $w' = VI \cos \phi$, and also $\sin \phi$ = nearly 1 for small values of $\cos \phi$—

$$w = w' - w' \tan \phi \tan \theta = w' - VI \times \tan \theta$$

The above results may be exhibited very clearly by the aid of the simple vector diagram of Fig. 32, in which I is the vector of current in the main circuit, and V the p.d. vector. If there were no self-inductance in the fine-wire circuit, the current in this circuit would be proportional to V, and the reading to $IV \cos \phi = IV'$. By reason of self-inductance, the effective or resultant e.m.f. which maintains the current through the resistance of the fine-wire circuit is not V, but V_r, and hence the reading is proportional to IV'_r. The error is thus $I(V'_r - V')$. It is at once evident that: (1) the instrument reading is too low if $\phi < \theta$; (2) the instrument reading is correct if

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* *The Electrician*, vol. xlvi. p. 774 (1901).

$\phi = \theta$; and (3) the instrument reading is too high if $\phi > \theta$, and the error increases with increase of θ.

It may be pointed out that it is easier to render the error arising from the self-inductance of the fine-wire coil negligible in wattmeters intended to be used on a high than in those for use on a low voltage circuit, since in the former case the non-inductive resistance may be made relatively higher, and so tan θ may be reduced.

The main value of the formulæ obtained above lies, not so much, perhaps, in the fact that they enable us to apply a correction, as in showing under what conditions the correction may be safely neglected.

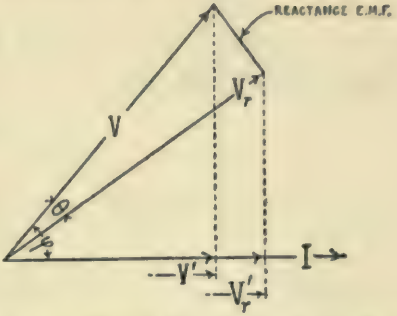

Fig. 32.—Effect of Reactance in Shunt Circuit of Wattmeter.

§ 22. Effect of Capacity

We may now consider the effect of capacity in the non-inductive resistance. Imagine this resistance not subdivided—as is generally done—but wound double in a single coil. Suppose next, for a moment, that the middle point of the resistance, where the wire is doubled back on itself, is cut. Then clearly the two halves of the winding constitute a condenser, and if an alternating p.d. is maintained between them, a definite capacity current or condenser current will flow through the dielectric between them.

Each half of the winding will at every instant be at a practically uniform potential. Let the cut ends be now joined. We then have, in addition to a capacity current, a conduction current, and a continuous drop of potential along the wire. Now, this drop of potential will obviously reduce the capacity current to *half* the value which it had when the resistance was cut, since the *mean* p.d. between the two halves of the resistance is now only half the p.d. across its ends. The *equivalent* capacity is thus half the true capacity, and the effect due to it is clearly the same as if the resistance were devoid of capacity, but were shunted by a condenser of capacity C equal to half the true capacity of the resistance, as shown by the diagram of Fig. 33.

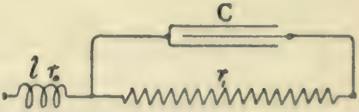

Fig. 33.—To illustrate Capacity Effect in Wattmeter.

Applying to the branched portion of the circuit of Fig. 33 the rules established in § 9 for parallel arrangements of impedances, we find for the joint resistance of the branched portion—

$$\frac{r_1}{1 + r_1^2 C^2 p^2}$$

and for the joint reactance—

$$-\frac{Cpr_1^2}{1 + r_1^2 C^2 p^2}$$

From this we find the total resistance of the fine-wire circuit to be—

$$r_0 + \frac{r_1}{1 + r_1^2 C^2 p^2}$$

and its total reactance—

$$lp - \frac{Cpr_1^2}{1 + r_1^2 C^2 p^2}$$

The current in the fine-wire circuit therefore lags behind the p.d. by an angle—

$$\tan^{-1}\frac{lp - \dfrac{Cpr_1^2}{1 + r_1^2 C^2 p^2}}{r_0 + \dfrac{r_1}{1 + r_1^2 C^2 p^2}}$$

or, approximately—

$$\tan^{-1}\frac{l - r_1^2 C}{r}p$$

where $r = r_0 + r_1$.

From this we see that the capacity of the non-inductive resistance, if not excessive, is beneficial, as it tends to neutralize the effect of the self-inductance of the fine-wire coil.

The formulae established in § 21 in connection with the effect of self-inductance alone are immediately available for the combined effect of self-inductance and capacity, provided that for θ we substitute $\tan^{-1}\frac{l - r_1^2 C}{r}p$.

The capacity effect is greatly reduced by subdividing the non-inductive resistance into a number of sections. Let there be n sections, each wound non-inductively as before. If we imagine each section bisected, we obtain n condensers in series, each of capacity $\frac{1}{n}$C, where C is the capacity of the resistance when wound in a single section. But since the joint capacity of n equal condensers arranged in series with each other is $\frac{1}{n}$th that of one of them, we see that by subdividing the resistance we have reduced the capacity to $\frac{1}{n^2}$ of its original amount. The subdivision of the resistance therefore affords a simple means of reducing the capacity to any desired extent.

§ 23. Effect of Eddy Currents

In most of the earlier wattmeters there was a source of error whose importance was not realized, and whose very existence, in fact, remained for a long time unknown. It is largely due to this source of error that a strong prejudice was established against the wattmeter, a prejudice which was only gradually overcome as the construction of the instrument underwent improvement. The error referred to is that due to *eddy currents* induced in any solid masses of metal used in the construction of the instrument—such as a metal case, metal supports for coils, or even a heavy winding for the main coil. The error due to eddy currents may be regarded as a twofold one. In the first place, the field due to the main coil is distorted by the action of the eddy currents, and so the torque acting on the movable coil is altered; and secondly, there will be, as explained in § 11, an additional torque (which may correspond to either an increase or a decrease of torque, according to circumstances) acting on the movable coil, due to the presence in its neighbourhood of large conducting masses.

In order to avoid this source of error, the case of the dynamometer type of wattmeter is invariably constructed of some insulating material, such as ebonite or ambroin, and the same material is also employed for the supports of the coils, etc. For large currents, the current coil should be carefully stranded.*

§ 24. Effect of Mutual Inductance

The errors arising from self-inductance, capacity, and eddy currents are the only ones present in the zero type of dynamometer wattmeter, in which the movable coil always occupies the same position when a reading is taken. But since this type of instrument requires a preliminary adjustment, it is not very convenient to use, and is not much used commercially, most modern forms being indicating instruments, in which the fine-wire coil is allowed to move, a suitable spring furnishing the controlling couple. Now, as soon as the fine-wire coil is displaced from its position at right angles to the main coil, there is, on account of the mutual inductance between the coils, an e.m.f. induced in it, and this e.m.f. is a variable one, depending on the relative positions of the two coils. Thus a further possible source of error arises in connection with indicating instruments.

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* Another solution of the difficulty is to use a suitably designed current transformer, so that the current coil may be wound with thin wire.

In order to investigate this error, we may conveniently use a diagram similar to that of Fig. 32. With no mutual inductance between the coils, the reading would be proportional to V′ in Fig. 34. Owing to mutual inductance, the e.m.f. V_m due to it must be compounded with V, and the resultant treated as if it represented the p.d. impressed on the fine-wire circuit. It is thus evident that the reading will be proportional to V_m'. Now, in the perfect instrument the reading would be proportional to V′, so that the error when mutual inductance is taken into account is proportional to $V_m' - V'$. With no mutual inductance present, the error would be proportional to $V_r' - V'$, *i.e.* in the case shown in the diagram it would be greater. Thus mutual inductance, in the case shown, has the effect of *reducing* the error due to self-inductance.* It is easy to see that for negative values of ϕ, *i.e.* for leading currents, the error due to self-inductance would be *increased* by mutual inductance.

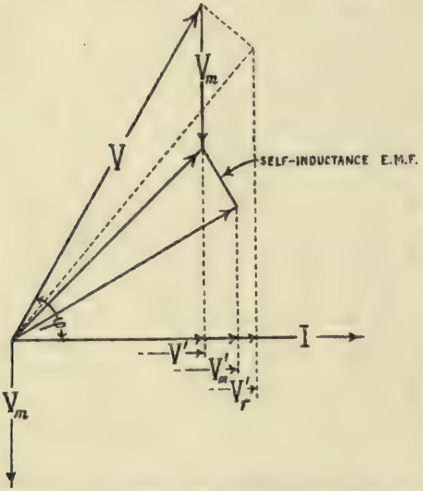

FIG. 34.—To illustrate Effect of Mutual Inductance.

If, however, the self-inductance and capacity effects are quite negligible, then it is evident from the diagram that ($V' = V_m' = V_r'$) mutual inductance does not introduce any error.

We may summarize the above results by saying that mutual inductance tends to *correct* the errors of a wattmeter so long as the current is a *lagging* one, and to *exaggerate* the errors if the current is a *leading* one.

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* Or capacity, or self-inductance and capacity combined.

§ 25. Correction for Loss in Wattmeter

Returning to the diagram of Fig. 31, which shows the connections of the wattmeter circuits, we notice that when the fine-wire or shunt circuit is arranged as shown by the full line, the power measured includes that wasted in the shunt circuit of the wattmeter, whereas with the dotted line connection the power measured includes that wasted in the current coil of the wattmeter. A correction is easily made in either case by calculating the power wasted from the known resistances of the circuits and the currents passing through them The error may also be compensated for by providing an auxiliary fine-wire winding on the main coil, having a number of turns equal to that on the main coil, and arranged so as to oppose the effect of the main winding; the auxiliary winding being included in the fine-wire circuit. This arrangement suffers, however, from the disadvantage of increasing the self-inductance of the shunt circuit.

§ 26. Power Measurement in Three-phase Circuits

The measurement of power in a single-phase circuit is readily effected, as explained in § 21, by the use of a wattmeter. A similar method, involving the use of *two* wattmeters, is applicable to a two-phase circuit, one wattmeter being included in each phase. If the phases are coupled, the wattmeter connections remain unaltered, and are as shown in Fig. 35, where C_1 and C_2 denote the current coils of the wattmeters, p_1 and p_2 their pressure coils, and r_1 and r_2 the high non-inductive resistances connected in series with the pressure coils. The sum of the two wattmeter readings gives the total power in the two-phase system. If only a single wattmeter is available, it may be used to obtain the two readings in succession; this, however, is not so satisfactory as a simultaneous reading of two wattmeters.

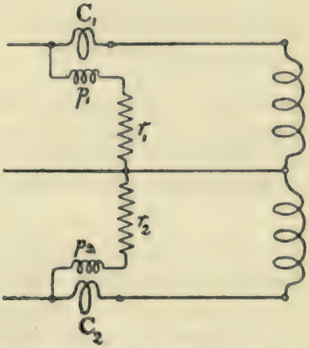

FIG. 35.—Power Measurement in Two-phase Circuit.

The measurement of power in a three-phase circuit is a much more complicated matter, and it is only in certain special cases that it approaches the simplicity of single-phase power measurement. We shall first consider the most general method of finding the power in a three-phase circuit, a method which applies to every possible case, and which only involves the use of *two* wattmeters. For this reason it is known as the *two-wattmeter method.*

A large number of methods of measurement and calculation in connection with three-phase systems is based on various more or less arbitrary and frequently unjustifiable assumptions, such as sine waves of p.d. and current, perfect balance of the system, accessibility of the neutral point, or phase differences of exactly 120° between the three p.d.'s or the three currents. Now, when the load is not a balanced one, not only are the three p.d.'s in general unequal, but their phase differences are also unequal, and different from 120°.

The two-wattmeter method of power measurement is entirely free from any such arbitrary assumptions, as will be evident from a study of its theory. In Fig. 36, A, B, and C are the three line wires conveying power to the three terminals A', B', C', to which the load is connected. The load may be connected either delta- or star-fashion. In either case, we may always arrange a star-connected load, as shown in Fig. 36, having a neutral point O, which is equivalent in every respect to the actual load. Let the instantaneous values of the currents along the three wires (reckoned positive when flowing from left to right) be i_1, i_2, and i_3, and let v_1, v_2, and v_3 be the instantaneous p.d.'s between the neutral point O and the line wires (reckoned positive if directed towards O). The wattmeter connections are arranged as shown. It is to be noted that the series coils of the wattmeters may be placed in *any* two of the line wires, provided the free ends of their shunt coils are in connection with the remaining line wire. If w = total instantaneous power, we evidently have—

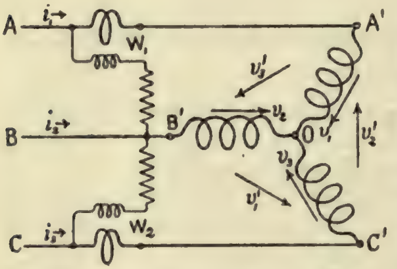

FIG. 36.—Power Measurement in Three-phase Circuit.

$$w = v_1 i_1 + v_2 i_2 + v_3 i_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, i_1 and i_3 are the currents traversing the series coils of the wattmeters, while i_2 does not flow through any wattmeter coil. It will, therefore, be convenient to eliminate i_2 from the expression for the instantaneous power. This is easily done by means of the equation—

$$i_1 + i_2 + i_3 = 0$$

which must obviously be true in every possible case. Putting $i_2 = -i_1 - i_3$ in (1), and rearranging the terms, we find—

$$w = (v_1 - v_2)\, i_1 + (v_3 - v_2)\, i_3 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But since $v_1 - v_2$ is the instantaneous p.d. across the shunt circuit of the wattmeter W_1,* and $v_3 - v_2$ the instantaneous p.d. across the shunt circuit of W_2, it follows that the mean total power over a period (*i.e.* the mean value of w) is equal to the *algebraical sum of the wattmeter readings.*

§ 27. Relations of Line P.D.'s and Currents

On account of the importance of the two-wattmeter method, we shall investigate the phase relations of the line p.d.'s and line currents somewhat more closely in the general case of an unbalanced inductive load. In Fig. 37 is given a vector diagram of the line currents, the star p.d.'s, and the line p.d.'s.† The three vectors I_1, I_2, and I_3 represent the line currents. These lag behind their respective star p.d.'s V_1, V_2, and V_3 by the angles θ_1, θ_2, and θ_3, and for the sake of generality we have supposed these angles to be all different. The line p.d.'s, whose instantaneous values are denoted by v_1', v_2', and v_3' in Fig. 36, and which are reckoned positive when they have a counter-clockwise direction around the mesh or triangle A'B'C' in Fig. 36, are easily obtained by noticing that $v_1' = v_2 - v_3$, $v_2' = v_3 - v_1$, and $v_3' = v_1 - v_2$. Hence in the vector diagram of Fig. 37 these line p.d.'s are represented by the sides of the triangle formed by joining the extremities of V_1, V_2, and V_3; their directions are indicated by the arrow-heads.

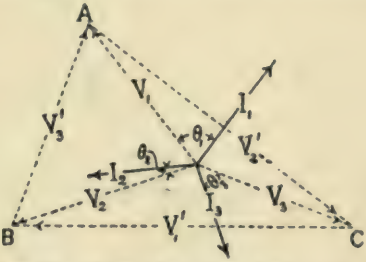

FIG. 37.—Vector Diagram of Three-phase Circuit.

For the sake of clearness in the diagram, it will be convenient to draw the triangle of line p.d.'s as in Fig. 38, and from the vertices of this the current vectors I_1, I_2, and I_3.

We next notice that the connections of the two wattmeters W_1 and W_2 in Fig. 36 were supposed to be so arranged that a positive couple was obtained when the current and p.d. at any instant were both acting away from (or towards) the junction of the current and pressure coils. Now, from Fig. 36 it will be seen that the positive direction of v_3' is the same as that of v_1, and since the reading of W_1 is equal to the mean value of $+ v_3'i_1$, it is represented, in Fig. 38, by $V_3'I_1 \cos \phi_1 = V_3' \times AD$. On the other hand, considering the wattmeter W_2, we notice that, since in Fig. 36 the positive direction of

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* Cf. § 16.

† This diagram should be compared with Fig. 36.

v_1' is opposed to the positive direction of v_3, the wattmeter reading will represent the mean value of $-v_1'i_3$, and hence will, in Fig. 38, correspond to $-V_1'I_3 \cos \phi_3 = V_1'I_3' \cos \phi_3' = V_1' \times CE$.

Now, it is evident that under certain conditions of load CE may be of opposite sign to CB or V_1', *i.e.* the reading of W_2 may become *negative.* If the wattmeter is only capable of reading to one side of zero, this will necessitate a reversal of its shunt-coil connections, and the total power will then be given by the *arithmetical difference* of the readings of the two wattmeters. This case, although possible, is not of frequent occurrence, and, as may be seen by an examination of the vector diagrams, requires a very large angle of lag of I_3 behind V_3.

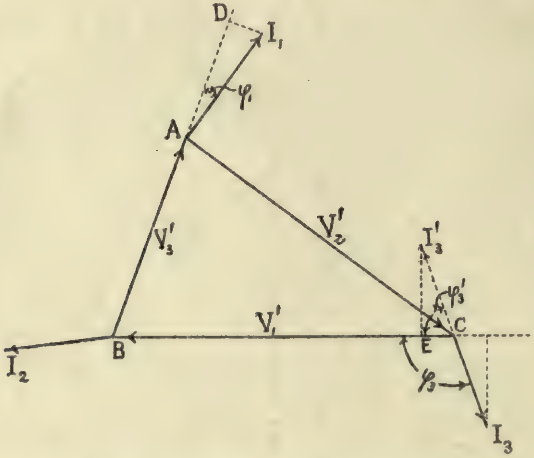

Fig. 38.—Vector Diagram of Three-phase Circuit.

We have already (§ 7) defined the *power factor* of a single-phase circuit as the ratio of the true power to the volt-amperes, and the question now arises as to what meaning is to be attached to this term in the case of an unbalanced inductive three-phase load. When, as in Fig. 37, the currents all lag by different amounts behind their respective p.d.'s, it is clear that the term "power factor" can have no definite physical meaning.

Having considered the most general case of power measurement in three-phase circuits, we may next examine certain special cases. The most interesting of these, from a practical point of view, are (1) that of a balanced circuit, and (2) that of an unbalanced non-inductive load. The first may be closely approached by either a symmetrical lamp load or a load of induction motors, and the second frequently arises in connection with a lamp load.

§ 28. Case of Balanced Load. Tangent Formula

When the circuit is balanced, the line p.d.'s and line currents will, in the diagram of Fig. 37, be represented by lines of equal length. Hence ABC will form an equilateral triangle. Further, the vectors V_1, V_2, and V_3, representing the star p.d.'s, will radiate towards the vertices of the triangle from its middle point, making angles of 30° with its sides, *i.e.* with the line p.d. vectors. The star of currents will consist of three equal arms, making angles of 120° with each other, and displaced relatively to the p.d. star through an angle θ. The cosine of this angle, which by supposition is the same for each phase, is the *power factor of the balanced three-phase load.* The diagram of Fig. 38 now takes the shape shown in Fig. 39, and it will be seen that the reading of the wattmeter W_1 in Fig. 36 is $V_3'I_1 \cos (30° - \theta)$, while that of W_2 is $V_1'I_3' \cos (30° + \theta)$. Since $V_3' = V_1' = V'$ say, and $I_1 = I_3 = I$ say, we have, using W_1 and W_2 to denote the two wattmeter readings—

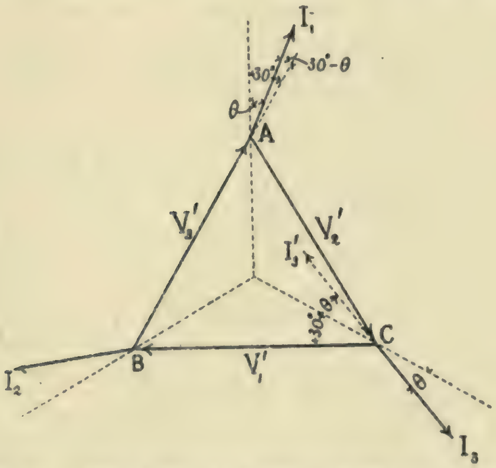

FIG. 39.—Vector Diagram of Balanced Three-phase Circuit.

$$W_1 = V'I \cos (30° - \theta)$$
$$W_2 = V'I \cos (30° + \theta)$$

Hence—

$$W_1 + W_2 = 2V'I \cos 30° \cos \theta$$
$$W_1 - W_2 = 2V'I \sin 30° \sin \theta$$

Dividing the second of the last two equations by the first, we find—

$$\tan 30^\circ \tan \theta = \frac{W_1 - W_2}{W_1 + W_2}$$

or—

$$\tan \theta = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}\text{*}$$

By using this formula, we are enabled to calculate the power factor cos θ of the balanced three-phase system from the readings of the two wattmeters. The formula is frequently referred to by continental writers as the "tangent formula." It must be carefully noted, however, that the tangent formula is based on the assumption of *sine waves* of p.d. and current,† and cannot be relied upon when the wave-forms are greatly distorted.

If the power factor of a balanced three-phase system is unity, the load being entirely non-inductive, and $\theta = 0$, then it is evident, either from the vector diagram of Fig. 39, or from the tangent formula just established, that $W_1 = W_2$, *i.e.* the readings of the two wattmeters are equal. This is, in fact, the only case in which the two instruments give identical readings.

As the angle of lag θ increases, it is evident from Fig. 39 that, the currents being assumed to retain their original values, the reading of W_1 will increase, and that of W_2 decrease, until θ becomes equal to 30°. At this stage, the reading of W_1 reaches a maximum value for a given value of the line currents. Any further increase of θ will cause both readings to decrease (it is evident from the diagram that the reading of W_2 will decrease more rapidly than that of W_1). When θ reaches 60°, we have $30^\circ + 60^\circ = 90^\circ$, and the current I_3 is in quadrature with V_1', hence W_2 will give a zero reading. A still further increase in θ will cause the reading of W_2 to become *negative*.

A very special case of a balanced three-phase load is that in which the load is star-connected, with an accessible neutral point (a star connection is very common, but the neutral point is not generally accessible). A *single* wattmeter may then be used to measure the power. The current coil of the wattmeter is introduced into one of

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* This may also be written in the form—

$$\tan \theta = \sqrt{3}\frac{\frac{W_1}{W_2} - 1}{\frac{W_1}{W_2} + 1}$$

† It may be shown that, with distorted wave-forms, the star p.d.'s of a three-phase system cannot, in general, be represented by three co-planar vectors. Hence the ordinary graphical method fails (see, in this connection, a paper by E. Orlich in *Elektrotechnische Zeitschrift*, vol. xxiv. p. 59).

the line wires, while its pressure coil is connected between this line wire and the neutral point. The wattmeter will then evidently measure the power in one phase, and since the system is balanced, the total power is given by three times the reading of the instrument.

In this connection it may be mentioned that, with an accessible neutral point, the total power of an *unbalanced* system may be measured by means of three wattmeters, connected as just explained, each wattmeter measuring the power in one phase. The total power is the sum of the three wattmeter readings. This method, whose applicability is limited by the accessibility of the neutral point, is known as the *three-wattmeter method.*

§ 29. Power Measurement by Ammeters and Voltmeters

The case of an unbalanced non-inductive load is of considerable interest, not only on account of its frequent occurrence in practice, but also because in this case the power may be found from the readings of three voltmeters and three ammeters, connected so as to read the three line p.d.'s and three line currents, without the use of any wattmeter.

A reference to the vector diagram of Fig. 37 shows that the directions of the current vectors must in this case coincide with those of the star p.d. vectors, with which they are co-phasal. From this it is evident that, if the current vector diagram (showing the magnitudes and phase relations of the line currents) be given in the form of three vectors radiating from a point, it must be possible to so fit this diagram into the triangle ABC, Fig. 37, of the line p.d. vectors as to cause the current vectors, or those vectors produced, to pass through the angular points of the triangle.

Assuming, therefore, that the three line voltages and three line currents have been measured, we may proceed to find the power as follows. Construct the triangle ABC, Fig. 40 (*a*), of the line voltages, and the triangle of line currents, Fig. 40 (*b*). From this latter obtain the star of currents, Fig. 40 (*c*). Produce the rays of the star, if necessary, and, having made a tracing of it, fit it over the triangle of voltages. By means of a needle-point, mark the position of the centre O of the star inside the triangle. Remove the tracing, and from each of the vertices A, B, and C of the angle draw lines radiating from O; along these lines lay off lengths representing the line currents. Then the power is given by—

$$V_3' \times I_1 \cos \phi_1 + V_1' I_3 \cos \phi_2 = AB \times AD + CB \times CE$$

since this would represent the sum of the two wattmeter readings if these instruments were employed. There is, however, another way of expressing the power. A reference to Fig. 37 shows that OA, OB,

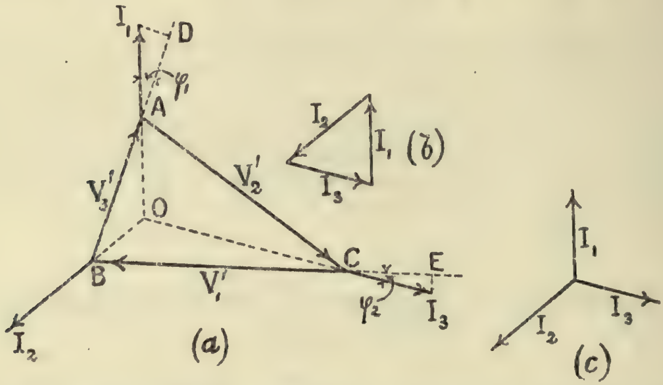

FIG. 40.—Vector Diagram of Unbalanced Non-inductive Three-phase Load.

and OC give the star voltages of the equivalent star-connected load. Hence, the load being non-inductive, and the currents in phase with the star voltages, we have for the total power—

$$OA \times I_1 + OB \times I_2 + OC \times I_3$$

§ 30. Measurement of Energy

For the measurement of the *energy* supplied to a three-phase circuit, two single-phase energy meters of suitable construction — such as the Aron clock meter or the Elihu Thomson meter—may be employed, the total energy being given by the sum of the readings of the two instruments. The connections of the current and pressure coils of these instruments are precisely similar to those used in the two-wattmeter method of power measurement shown in Fig. 36. For a balanced three-phase circuit, a single instrument will suffice if, by the creation of an artificial neutral point, this point is made accessible; the instrument then has its current coil in series with one of the mains, and

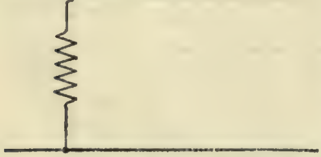

FIG. 41. — Connections of Energy Meter for Balanced Load.

its pressure coil forms one of the arms of an artificial star (all the arms being of equal resistance), the connections being as shown in Fig. 41.* Again, two meters connected as in the two-wattmeter method of power measurement may be combined into a single instrument, the rotating parts being mounted on a common axis; this has the advantage of compactness as compared with two ordinary single-phase meters.

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* If the meter is arranged to give correct readings on a single-phase circuit, its readings when transferred to a three-phase circuit and connected as above give one-third of the total energy.

CHAPTER V

§ **31.** General conditions to be satisfied by alternate-current instruments—§ **32.** Hot-wire instruments—§ **33.** Electrostatic instruments for low voltages—§ **34.** Methods of extending range of low-reading electrostatic voltmeters—§ **35.** High-voltage electrostatic instruments—§ **36.** Wattmeters—§ **37.** Power factor indicators—§ **38.** Oscillographs.

§ 31. General Conditions to be satisfied by Alternate-current Instruments

SOME of the instruments employed in connection with continuous currents are also available, without any modification, for use on alternate-current circuits, while others would give totally different readings on the two circuits. Permanent magnet instruments, which for accuracy and convenience of use are unsurpassed so long as we are dealing with continuous currents, are, of course, out of the question for alternating-current measurements.

The values of the p.d. and current with which we are concerned are their r.m.s. values. It is, therefore, evident that if a measuring instrument is to read correctly on both continuous and alternating-current circuits—*i.e.* if the readings are to be independent of frequency—the particular effect on which the reading of the instrument depends must be proportional to the *square* of the current or p.d. to be measured.

Considering instruments intended to measure currents, we have at our disposal several arrangements in which the force or couple, as the case may be, acting on the movable part of the instrument is determined by the square of the current.* The most important of these arrangements are: (1) A very thin wire traversed by the current, and fitted with suitable magnifying gear for indicating the amount of elongation due to rise of temperature. The elongation depends solely on rise of temperature, and the latter is completely determined if the mean value of the *square* of the current be given, it being quite immaterial as to whether the current is alternating or continuous. (2) An arrangement of two coils, with their planes inclined at any suitable angle, connected in series with each other and traversed by the current to be measured. The stress between the two coils is proportional to the product of their currents, *i.e.*, the currents being the same, to the square of the current. (3) A coil acting on a *feebly* magnetized core. So long as the magnetization is kept sufficiently low, the permeability may be regarded as constant, and hysteresis negligible. Thus for given relative positions of coil and core the stress between them will be proportional to the mean square of the current.

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* And is independent of the *kind* of current used, *i.e.* whether alternating or continuous.

Passing now to the measurement of p.d.'s, all the above arrangements may be employed, provided the instrument circuit is arranged so that the current in it is proportional to the p.d., and *independent of the frequency* of the p.d. This latter condition involves negligible reactance in comparison with the resistance of the instrument circuit. There is, however, a further arrangement available for the measurement of p.d.'s, which is by far the most satisfactory. It consists of a fixed and a movable system of plates, the two systems being in connection with the points across which it is desired to measure the p.d. For a given relative position of the two systems, the electric field intensity will at any instant be in simple proportion to the p.d. But since the stress between the plates varies in proportion to the square of the electric field intensity, it follows that the mean value of this stress will be proportional to the mean square of the p.d.

We shall now describe a few important types of alternating current instruments whose action is based on the above principles.

§ 32. Hot-wire Instruments

The prototype of all hot-wire instruments is the at one time well known and widely used, but now entirely obsolete, Cardew voltmeter. In Fig. 42 is shown a modern type of hot-wire instrument, designed by Messrs. Hartmann and Braun, and manufactured in this country by Messrs. Johnson and Phillips. The "hot wire" is a thin wire of platinum-silver stretched between two supports, one of which is a fixed pillar, while the other is provided with an adjustment for varying the tension of the wire, and so altering the zero position of the pointer. Platinum-silver is used as the material for this wire because of its high resistivity and non-oxidizable property, which enables it to stand the action of comparatively high temperatures when freely exposed to the air. The "hot wire" is maintained in a state of tension and made to sag by the side pull exerted on it by a phosphor-bronze wire attached to a point about its middle. The phosphor-bronze wire (which is *not* traversed by any current) is in its turn maintained in a state of tension by a silk fibre attached to it and passing round the circumference of a small pulley, on whose axle is mounted the pointer of the instrument, the other end of the silk fibre being fixed to the end of a bent flat steel spring. The silk fibre which passes round the pulley is prevented from slipping by a small screw. It is a well-known fact that a very slight amount of elongation in a suspended wire produces a relatively enormous increase of sag. Now, in the instrument under consideration we have a double magnifying arrangement of this description, the elongation of the hot wire being magnified by the phosphor-bronze wire, and the sag of this latter giving a still further magnification. A comparatively small elongation of the hot wire thus results in a large angular displacement of the pointer.

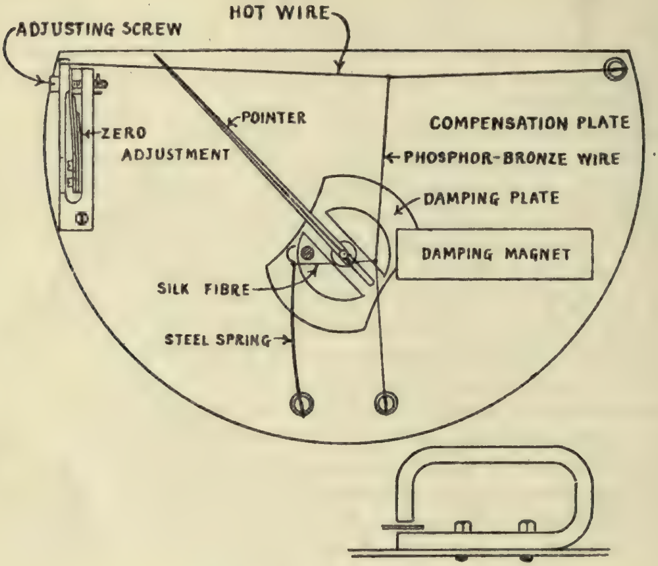

Fig. 42.—Hot-wire Instrument.

It is evident that if the hot wire were mounted on a plate whose temperature-coefficient of expansion is different from that of the wire itself, the instrument would act as a thermometer, and its zero would change with any change of temperature in its surroundings. In order to prevent this effect, and render the readings independent of the surroundings, the plate supporting the wire (marked "compensation plate" in Fig. 42) is made of bronze, of a composition which gives a temperature coefficient of expansion nearly equal to that of the wire.

It is hardly necessary to explain that one of the supports for the wire must be insulated from the compensation plate, so as not to be short-circuited by it.*

In the case of a hot-wire voltmeter, the hot wire would be connected in series with a high non-inductive resistance mounted inside the case of the instrument, at the back of the compensation plate. In an ammeter (unless for exceptionally small currents) the hot wire would form a shunt across a resistance, which in instruments for currents up to 200 amperes is contained in the instrument case, at the back of the compensation plate, and in instruments for currents above this value is separate from the instrument proper, special connecting leads being provided for connecting the instrument across the shunt.

Hot-wire instruments are necessarily more or less sluggish in their action, as the final temperature of the wire is reached gradually. The makers provide an eddy-current damping plate (of aluminium), as shown in Fig. 42. For an instrument not subject to mechanical vibration, this damping arrangement is entirely useless. But where violent vibration is present, the damping serves to check the rapid vibrations of the pointer due to purely mechanical causes.

The shaded circle shown in Fig. 42 to the left of the pulley corresponds to the cross-section of the pillar supporting the front jewelled bearing, which is shown removed. Another view of the damping magnet is given at the right-hand lower corner of the figure.

A serious disadvantage of the hot-wire type of instrument is its liability to be fused by a momentary excess of current.

Instruments constructed on the principles considered under (2) and (3) in § 31 are used to a considerable extent. As, however, details of their construction are fully described in treatises on continuous currents, we shall not refer to them further here. As examples of such instruments, we may mention Lord Kelvin's balances; the Weston standard voltmeter for alternating p.d.'s; and the numerous forms of "soft-iron" instruments, whose great advantage is their cheapness.

§ 33. Electrostatic Instruments for Low Voltages

On account of the importance of the electrostatic type of instrument, we shall devote some space to a description of several examples of this type. One of the best known is Lord Kelvin's multicellular voltmeter, the construction of the working parts of which will be understood by a study of Figs. 43 and 44. In Fig. 43 is shown the movable system of vanes, or "needles," which are threaded on a rod carrying a damping vane at its lower end, and suspended by a fine wire from a coach-spring attached to a torsion-head provided with a tangent screw for adjusting the zero of the instrument. The object of using a coach-spring between the torsion-head and the suspension is to prevent the latter from being broken by a sudden accidental jerk or blow. If, in fact, the vanes are jerked downwards, the coach-spring yields sufficiently to allow the safety-sleeve to come into contact with the guide-stop before the suspending wire is overstrained. By means of a clamping-spring, which may be raised by a screw provided with a milled head which projects through the bottom part of the case, the movable system may be raised until the conical portion shown above the pointer comes against the guide-stop. The suspension is then quite slack, and the instrument is capable of standing a considerable amount of ill usage without damage. The perforated damping-vane fits into a glass vessel filled with oil, the oil being at a sufficiently high level to completely cover the vane.

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* In the case, however, of ammeters intended for the measurement of large currents, the hot wire is divided into two (or more) sections connected in parallel, by means of a very thin strip (or strips) of silver foil attached to a point (or points) in the wire. The ends of the wire are then at the same potential, and need not be insulated from each other.

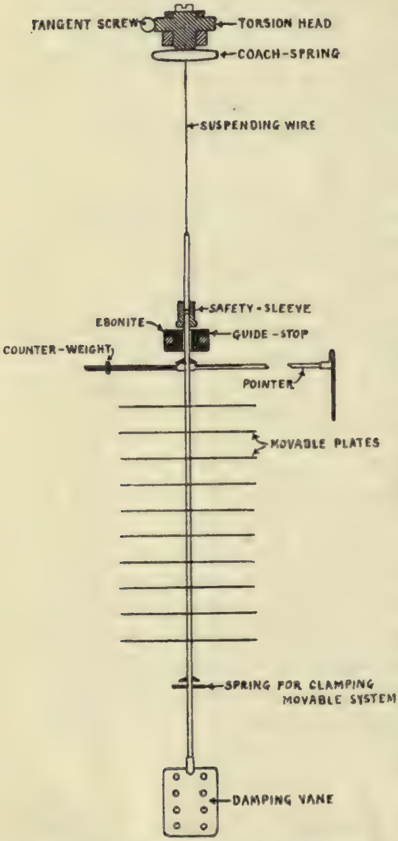

Fig. 43.—**Multicellular Voltmeter.**

The movable vanes are in connection with one terminal of the instrument. The other terminal is connected to the two sets of fixed plates, one set on each side of the movable plates, as shown in Fig. 44. The fixed plates are not shown in Fig. 43 for the sake of simplicity. Each set of fixed plates contains one more plate than the movable system, there being a fixed plate above each half of the uppermost moving plate, and a fixed plate below each half of the lowermost moving plate.

In the switchboard type of instrument the end of the pointer, bent down as shown in Fig. 43, is arranged to move over a cylindrical scale whose axis coincides with that of the moving system.

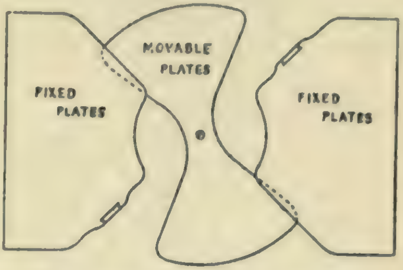

FIG. 44.—Multicellular Voltmeter.

Multicellular voltmeters of this type are constructed to read up to 600 volts. For higher voltages a modified form of the instrument, known as the "dial pattern," and illustrated in Fig. 45, is used. In this, there are only two movable plates, the axis of rotation is horizontal, and knife-edge supports take the place of the suspension used in the former or "vertical scale" pattern. The controlling couple is furnished by gravity, and oil damping is used. Instruments of this kind are made to read up to 8500 volts.*

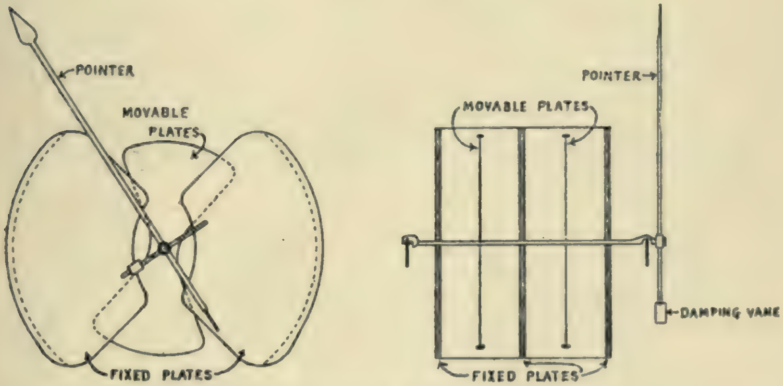

FIG. 45.—Dial Pattern of Multicellular Voltmeter.

A somewhat different type of instrument is the Ayrton and Mather electrostatic voltmeter, the working parts of which are shown in Fig. 46. The plates, both fixed and movable, are cylindrical, and electromagnetic damping is employed. When a p.d. is established between the movable and fixed portions, the former are sucked into the spaces between the latter. The controlling couple is furnished by gravity, but in instruments for low voltages spring control is used. The main feature of the instrument is the cylindrical shape of the plates, as a result of which the stress at every point of the movable plate acts at the same radius, so that for a given area of plate a much larger deflecting couple is obtained than with a flat-plate instrument, in which the stress acts at varying radii, and so gives rise to a much smaller couple.

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* In instruments of the multicellular type, the moving vanes are, in their zero position, displaced from the position of symmetry; otherwise they would be in a position of instability, and the direction of displacement would be indefinite, being determined by the accidental displacement, at the instant when the p.d. is applied, of the movable vanes from the exact position of symmetry.

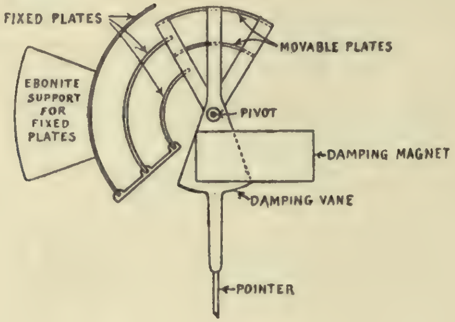

Fig. 46.—Ayrton-Mather Voltmeter.

§ 34. Methods of extending Range of Low-reading Electrostatic Voltmeter

It is frequently convenient to be able to extend the comparatively narrow range of an electrostatic voltmeter, and various methods of doing this are available. The most satisfactory of these is the subdivision of the unknown p.d. by means of a resistance. Such resistances, known as "multipliers," are supplied by the makers of electrostatic voltmeters, and by means of them the range of an instrument may be greatly extended.

This extension may be carried on indefinitely, as it is simply a matter of providing suitable resistances. But for very high voltages the cost of the resistance becomes prohibitive, and the resistance itself becomes bulky. Other methods of adapting a low-reading voltmeter to the measurement of high voltages have therefore been used. By means of a transformer, the unknown p.d. may be transformed down to a fraction of its original value, and this fraction may be calculated with fair accuracy (provided the transformer is of suitable design) from the number of turns in the primary and secondary coils (§ 49). This method is also limited by the fact that a suitably designed transformer for this purpose is both bulky and costly when the voltages to be dealt with are very high. Another method, originally suggested by Ayrton, subsequently by Peukert, and recently reinvented by Marchant and Worrall, consists in subdividing the p.d. by means of condensers. This method has been used to some extent, and is cheaper than the other two. It is, however, by no means free from difficulties, chief among which are leakage and variation of capacity with p.d. or frequency, or variation of capacity due to accidental external circumstances capable of affecting the capacity.

Any one of the methods described may be employed satisfactorily so long as the voltage is not excessive. But for very high voltages it is better to use specially constructed instruments. A good deal of attention has recently been devoted to the construction of such instruments, and in what follows we shall briefly describe a few typical forms.

§ 35. High-voltage Electrostatic Instruments

In Fig. 47 is shown Lord Kelvin's *Volt Balance*. This instrument is portable, and is suitable for pressures up to 30,000 volts (r.m.s.). The fixed plate is insulated by a thick disc constructed of mica. Above it is suspended an aluminium pan, which forms the movable system. The vibrations are damped by a thin aluminium vane arranged to swing between the poles of a damping magnet as shown in the figure. The bulk of the dielectric between the fixed and movable portions of the instrument is air.

Fig. 48 shows an electrostatic voltmeter designed by Prof. A. Grau,* and capable of reading up to 100,000 volts. The general construction of the instrument will be easily understood from the sketch. C is a fixed sheet-metal cylinder supported by a glass plate. The movable system consists of a hollow metal cylinder B provided with hemispherical ends, and weighted (so as to maintain it accurately vertical) by means of a lead weight L. From this weight there projects downwards a glass guide-rod which passes between three guide-pulleys (only one of which is shown) arranged at angles of 120°. Between B and L is interposed a damping vane of mica. The cylinder B carries a glass tube GT, through which passes a suspending band. This is taken round a pulley, and supports a counterweight. The glass vessel containing the fixed and movable systems is completely filled with resin oil, which provides the dielectric strength necessary to prevent a disruptive discharge at these high voltages.

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* *Elektrotechnische Zeitschrift*, vol. xxvi. p. 269 (1905).

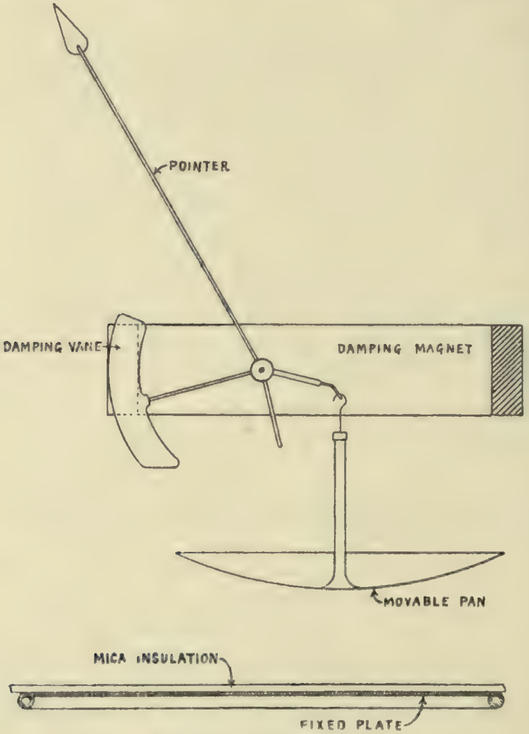

Fig. 47.—Volt Balance.

In Fig. 49 is shown a further recent example of an instrument intended to measure very high voltages. As in the case of the instrument just described, the working parts are completely immersed in oil. The instrument is constructed by the Westinghouse Co., and has recently been described by Kintner.* The movable system consists of two hollow cylinders with hemispherical caps attached to the ends of a light cross-arm; spring control is used. The fixed

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* *Electrical World and Engineer*, vol. xlv, p. 1176 (1905).

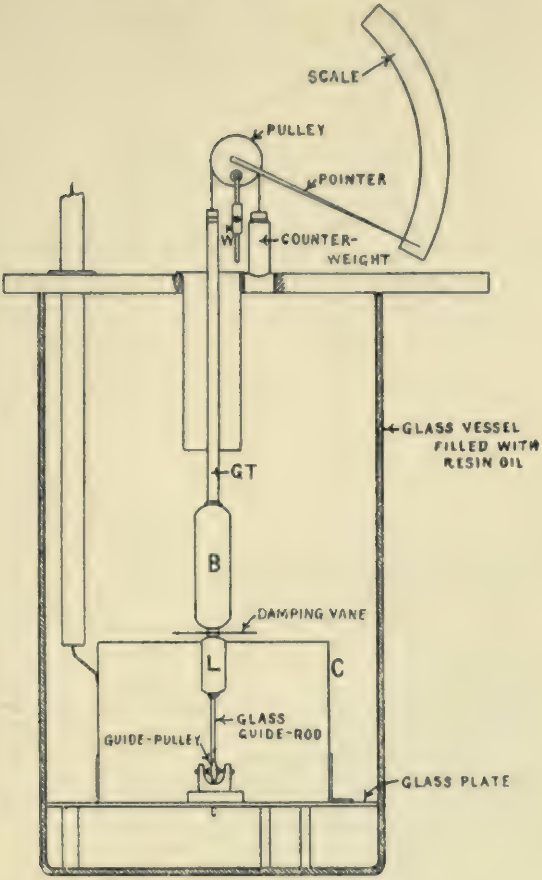

FIG. 48.—Grau's Voltmeter.

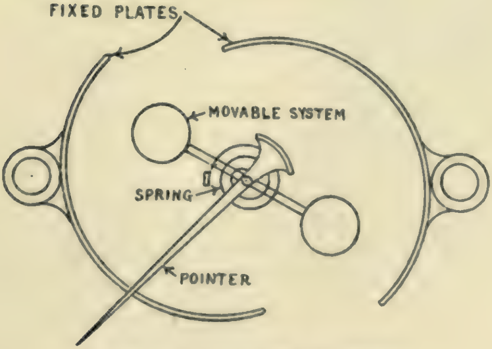

FIG. 49.—Westinghouse Voltmeter.

plates are shaped so that as the movable system is rotated the distance between the cylinders and the fixed plates decreases. Owing to the buoyancy of the movable system due to the oil immersion, the frictional error is said to be very small.

§ 36. Wattmeters

Owing to the inconvenience of having to make an adjustment before taking a reading, the zero type of dynamometer-wattmeter has practically disappeared (except for laboratory purposes), all commercial types of wattmeter being direct-reading instruments. The usual arrangement of fixed and movable coils is shown in Fig. 50. The current is led into and out of the movable coil by means of two hair-springs of phosphor-bronze, which also furnish the controlling couple.

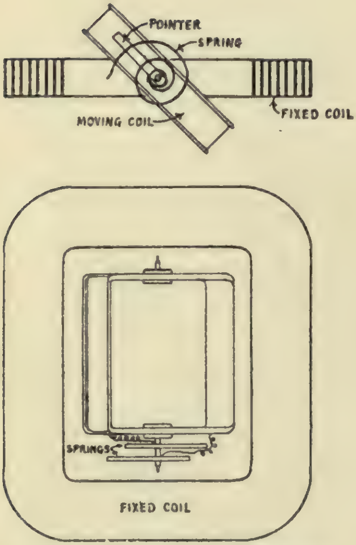

Fig. 50.—Indicating Wattmeter.

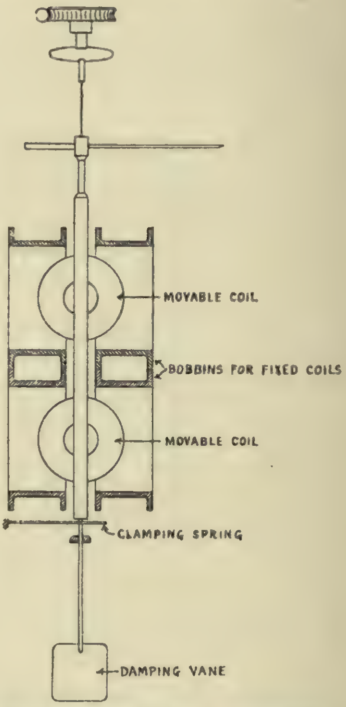

Fig. 51.—Kelvin Astatic Wattmeter.

A wattmeter of the simple form shown in Fig. 50 is subject to the error due to stray magnetic fields, as there is no arrangement for compensating the couple due to an external field. In order to remedy this defect, Lord Kelvin has introduced an *astatic* type of wattmeter, the latest form of which is shown in Fig. 51. The movable system consists of two coils, similar in every respect, but connected up so that the current circulates round them in opposite directions. Thus the couples exerted by a uniform external field on the movable coils are in opposite directions, and balance each other. Corresponding to each movable coil there is a fixed coil (wound in two sections, one on each side of the axis of rotation). Oscillations are prevented by an oil-damping arrangement precisely similar to that employed in the multicellular type of voltmeter, and the method of suspension is also identical with that used in this latter type of instrument.

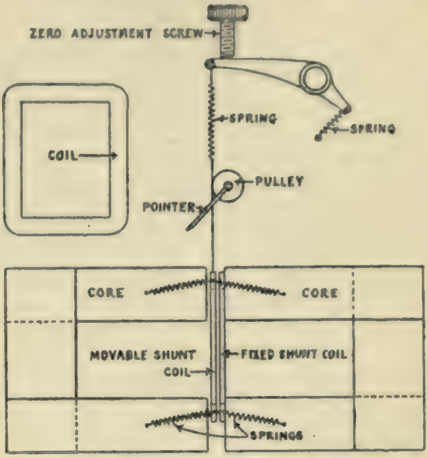

Fig. 52.—Drysdale Wattmeter.

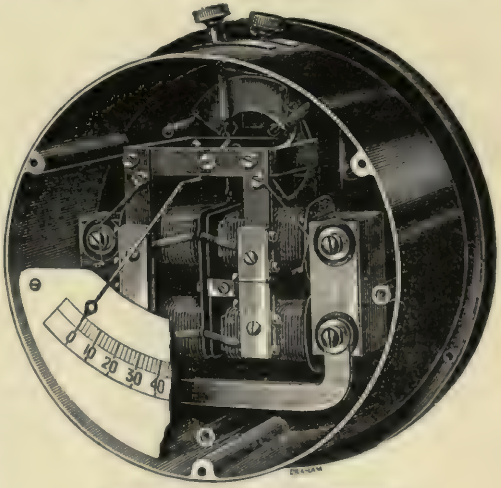

Fig. 53.—Drysdale Wattmeter.[*]

An astatic arrangement of movable coils, though rendering the reading of the instrument independent of a *uniform* external field, would still be liable to disturbance by a field of varying intensity. In general, however, it is possible to place the instrument so as to render the error due to non-uniformity of field very small.

An extremely ingenious form of wattmeter, which possesses the double advantage of being practically non-inductive and unaffected by external fields, has recently been devised by Dr. C. V. Drysdale, and is shown in Figs. 52 and 53. The general arrangement of the working parts will best be understood by reference to Fig. 52. The main or series coils are wound on laminated cores which form a nearly closed magnetic circuit, there being only two small air-gaps. In these air-gaps are arranged the two shunt coils, one of which is fixed against the ends of the right-hand core, while the other is supported by springs in the manner shown, and possesses freedom of motion up and down. The current is arranged to circulate in opposite directions around the shunt coils, so that their self-inductance is practically wiped out. Further, by reason of the very intense field due to the series coils, the instrument is not liable to be disturbed by external fields, and a still higher degree of security in this respect is obtained by enclosing the instrument in a cast-iron case. Fig. 53 shows the actual instrument with the front cover and a portion of the scale removed so as to exhibit the mechanism of the instrument.

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[*] The block for this illustration has been supplied by makers of instrument (Messrs. Nalder Bros. & Thompson, Ltd.).

§ 37. Power Factor Indicators

The power factor (§ 7) can always be calculated from the readings of a voltmeter, ammeter, and wattmeter. Instruments have, however, been devised for indicating directly the value of the power factor. Such instruments are known as *power factor indicators.* In Fig. 54 is illustrated the general arrangement of a type of power factor indicator devised by Punga, and manufactured by Messrs. Everett, Edgcumbe & Co. It is intended for use on a three-phase circuit, and consists essentially of two systems of coils, a fixed system of current coils and a movable system of pressure coils, the coils of each system forming angles of 120° with each other. Currents are led into and out of the movable system by means of very weak strips of phosphor-bronze, which exert no appreciable controlling couple. Each system gives rise to a rotating magnetic field when traversed by three-phase currents (§ 15). So long as the two fields coincide, no torque will be exerted on the movable system. But any change in the relative phase of the currents will cause a displacement of the

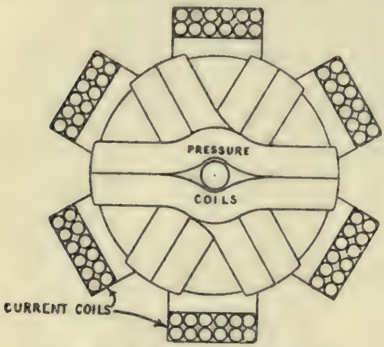

Fig. 54.—Power Factor Indicator.

fields, giving rise to a torque which will rotate the movable system through an angle sufficient to restore coincidence of fields. Corresponding, therefore, to each value of the phase difference between the currents in the two systems, there is a perfectly definite position of equilibrium for the movable system. The value of the phase difference ϕ, or of the corresponding power factor cos ϕ, is read off on a scale over which moves a pointer attached to the movable system. The two systems of coils are connected up like the circuits of three wattmeters, the current coils being placed in series with the three line wires, and the three star-connected pressure coils being joined in series with three large non-inductive resistances, and then connected to the line wires. If the three-phase circuit on which the instrument is used is unbalanced, the reading corresponds to the mean value of the power factor for the three phases.

§ 38. Oscillographs

In many cases, it is important to know the exact shape of the p.d. or current wave. Numerous methods of determining the wave-shape are available, but by far the quickest and most convenient is that involving the use of a special instrument devised for this purpose, and known as an oscillograph. We shall select for description a form of oscillograph due to Blondel.

Every oscillograph is in reality a galvanometer whose moving system is capable of extremely rapid vibration, and which is fitted with a suitable arrangement for recording the vibrations. This arrangement is shown diagrammatically in Fig. 55, and is, except as regards details of construction, the same in all forms of oscillograph. Light coming from an arc passes through a cylindrical lens, and falls on the tiny mirror attached to the moving part of the oscillograph galvanometer. The vibrations of the mirror cause the reflected beam

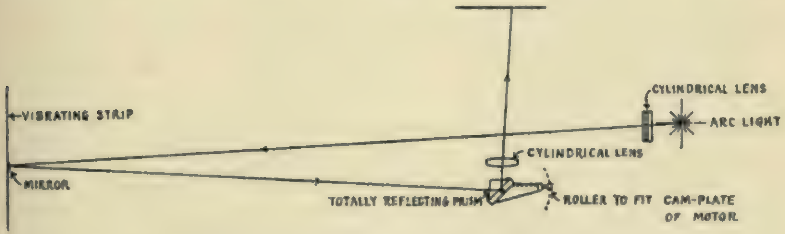

Fig. 55.—Arrangement for recording Vibrations of Oscillograph.

to sway in a horizontal plane. This reflected beam is received by a totally reflecting prism (or by a mirror), which is kept oscillating about a horizontal axis by means of a synchronous motor (§ 68) fitted with a suitable cam-plate, which engages the roller mounted on the tail-piece of the prism. The shape of the cam-plate is such that while the prism is receiving the reflected beam its angular velocity is constant, so that the displacement of the reflected beam in a vertical plane is directly proportional to time. When the prism reaches its maximum displacement, a sector mounted on the motor axle cuts off the light, and during this time the prism executes a quick-return motion, to resume its constant angular velocity in a forward direction as the sector passes out of the path of the beam.

On leaving the prism or mirror, the light is rendered convergent by a second cylindrical lens, and forms a tiny speck of light on the ground-glass screen which is provided to receive it. This speck is subject to two displacements at right angles to each other, one being due to the vibrations of the oscillograph galvanometer, and the other to the steady forward displacement of the prism (during the return motion, the light is cut off as already explained). Thus the curve traced out on the screen shows the wave-shape at a glance. If it is desired to obtain a permanent record of the wave-shape, a photographic dark slide is substituted for the screen, and an exposure made in the ordinary way, by means of a shutter placed behind the first cylindrical lens.

Fig. 56 shows the form of cam-plate used in the Blondel oscillograph. The axle of the synchronous motor carries a gun-metal disc, and screwed to this are the two portions of the hardened steel cam-plate. It is important to use hardened steel for both the cam-plate and the roller at the end of the prism tail-piece in order to prevent a distortion of the curve due to wear of these parts.

The controlling field for the vibrating portion of the oscillograph galvanometer is provided by means of a powerful permanent magnet, shown in Fig. 57. This magnet is fitted with laminated tapering pole-pieces and core, which are separated from each other by two air-gaps, in which are placed the vibrating portions of the instrument. A *single* gap is quite sufficient if only one wave at a time is required. But frequently it is desirable to obtain simultaneously the records of *two* waves (such as a p.d. and a current wave), so as to determine their phase relation. Hence most oscillographs are made double.

In each gap is placed an extremely thin and narrow vertical band or strip of soft-iron, with its width along the lines of force—a position which the band would naturally take up if free to rotate about a vertical axis. The tension of the band may be adjusted to the desired amount, and so its natural rate of torsional vibration about its axis controlled. This rate of vibration may correspond to a frequency as high as 40,000 per sec., so that the band is easily able to follow the comparatively slow vibrations due to the current whose wave-shape is required.

The powerful permanent magnet shown in Fig. 57 corresponds to the controlling magnet of an ordinary needle galvanometer, the soft-iron band corresponds to the needle or needles, and the deflecting coil is represented by two small coils (not shown in Fig. 57), one on each side of the gap, the common axis of the coils being normal to the field of the magnet (*i.e.* normal to the plane of the paper in Fig. 57). Each gap is, of course, fitted with its own coils. If a current is sent through either set of coils, the field of the corresponding gap is distorted, the band tends to take up a position with its width along the resultant field, and is thus twisted. The angle of twist is greatest at the middle point of the band, and it is to this point that the tiny mirror is attached.

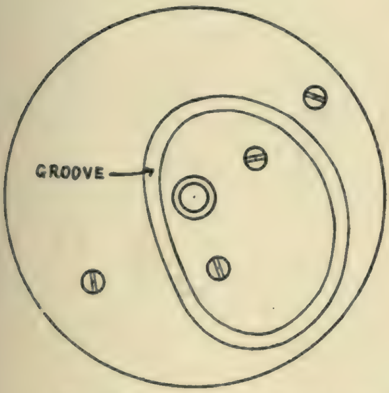

FIG. 56.—Cam-plate of Synchronous Motor.

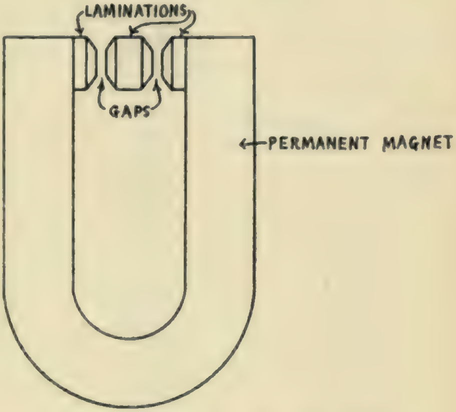

FIG. 57.—Magnet of Blondel Oscillograph.

Fig. 58 shows the details of construction of the frame which supports the vibrating band. This frame is made of German-silver in order to avoid excessive eddy currents. On each side of the vibrating band is a tapering soft-iron pole-piece, an arrangement which gives a very intense field. The methods of fixing the ends of the strip and of adjusting its tension are clearly shown in the figure.

The entire lower portion of the frame supporting the band fits into a glass tube filled, or partially filled, with pure castor-oil. This provides the necessary damping, so that the band merely responds to the forced vibrations impressed upon it by the current, but is practically free from superposed natural vibrations (which would cause ripples in the curve). In order to prevent distortion of the luminous speck formed on the screen, the glass tube is fitted with a small lens, through which the light enters and leaves the tube. The tubes containing the bands slip into the spaces between the laminated pole-pieces and central core of the controlling magnet shown in Fig. 57.

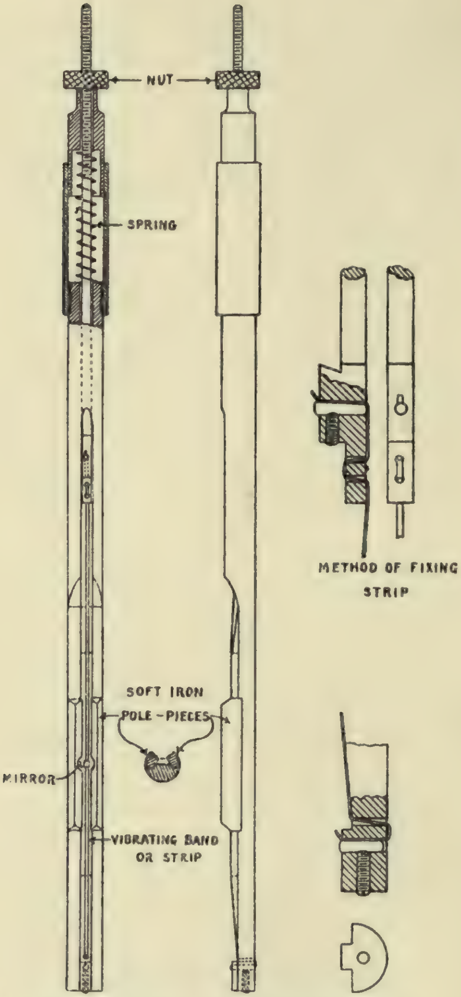

Fig. 58.—Details of Blondel Oscillograph.

Besides the oscillograph just described, Blondel devised another type, which has a moving-coil galvanometer. The development of this type in England we owe to Mr. Duddell, and the Duddell oscillograph is the best-known form of the instrument in this country. A powerful electromagnet is used, between whose pole-pieces are placed, in the narrow air-gap,* two parallel strips of phosphor-bronze whose tension may be adjusted. The strips are stretched over bridge-pieces, the plane containing them being along the field. Across the two strips is fixed a small mirror. The current whose wave-form is required passes up one strip and down the other. As a result, the strips move across the field in opposite directions, thereby deflecting the mirror. It is to be noted that here the strips vibrate bodily across the field, like the strings of a violin, while in the particular Blondel instrument which we have described more in detail, there is no bodily vibration of the soft-iron strip, but simply a torsional vibration, the axis of the strip retaining a fixed position in space.

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* Two air-gaps are provided in the double oscillograph.

CHAPTER VI

§ **39**. Dynamo used as alternator—§ **40**. Constructional differences between dynamos and alternators—§ **41**. Standard type of alternator—§ **42**. Construction of turbine-driven alternator—§ **43**. Armature windings of alternators—§ **44**. Calculation of armature e.m.f. Effect of varying length of polar arc—§ **45**. Effect of varying number of slots—§ **46**. General formula for e.m.f. of alternator—§ **47**. Armature reaction in alternators—§ **48**. Walker compounded alternator.

§ 39. Dynamo used as Alternator

ALTHOUGH the machines employed for the generation of alternating currents differ somewhat in details of construction from continuous-current dynamos, the principle of action is the same—the motion of a conductor across a magnetic field—and, in fact, any continuous-current generator may be easily modified so as to act as an alternator. The only addition necessary is the fitting of a suitable number of insulated contact-rings or "slip-rings" which are in *permanent* connection with certain points in the winding. By means of brushes making contact with the slip-rings, alternating currents may be supplied to an external circuit. If the commutator is retained, then the same machine may be made to supply at the same time both continuous and alternating currents, the continuous current being obtained as usual from the commutator, and the alternating currents from the slip-rings. Some very large generators of this kind are in use in the United States, and are known as *double-current generators.*

The construction of the typical modern alternator is, however (for reasons which will be explained presently), different from that of the typical modern continuous-current dynamo. We shall commence our study of alternators by considering the action of a dynamo as an alternator when fitted with slip-rings as explained.

In the diagrams of Fig. 59 is shown a two-pole armature fitted with two slip-rings, diagram (*a*), connected to two diametrically opposite points, C and D, of the winding. For the sake of simplicity, the winding itself is indicated by a simple circle. Let the arrangement of the winding and the direction of rotation be such that e.m.f.'s are induced in the two halves ACB and ADB of the armature having a direction from A to B inside the winding, as indicated by the arrows, and hence producing a p.d. from B to A across an external circuit connected to these two points. In the diagrams, the points B and A are supposed to be fixed *in space*, and to correspond to the position of the brushes of a continuous-current machine. The two diametrically opposite points C and D are, on the other hand, points fixed *in the winding*, and are carried round in space, as shown in the consecutive diagrams, during the rotation of the armature. We shall suppose C and D to be in permanent connection with two slip-rings provided with brushes as shown in Fig. 59 (*a*),* by means of which permanent contact is maintained between the terminals of an external circuit and the points C and D in the winding. In the position of the armature corresponding to (*a*), it is clear that there will be zero p.d. between C and D, since the clockwise e.m.f. induced in the quadrant AC of the winding is exactly balanced by the counter-clockwise e.m.f. induced in the quadrant AD; similarly, the e.m.f. in DB balances that in CB. When the armature reaches the position shown in diagram (*b*), the e.m.f.'s in the sections AC and DB of the winding overpower those in the smaller sections AD and BC respectively, so that there is now a p.d. acting from C to D. Since the first two sections (AC and DB) steadily increase in length, while the last two (AD and BC) steadily decrease, the p.d. acting from C to D will increase until, as shown in diagram (*c*), the points C and D coincide with B and A respectively—when it reaches its maximum value. Beyond this position, the p.d. will steadily *decrease*, since, as is evident from diagram (*d*), the e.m.f.'s in the sections AC and DB are now decreasing, while those in CB and AD are increasing.

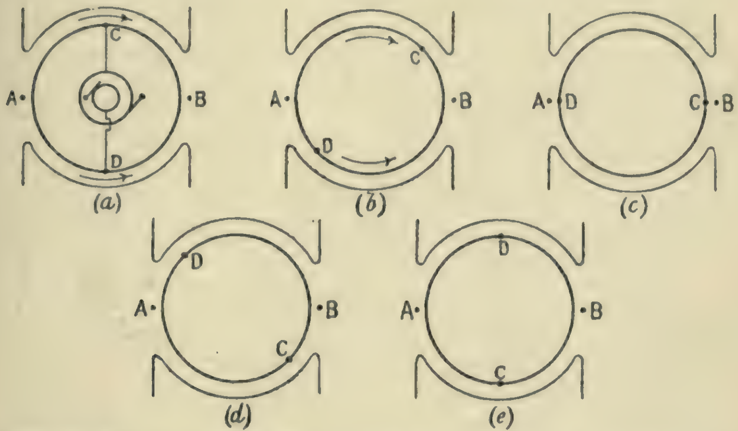

FIG. 59.—To illustrate Action of Continuous-current Armature when generating Single-phase Current.

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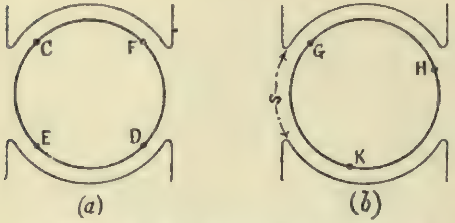
* In order to simplify the figures as much as possible, the slip-rings are omitted from the diagrams (*b*) to (*e*).

The p.d. reaches a zero value in the position shown in diagram (*e*), the points C and D having now come halfway between A and B. It is evident that since (*a*) differs from (*e*) only in that C and D have now changed places, a similar series of changes to that just considered will take place in the p.d. across CD during the next half-revolution, the p.d. being now, however, directed from D to C instead of from C to D as during the first half-revolution. We thus get an *alternating p.d.* between the slip-rings connected to the diametrically opposite points C and D in the winding. Such an arrangement would, therefore, constitute a *single-phase generator*, or *single-phaser*.

It is clear that by selecting two pairs of diametrically opposite points, such as C, D and E, F in Fig. 60 (*a*), so arranged that the diameter CD is perpendicular to the diameter EF, we get, during the rotation of the armature, an alternating p.d. between C and D which differs 90° in phase from the alternating p.d. between E and F, the p.d.'s being necessarily of the same frequency. By using *four* slip-rings connected to the points C, D and E, F, and connecting one external circuit to C, D, and another to E, F, we obtain a *two-phase system*. The machine now becomes a *two-phase generator*, or *two-phaser*.



(*a*) (*b*)

FIG. 60.—Slip-ring Connections for Two- and Three-phase Currents.

Lastly, by taking three points, G, H, and K, in the winding, 120° apart, as in Fig. 60 (*b*), and attaching them to three slip-rings, we get, between any pair of slip-rings, an alternating p.d. which differs by 120° in phase from the p.d.'s between the two remaining pairs of rings. Three conductors connected to the slip-rings would thus form the mains of a *three-phase system*, the machine now acting as a *three-phase generator*, or *three-phaser*. The arrangement of the armature winding in this case would, it may be noted, correspond to a delta or mesh connection (§ 16).

It will now be evident that, by the simple addition of a suitable number of slip-rings, any ordinary continuous-current dynamo may be converted into an alternator. Such a mode of construction is used in practice when either (1) the same machine is required to generate both continuous and alternating current, the former being obtained from the commutator and the latter from the slip-rings; or (2) when a transformation from alternating to continuous current, or *vice versâ*, has to be effected. In the first case, the machine forms a *double-current generator*; in the second, a *rotary converter*.

§ 40. Constructional Differences between Dynamos and Alternators

If, however, we want a machine which is only required to generate alternating current, the above arrangement admits of considerable improvement. A glance at the diagrams of Fig. 59 will show that with a uniformly distributed winding such as is used in continuous-current armatures, we have *opposition of e.m.f.'s* in the different parts of the winding. Thus, in Fig. 59 (*b*) the e.m.f. in the section AC of the winding is opposed by the e.m.f. in the section AD ; it is thus evident that in this particular position of the armature the conductors forming the section AD are not only useless, but positively harmful in (1) unnecessarily adding to the resistance and reactance of the armature, and (2) reducing its e.m.f. Such opposition of e.m.f.'s in different parts of the winding is conveniently spoken of as *differential action.* In order to eliminate differential action, the winding must be arranged so that all the conductors forming a group of the winding between two contact-rings shall always be moving simultaneously across a field of the same polarity. In order to secure this result, the width of a band of conductors forming such a group must evidently not exceed the width of the interpolar space—*i.e.* the distance between the neighbouring polar horns of two consecutive pole-pieces (*s* in Fig. 60 (*b*)). Now, in continuous-current machines the interpolar arc is only about 30 per cent. of the pole-pitch (the pole-pitch is the distance between the middle points of two consecutive pole-pieces, which corresponds to half the armature circumference in a two-pole machine), so that if the same ratio of length of interpolar arc to pole-pitch were retained in alternators, we should, in a single-phase machine, utilize only 30 per cent. of the available winding space on the surface of the armature. In order to increase the width of the available winding space without at the same time introducing any appreciable amount of differential action, it is usual to make the lengths of polar and interpolar arcs equal, so that each of them, as well as the width of a group of the winding, is equal to half the pitch of the pole-pieces. In this reduction of the polar arc, and increase of interpolar space, we have one of the characteristic constructional differences between single-phase alternators and continuous-current dynamos.

Since with the arrangement indicated only 50 per cent. of the available winding-space is utilized with a *single-phase* armature, we have, as regards the armature winding, another important constructional difference between continuous-current machines and *single-phase* alternators; the winding in the latter, instead of being uniformly distributed as it is in the former, is broken up into groups separated by spaces devoid of conductors.

If, however, our alternator is to be a *two-phaser*, we may fill up the vacant spaces on our armature with the conductors forming the second phase of the winding, thereby utilizing the entire available winding-space.

In a *three-phaser*, the pole-pitch is divided into three equal parts, and the width of each of these represents the width of a group of conductors belonging to the same phase. Here, again, the entire available winding-surface is utilized.

Since in a three-phase alternator the width of a group of conductors belonging to one phase does not exceed $\frac{1}{3}$rd of the pole-pitch, it follows that the pole-arc may be made equal to $\frac{2}{3}$rds of the pole-pitch without introducing any appreciable amount of differential action. This, accordingly, is the generally adopted value for the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ in *three-phase* alternators, and it is about the same as that commonly employed in continuous-current machines.

§ 41. Standard Type of Alternator

The standard modern type of alternator is one in which the armature is stationary, and the field revolves. The general construction of this type will be understood from Fig. 61. The armature laminations are supported by a casting which forms the "yoke-ring,"

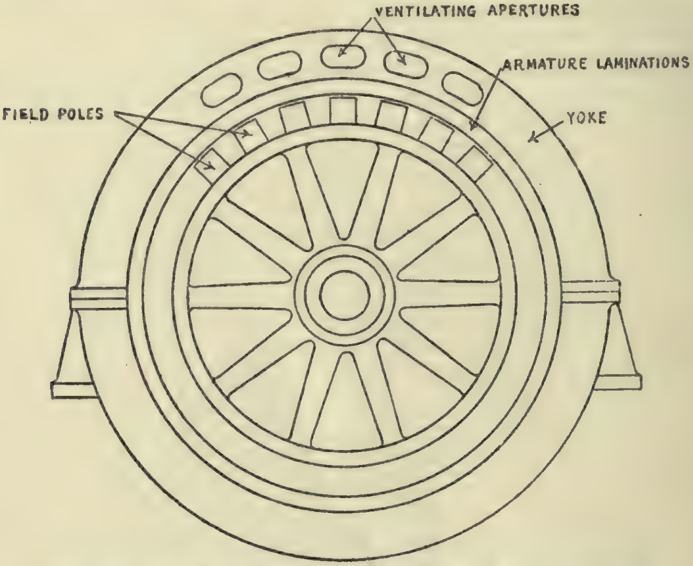


Fig. 61.—General Arrangement of Revolving Field Alternator.

and which is generally divided into two sections, an upper and a lower one, bolted together. The lower section is provided with two feet, by means of which it rests on a foundation-plate. In very large machines, however, the yoke-ring is made in four sections. This ring is provided with numerous ventilating apertures, only a few of which are shown in Fig. 61. The revolving field consists of a fly-wheel or spider, supporting a ring to which are bolted the field poles. In large machines the ring is a separate structure, bolted to the spider arms; or the boss of the spider is split, and after having been bored out to fit the shaft, is strengthened by two steel rings, which are shrunk on. Either of these expedients ensures freedom from any dangerous strains which may be set up in the arms of a large wheel during cooling when cast in one piece.

In order to render the armature ring more rigid, the laminations are arranged to overlap or break joint, as shown in Fig. 62, where the full radial lines indicate the laminations of one layer, and the dotted lines those of the next. The thickness of the core-plates or laminations is from 0·014 inch to 0·018 inch.

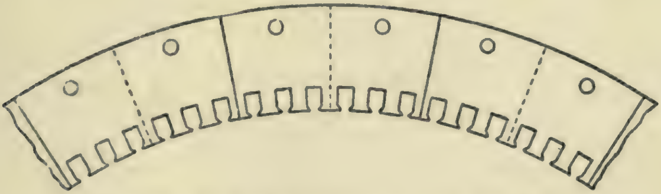


Fig. 62.—Method of building up Armature Core.

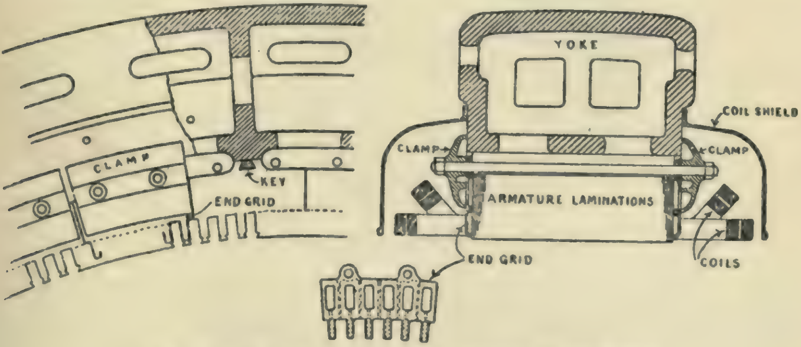


Fig. 63.—Method of supporting Armature Laminations.

There are various methods of supporting the laminations from the yoke-ring. One of these is illustrated in Fig. 63. As will be seen, the laminations are held together by means of bolts between two sets of clamps, end grids being interposed between the clamps and plates. These end grids are provided with teeth, which fit over the teeth of the plates or laminations, and prevent them from spreading outwards. The core-plates are, in the design of Fig. 63, provided with key-way projections, which fit over keys screwed to the yoke-ring. The end grids serve to ventilate the core, and additional ventilation is secured by providing gaps, or ventilating ducts (see Fig. 65), in the core, these gaps being formed by the interposition of ventilating space-blocks. One of the simplest forms of space-block is shown in Fig. 64. It consists of a steel plate, about $\frac{1}{32}$ inch thick, shaped like one of the core-plates, and having semicircular pieces cut and bent at right angles to the plane of the plate. These support bent radial steel strips as shown, which act as distance-pieces between the core-plates. The width of ventilating duct varies from about $\frac{1}{3}$ inch to $\frac{3}{4}$ inch. It is usual to provide a ventilating duct to every 5 inches or 6 inches of core length.

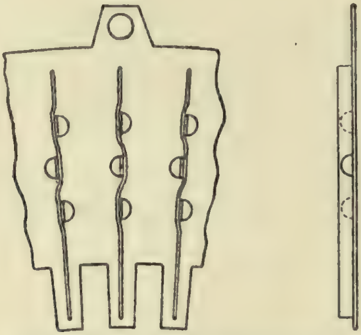


Fig. 64.—Ventilating Space-block.

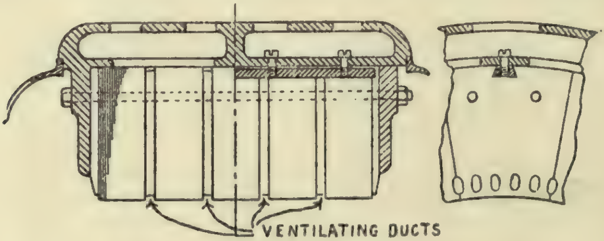


Fig. 65.—Armature Core Construction.

Another mode of supporting the core-plates is shown in Fig. 65. Instead of having clamps on both sides, there is a ring cast on to the yoke, which acts as a clamping-ring. The core-plates are further secured by means of keys screwed to the yoke casting as shown.

In Fig. 66 is shown a similar construction for a very narrow core, in which case the keys may be dispensed with.

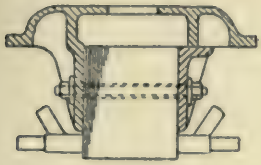


Fig. 66.—Type of Construction for Narrow Armature.

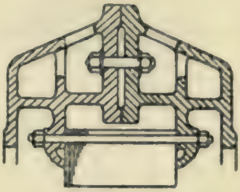


Fig. 67.—Yoke Construction.

A somewhat different design is illustrated in Fig. 67. Here the yoke-ring itself is divided by a plane normal to the shaft, the two halves of the ring forming clamps for the core-plates.

Three methods of securing the field-poles to the rim of the field-wheel are shown in Figs. 68, 69, and 70. In Fig. 68, each pole is secured by two bolts passing through the rim, while in Fig. 69 additional safety is obtained by providing the rim with a clamping projection on one side and the field-pole with a similar projection on the other. For very high peripheral velocities, the field-poles may be dove-tailed into the rim as in Fig. 70, and secured by means of wedges.

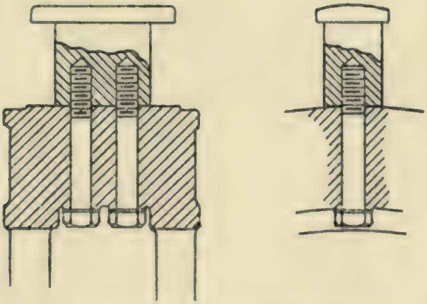
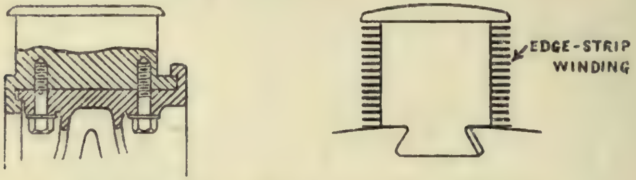


Fig. 68.—Field-pole Construction.

Owing to the high peripheral velocities (7000 feet per min.) common in alternators, ordinary wire winding is inadmissible for the field-poles, as the wires would roll over each other. For this reason, the usual form of field winding consists of thin strips of copper wound on edge in one or two layers. This kind of winding is known as *edge-strip winding*, and is shown diagrammatically in Fig. 70. Besides being mechanically strong, it possesses the further advantage of rapidly conducting the heat to the outside of the coil, and so maintaining it at a uniform temperature, thereby preventing excessive rises of temperature in the interior of the coil.



Figs. 69 and 70.—Methods of supporting Field-poles.

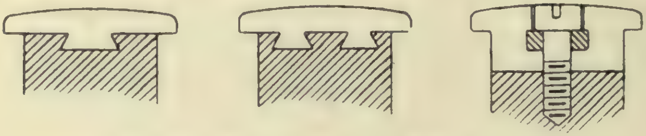


Fig. 71.—Types of Laminated Pole-shoes.

In order to prevent excessive eddy-current loss in the ends of the pole-pieces, these are frequently laminated. In Fig. 71 are shown various ways of securing the laminated pole-shoes to the field-cores.

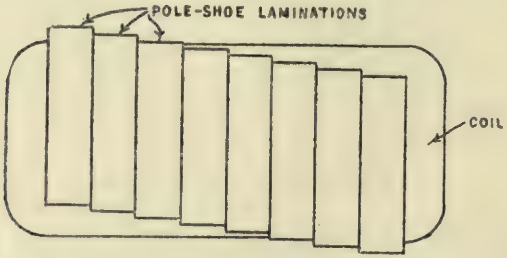


Fig. 72.—Showing construction of Skewed Pole-shoe.

For the purpose of securing a smooth wave-shape forming a close approximation to a sine wave, the Oerlikon Co. employs laminated pole-shoes, consisting of a number of sets of core-plates displaced relatively to each other, or "staggered," so as to form a series of steps, as shown in Fig. 72.

§ 42. Construction of Turbine-driven Alternator

Steam turbine-driven alternators present many constructional difficulties. Owing to their high speed,* the number of field-poles for the usual frequencies (25, 30, 50, and 60) is small, and as a result each field-pole is relatively very large, and the difficulty of securing it satisfactorily correspondingly great. Where the speed is moderately high, the poles may be dove-tailed into the field-ring as in Fig. 70. But for very high speeds special forms of construction must be used.

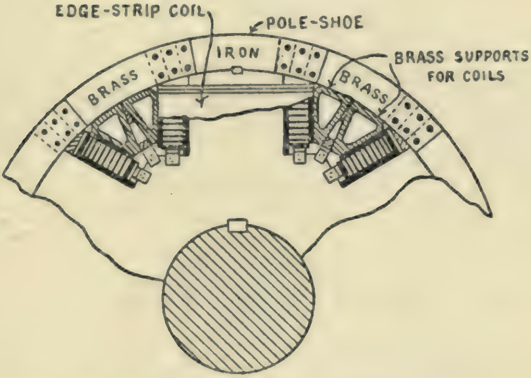


FIG. 73.—Field Construction for Turbo-generator.

One of these, used by the Oerlikon Co., is shown in Fig. 73. The entire field structure is laminated, the core-plates being directly threaded on the shaft. The edge-strip winding is prevented from bulging sideways by means of wedge-shaped brass supporting frames. In order to prevent noise—always a troublesome feature in turbine-driven machines—the field is surrounded by a laminated ring, built up of alternate segments of brass and iron made to overlap and riveted together, the iron portions of the ring forming the pole-shoes. This ring presents a perfectly smooth outer surface, which is conducive to silent running.

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* The peripheral velocity may be as high as 18,000 feet per min.

§ 43. Armature Windings of Alternators

As regards the armature winding, by far the greatest number of alternators are provided with six slots per pole (though sometimes as many as twelve slots per pole are used). One advantage of choosing this number lies in the possibility of standardizing the core-plates, so that the same core may be used for single-, two-, or three-phase machines. If the machine is to be a single-phaser, only three or four slots per pole would be utilized for the armature winding, the remaining slots being left empty. These vacant slots tend to improve the ventilation of the machine, and also offer the further advantage that a supplementary winding may, if necessary, be placed in them, thus enabling the machine to supply a small two-phase load in addition to the main single-phase load.*

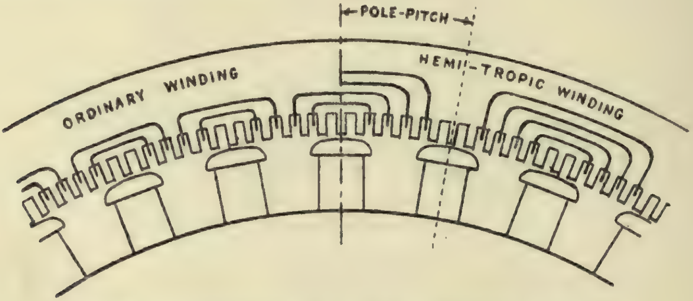

Fig. 74.—Illustrating Two Types of Armature Winding.

If we agree to use four out of the six slots per pole for a single-phase winding, then there are two ways of arranging the armature coils. These two methods are shown in Fig. 74. The first method, which we may describe as the ordinary method, consists in using *one coil per pole*, each coil being distributed over four slots. This method is illustrated by the left-hand side of Fig. 74. The second method, for which Prof. S. P. Thompson has suggested the term *hemi-tropic winding*, consists in using one coil per *pair* of poles. Each coil is now embedded in eight slots, as shown by the right-hand side of

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* If a single-phase generating station is required to supply current to a short tramway line, the transformation from single-phase to continuous current may be effected by means of a synchronous motor-generator set. A single-phase rotary converter, on account of its unsatisfactory performance, would not be used. But two-phase converters have been used for this purpose, the alternating currents corresponding to the second phase being obtained from the supplementary winding in one or more of the main generators, the winding being placed in some of the unused slots.

Fig. 74. If we consider only a single-phase winding, then it is immediately evident from Fig. 74 that the hemi-tropic winding, by reason of the greater mean length of turn, requires more copper and leads to a higher armature resistance than the ordinary winding. On the other hand, it offers somewhat greater flexibility in the matter of distributing the winding. Thus with the six slots per pole which we have assumed, each half or side of a coil must be embedded in either one or two slots if the ordinary winding is employed; whereas, with a hemi-tropic winding, each side of a coil may occupy one, two, three, or four slots.

Considering next a two-phase armature winding, and assuming as before six slots per pole, we have three slots available per pole per phase. The winding must therefore necessarily be hemi-tropic, as shown in Fig. 75, in which the winding of one phase is shown by the full lines, and that of the other by the dotted lines. The positions of the field-poles are indicated by the letters N and S. Owing to the overlapping of the coils, the coil-ends of one phase are brought out straight, and lie on a cylindrical surface (as in the typical modern continuous-current armature with a barrel or "straight-out" winding), while those of the other are bent up so as to lie on a conical surface. The bent-up ends of the armature coils are shown in Figs. 63 and 66, and are still more clearly visible in Fig. 77.

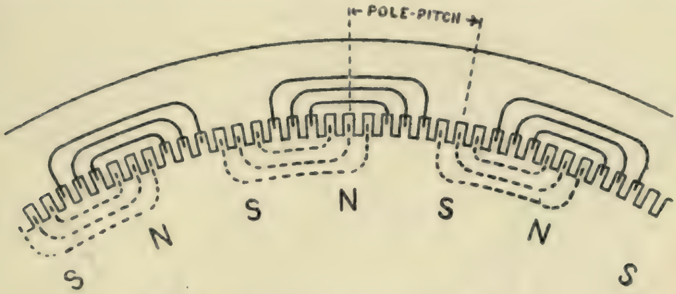

Fig. 75.—Two-phase Armature Winding.

If the winding is to be a three-phase one, we have, assuming as before six slots per pole, two slots available per pole per phase. The winding might therefore be either of the ordinary or the hemi-tropic type. The adoption of the ordinary winding (with one-half of a coil in a single slot) would, however, necessitate a somewhat complicated arrangement of coil-ends, as these would have to be arranged so as to lie on three different surfaces, a different surface corresponding to each phase. By adopting the hemi-tropic type of winding, shown in Fig. 76, this difficulty is overcome, since the coil-ends may now be arranged to lie on two surfaces only (as with the two-phase winding), the coils being alternately straight and bent-up. The three phases in Fig. 76 are distinguished by full, dotted, and chain-dotted lines. It will be noticed that the coils belonging to any one phase are alternately straight and bent.

In the two-phase winding, the distance apart of two neighbouring coils belonging to different phases amounts to ½ the pole-pitch (corresponding to a phase difference of $\frac{\pi}{2}$, or 90°), and in the three-phase winding to ⅓ the pole-pitch. Now, a distance of ⅓ the pole-pitch corresponds to a phase difference of only 60°. Thus, if we consider three consecutive coils, A, B, and C, there will be a phase difference of 60° between the e.m.f.'s in A and B, and B and C, and a phase difference of 120° between the e.m.f.'s in A and C. But by reversing the connections of B we alter the phase of its e.m.f. relatively to the other two by 180°, and so obtain three e.m.f.'s differing 120° in phase, as required for a three-phase system.

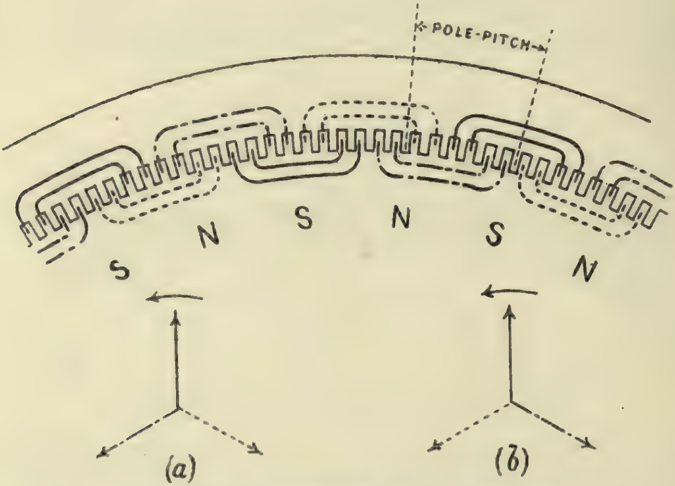

Fig. 76.—Three-phase Armature Winding.

A little consideration will show that if the direction of rotation of the field in Fig. 76 is counter-clockwise, or left-handed, the vector diagram of e.m.f.'s in the three phases is as shown at (*a*). If now the direction of rotation be reversed, the diagram assumes the form shown at (*b*), the dotted and chain-dotted vectors having changed places. Thus we see that a reversal of the direction of rotation of a three-phase generator is equivalent, so far as the external circuit is concerned, to interchanging the connections of two of the terminals.

From a manufacturing point of view, armature coils may be divided into two classes: former-wound and hand-wound. Former-wound coils require entirely open slots, whereas hand-wound coils may be placed in closed or semi-closed slots. Former-wound coils are cheaper, and are more easily replaced than hand-wound coils. On the other hand, in fitting a former-wound coil into position there is some risk of damaging the insulation of the coil or slots. The use of open slots results in an irregular wave-shape (unless some such expedient as skewing the pole-pieces is adopted, see Fig. 72), and necessitates, as a rule, the use of laminated pole-shoes in order to prevent excessive eddy-current loss. Owing to the fact that thoroughly reliable insulation is more easily obtained with hand-wound coils (the insulating lining of the slot in this case consisting of a seamless tube), this type would appear to be preferable in the case of generators designed for very high voltages.

The insulating lining of the slots consists of some form of mica insulation, such as micanite or "reconstructed mica," and in the case of hand-wound coils seamless tubes of the insulating material are fitted into the slots, as shown in Fig. 77. In order to prevent disruptive discharge between the coils and the frame, these tubes are carried well beyond the core-ends, as shown in the illustration. Since the end coils—*i.e.* those nearest the machine terminals—are subject to the greatest dielectric stress, it is advisable to provide these coils with additional insulation.

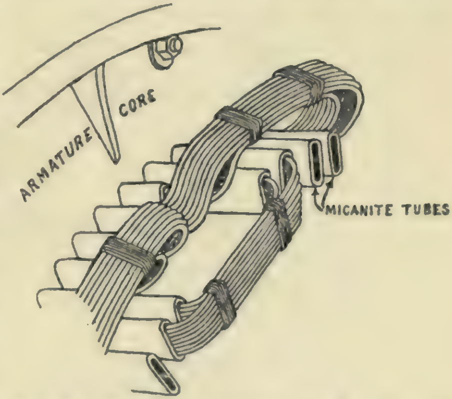

Fig. 77.—Showing Armature Coils in Position.

The actual thickness of the slot insulation depends, of course, on the voltage of the machine, and, according to Mr. H. M. Hobart,* the following table may be taken as representing modern practice—

R.m.s. voltage of alternator ...	500	1000	2000	4000	8000	12,000	16,000
Thickness of slot insulation, in mm.	1·30	1·75	2·47	3·35	4·6	5·67	6·67

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* *Electrical Review*, vol. lvi. p. 681 (1905).

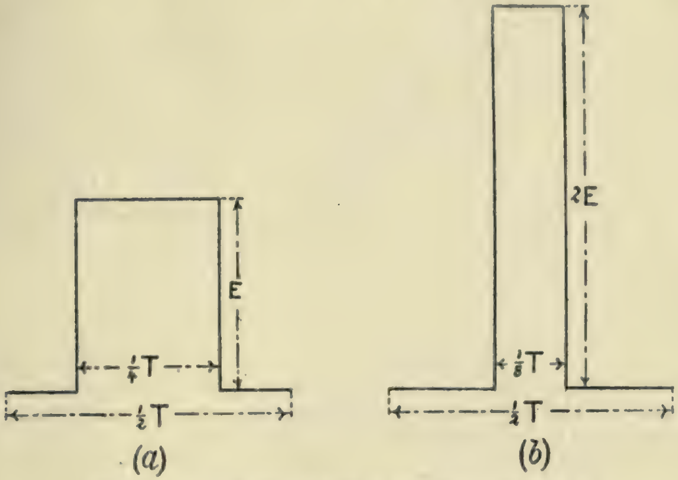
The insulation thickness in the above table is measured from copper to iron, and includes the insulation of the conductors.

Since with increasing voltage a greater thickness of insulation becomes necessary, either the size of slot must be increased—and this will involve an increase in the diameter of the machine—or an equivalent increase must be made in the length of core. High-voltage generators are therefore, for a given power and speed, larger and more expensive than low-voltage ones. Nevertheless, the extra cost of a high-voltage generator is much less than that of the step-up transformers (§ 50) which would be necessary with a generator of low voltage.

§ 44. Calculation of Armature e.m.f. Effect of Varying Length of Polar Arc

The calculation of the e.m.f. of an alternator is a more difficult and more uncertain matter than the calculation of the e.m.f. of a continuous-current dynamo. This arises from the fact that whereas in the latter case the e.m.f. (for a given speed) depends simply on the flux per pole and the number of conductors in series with each other between the brushes, in an alternator the r.m.s. value of the e.m.f. depends also on the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ and the number of slots per pole per phase. This will be evident from the following considerations.

Let the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ be $\frac{1}{2}$, and let us, for the sake of simplicity, suppose that the field is perfectly uniform under the pole-shoe, and ceases abruptly (without the formation of a magnetic "fringe") immediately outside it. It is then evident that the positive half-wave of e.m.f. induced in a single conductor is of the shape shown in Fig. 78 (*a*), the e.m.f. having a zero value during the first eighth of a period, a constant value, say E, during the next quarter of a period, and then suddenly falling to a zero value, which is maintained during the remainder of the half-period. It is easy to show that the r.m.s. value of the e.m.f. is $\frac{1}{\sqrt{2}}$E. Suppose next that the pole width is reduced to half its original value, the total flux remaining unaltered. On the same assumptions as before, the e.m.f. wave now takes the form shown in Fig. 78 (*b*). The r.m.s. value of the e.m.f. is now E instead of $\frac{1}{\sqrt{2}}$ E as in the first case, notwithstanding the fact that the total flux per pole has remained unaltered, and that the *arithmetic mean* of the e.m.f. in the conductor is the same as before. Thus for a given flux per pole, the r.m.s. value of the e.m.f. is increased by reducing the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$. From this one might at first sight conclude that there is a great advantage in making the pole-pieces as narrow as possible. It must be remembered, however, that *for a given flux* a reduction in the polar area involves an increase of field intensity in the gap, and this means an increase in the field copper. Beyond a certain point, therefore, the increase of e.m.f. per conductor and consequent decrease of armature copper would be more than counter-balanced by the increase of field copper. The gap induction in modern alternators ranges from 6000 to 9000 C.G.S. lines per sq. cm.



(a) (b)

Fig. 78.—To illustrate Effect of Pole-width on E.M.F.

§ 45. Effect of Varying Number of Slots

We have next to consider the effect of varying the number of slots in which the coil is embedded. If each side of a coil occupies only a single slot, the e.m.f.'s of the various conductors may be practically taken to be in phase with each other, so that the e.m.f. per coil is equal to the e.m.f. per turn, multiplied by the number of turns in the coil. But by distributing the coil over a larger number of slots, we introduce phase differences between the e.m.f.'s of conductors lying in different slots, and the addition of the e.m.f.'s of the various groups of conductors must be carried out *vectorially* instead of arithmetically. The vectorial sum being, in general, numerically less than the arithmetical one, an increase in the number of slots per pole per phase results in a lowering of the e.m.f. (other things remaining the same). In spite of this disadvantage of a distributed as compared with a uni-slot or concentrated winding, distributed windings are invariably used nowadays, as they tend to reduce armature reaction and give rise to a smoother e.m.f. wave.

If the shape and size of the pole-shoe are such as to give rise to a distribution of the flux in the gap according to the simple sine law, the value of the e.m.f. in any given case is easily calculated. By way of example, we shall calculate the e.m.f. in one phase of a three-phase generator having six slots per pole, or two slots per pole per phase.

Let N = flux per pole, n = frequency, and C = conductors per phase. Since each conductor cuts $2N$ lines per period, the arithmetic mean value of the e.m.f. induced in it is $2Nn$ (in C.G.S. units). The form factor of a sine wave being 1·11 (§ 3), the r.m.s. value, in volts, of the e.m.f. in each conductor is—

$$2{\cdot}22Nn \,.\, 10^{-8}$$

The conductors of each phase may be divided into two sets (corresponding to the two slots per pole per phase), such that the e.m.f.'s of all the conductors in one set are in phase with each other. There being $\frac{1}{2}C$ conductors in each set, the r.m.s. value per set is $1{\cdot}11CNn \,.\, 10^{-8}$. Now, since the distance apart of two neighbouring slots is $\frac{1}{6}$ of the pole-pitch, the phase difference between the e.m.f.'s in the two sets of conductors is $\frac{1}{6} \,.\, 180°$, or 30°. Hence, compounding the e.m.f.'s of the two sets vectorially, we find for the r.m.s. value of the e.m.f. per phase—

$$2 \,.\, 1{\cdot}11CNn \,.\, 10^{-8} \,.\, \cos 15° = 2{\cdot}14CNn \,.\, 10^{-8}\,*$$

In practice, however, the flux is not distributed according to the simple sine law, but is mainly concentrated under the polar arc, and in accordance with the explanation already given regarding the effect of narrowing the pole-piece, such a concentration of flux results in a *higher* value of the e.m.f. than that calculated on the assumption of a sine distribution.

§ 46. General Formula for e.m.f. of Alternator

The formula for the e.m.f. of each phase of an alternator may be written in the form—

$$E = kCNn \,.\, 10^{-8}$$

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* The e.m.f. between two machine *terminals* is, for a star-connected winding, equal to 1·73 times the phase e.m.f. (§ 16).

where k is a factor depending on the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$, on the number of slots per pole per phase, and on the shape of the pole-shoe.

The earliest attempt to calculate the value of k for various cases is due to Kapp, who, in 1889, determined k for various kinds of windings and various values of the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$, on the assumption that the field in the gap is uniform, and ceases abruptly at the edges of the pole-shoe. According to this assumption, the e.m.f. wave induced in each conductor is of the rectangular form shown in Fig. 78. The e.m.f. wave of the entire phase is obtained by superposing the waves corresponding to the various sets of conductors into which the winding of the phase may be divided; each set of conductors consisting of those conductors whose e.m.f.'s are coincident in phase.

The assumption of a rectangular wave-shape for each conductor is, however, unjustifiable, on account of the formation of a magnetic fringe around the polar edges. Owing to the greater spreading of the flux due to the formation of this fringe, the actual e.m.f. is *less* than that calculated on the rectangular wave assumption.

More reliable results are obtained by taking into account the effect due to the fringe, as was first done by Mr. C. C. Hawkins in 1900.* The following table gives the values of k calculated by Mr. Hawkins, for the case of two slots per pole per phase, the slots being spaced $\frac{1}{6}$th of the pole-pitch apart:—

$\frac{\text{pole-arc}}{\text{pole-pitch}}$	$= 0{\cdot}30$	$0{\cdot}50$	$0{\cdot}70$	$0{\cdot}90$
k	$= 2{\cdot}48$	$2{\cdot}26$	$2{\cdot}10$	$1{\cdot}93$

The above values are based on the assumptions of an air-gap length (single) equal to $\frac{1}{20}$th of the pole-pitch, of sharp rectangular polar edges, and of a width of pole-shoe equal to the width of pole or field-core.

The second of the above assumptions does not in general apply to practical cases, since the edges of the pole-shoe are always more or less rounded. Now if, retaining the same flux per pole and the same length of polar arc, we round off the edges of the pole-shoe, we thereby cause a greater concentration of the flux towards the middle portion of the pole-shoe, and a greater concentration of flux is, as we have seen (§ 44), accompanied by a rise of e.m.f. Thus for pole-shoes with rounded corners the value of k will be greater than that given by the above table.

The latest contribution to this subject is due to A. Müller,† who gives the following values of k for two slots per pole per phase, the slots being $\frac{1}{6}$th of the pole-pitch apart:—

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* *Electrical Review*, vol. xlvii. p. 655 (1900).

† *Zeitschrift für Elektrotechnik* (Wien), vol. xxiii. p. 31 (1905).

$\frac{\text{pole-arc}}{\text{pole-pitch}}$	= 0·4	0·5	0·6	0·7
k	= 2·58	2·42	2·3	2·17

These values are based on the following assumptions: single air-gap length = $\frac{1}{50}$th of pole-pitch; radius of curvature of rounded pole-shoe corner = $\frac{1}{20}$th of pole-arc; height of pole-shoe = $\frac{1}{8}$th of pole-arc.

§ 47. Armature Reaction in Alternators

When an alternator is loaded, the armature current reacts on the field, distorting and weakening (or, sometimes, strengthening) it. The nature of the armature reaction depends not only on the armature current, but also on its phase relation to the open-circuit or no-load e.m.f. The calculation of the exact effect produced by the armature current in any given case is an extremely laborious matter, and if accuracy is desired, must be carried out separately for each particular type of machine. But the general nature of the effects produced may be easily inferred from the following very simple considerations.

The currents in the armature windings of a polyphase generator may be assumed to give rise to a rotating wave of magnetic flux (§ 20). Since this wave travels around the stator periphery at the same speed as the rotating field-poles,* it will be stationary with respect to the poles. Its position relatively to them will clearly depend on the amount by which the armature current lags or leads, and the nature of the armature reaction will best be understood by considering the three cases of (1) zero phase difference, (2) lag of 90°, and (3) lead of 90°.

When the current is in phase with the e.m.f. in each phase of the winding, a reference to Figs. 75 or 76 shows that the crest of the rotating wave of flux falls exactly halfway between two pole-pieces, since at the instant when one of the component alternating waves reaches its maximum amplitude, it coincides in position with the resultant rotating wave (see concluding remarks of § 20). The relative position of the field-poles and the rotating wave of magnetic flux is shown in Fig. 79 (*a*); the field being supposed to rotate counter-clockwise as indicated by the arrow, and the radially outward direction of the flux being regarded as positive. It is evident at once that the effect in this case is a purely distorting or cross-magnetizing one (such as occurs in a continuous-current dynamo whose brushes have no lead), the field being weakened on the advancing side of each pole-piece, and strengthened on the other side. When the current lags by 90°, the rotating flux wave takes up the position shown in Fig. 79 (*b*), and it is at once evident that there is a powerful demagnetizing effect;

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* The speed of the rotating wave of magnetic flux is (§ 19) $\frac{\lambda}{T}$, λ being the wave-length and T the period. The wave-length, however, corresponds to twice the pole-pitch; hence $\frac{\lambda}{T}$ is also the speed of the field-poles, each field-pole advancing through a distance equal to twice the pole-pitch during a period.

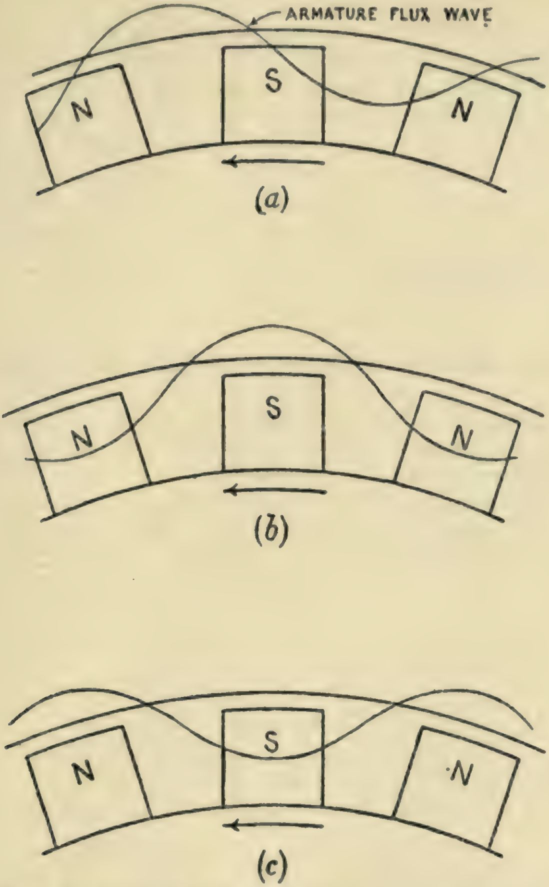

FIG. 79.—To illustrate Armature Reaction in Alternators: (*a*) Current in Phase with e.m.f.; (*b*) Current lagging by 90°; (*c*) Current leading by 90°.

whereas with a lead of 90°, Fig. 79 (*c*), there is an equally strong magnetizing effect. For any intermediate value of the phase difference, the distorting and demagnetizing (or, for a leading current, magnetizing) effects are both present.

In the case of a single-phase alternator, the armature current gives rise to a simple alternating flux wave (§ 18). Such a wave may, however, be resolved into two rotating waves (§ 19), travelling with equal speeds in opposite directions. One of these waves will be stationary with respect to the field-poles, and will produce the various effects (according to the phase difference of the current) already considered, while the other will move relatively to the field-poles with a speed equal to twice that of the poles themselves. Since the effect of this second rotating wave alternates from a magnetizing to a demagnetizing one as it sweeps past the poles, its mean reaction is zero, and it merely reduces the efficiency of the machine by causing additional eddy-current losses.

§ 48. Walker Compounded Alternator

With lagging currents such as have generally to be dealt with in practice, armature reaction is a troublesome feature, as it reduces the e.m.f. and thereby renders satisfactory regulation difficult. An extremely ingenious method of utilizing armature reaction in order to produce an automatic increase of e.m.f. with increase of load has recently been devised by Mr. Miles Walker,* and is shown diagrammatically in Fig. 80. Each field-pole consists of two parts, a larger one, which is surrounded by the exciting coil, and is strongly saturated, and a smaller one, which is unwound, and at no load is practically unmagnetized. In Fig. 80 are shown two such compound poles, and the dotted curve indicates the position of the rotating wave of magnetic flux due to slightly lagging currents. The field is supposed to rotate counter-clockwise. As will be seen, the armature currents tend to demagnetize the saturated portion of the pole, and to magnetize the

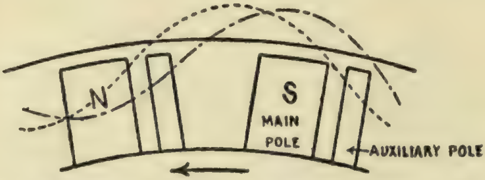

FIG. 80.—To illustrate Principle of Walker Compounded Alternator.

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* *Journal of the Institution of Electrical Engineers*, vol. xxxiv. [Part 2] p. 432 (1905).

unsaturated one. The latter effect is, however, much greater than the former one, owing to the high saturation of the main pole. Hence as the current increases the e.m.f. rises. This method of compounding an alternator, although capable of yielding excellent results so long as the power factor (§ 7) does not fall below about 0·85, is no longer applicable to highly inductive loads; for in this latter case, as shown by the chain-dotted curve of Fig. 80, the wave of rotating flux is shifted so far back relatively to the poles as to produce a magnetization of the auxiliary pole in the wrong direction, thus lowering the e.m.f.

CHAPTER VII

§ **49**. Transformers. Ratio of transformation—§ **50**. Constant potential transformer—§ **51**. Examples of transformer construction—§ **52**. Calculation of e.m.f. induced in transformer winding—§ **53**. Losses in transformer. Hysteresis loss—§ **54**. Calculation of eddy-current loss in core. Discrepancy between calculated and observed core losses—§ **55**. Copper losses. Best dimensions of core—§ **56**. Heating of transformers—§ **57**. Star and mesh connections for transformers. Comparison of single-phase and polyphase transformers for polyphase circuits—§ **58**. Auto-transformer or compensator—§ **59**. Phase transformers—§ **60**. Choking coils.

§ 49. Transformers. Ratio of transformation

One of the most important advantages of alternating over continuous currents is the extreme ease with which the transformation from a low to a high voltage, or *vice versâ*, may be accomplished. Such transformations are effected by means of *transformers*, whose efficiency exceeds that of any other known apparatus.

A transformer consists essentially of a laminated iron core surrounded by two windings: a *primary* winding or primary, which is supplied with alternating currents, an alternating magnetic flux being thereby produced in the core; and a *secondary* winding or secondary, in which an alternating e.m.f. is induced by the alternating flux. By connecting the terminals of the secondary to an external circuit, a current may be obtained in this circuit.

In order to reduce the ampere-turns necessary to produce the required magnetization to as low a value as possible, the core of a transformer is arranged so as to form a *closed magnetic circuit, i.e.* the path of the magnetic flux lies entirely in iron, air-gaps being avoided.

By far the greater part of the alternating magnetic flux in a well-designed transformer will become linked with both primary and secondary. A certain small fraction of it, however, representing *magnetic leakage lines,* will become linked with the primary alone. As will be seen at a later stage, it is important to adopt a form of construction which will reduce the magnetic leakage to the smallest possible amount.

As an approximation, we may provisionally assume that magnetic leakage is negligible. From this it immediately follows that the e.m.f. induced by the alternating flux in each turn of the primary is equal to that induced in each turn of the secondary, so that the ratio of the total primary to the total secondary e.m.f. is simply equal to the ratio of the number of primary turns to the number of secondary turns. Again, in order to avoid excessive loss by heating of the coils, their resistances are always so chosen as to reduce the resistance drop (even at full load) to a very small fraction of the p.d. (in both primary and secondary), so that the p.d.'s are nearly equal to the e.m.f.'s, and we have, approximately—

$$\frac{\text{primary p.d.}}{\text{secondary p.d.}} = \frac{\text{primary turns}}{\text{secondary turns}}$$

This ratio is sometimes spoken of as the *ratio of transformation.* If, *e.g.*, a transformer is required to transform from 10,000 to 2000 volts, then the primary will have to be wound with five times as many turns as there are in the secondary, and the ratio of transformation will be 5 : 1.

§ 50. Constant Potential Transformer

Let us suppose that the p.d. across the primary is maintained constant.* When the secondary circuit is open, the primary current adjusts itself to a value such that the e.m.f. due to the alternating flux just balances (neglecting the very small resistance drop) the primary p.d. Now, this balance of primary p.d. and primary (counter) e.m.f. must also—still neglecting the resistance drop—be maintained when the transformer is loaded, *i.e.* when the secondary circuit is closed. In order, therefore, to maintain the original value of the alternating flux corresponding to the given primary p.d., the resultant ampere-turns must be maintained constant. But this condition obviously implies that in the primary there must, in addition to the current which existed on open secondary circuit, be a further component of current—the *load* component—which has a value such that the ampere-turns due to it just suffice to wipe out the ampere-turns of the secondary, leaving a constant value for the resultant ampere-turns. We may speak of the inoperative primary ampere-turns (those balancing the secondary ampere-turns) as the *load ampere-turns,* and of the open-circuit ampere-turns as the magnetizing ampere-turns. We then have the relation—

$$\text{prim. load ampere-turns} = \text{second. ampere-turns}$$

or—

$$\text{prim. load current} \times \text{prim. turns} = \text{second. current} \times \text{second. turns}$$

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* A transformer whose primary is supplied at a constant p.d. is variously spoken of as a "constant potential" and a "pressure" transformer.

so that—

$$\frac{\text{primary load current}}{\text{secondary current}} = \frac{\text{secondary turns}}{\text{primary turns}}$$

Now, owing to the adoption of a closed magnetic circuit the open-circuit or magnetizing ampere-turns (or resultant ampere-turns of a loaded transformer) form a small fraction of the total primary ampere-turns at full load, so that the load ampere-turns corresponding to full load may be approximately taken to be equal to the total ampere-turns at full load. Hence, using the relation just established, we see that, approximately, *the primary and secondary currents are inversely as the turns in the windings.* For this reason the low-voltage winding will carry a heavier current than the high-voltage winding, and the cross-section of the conductor must be correspondingly greater. In practice the windings are designed so as to contain, roughly, equal amounts of copper.

Combining the result for the ratio of the p.d.'s with that for the *ratio* of the currents, we see that the product p.d. × current has approximately the same value for primary and secondary. In the case of a non-inductive load, this conclusion is also otherwise obvious from the fact that if the losses be negligible, then the primary watts must equal the secondary watts.

A transformer whose low-voltage winding is the primary, is spoken of as a *step-up transformer;* and one whose high-voltage winding is the primary, as a *step-down transformer.*

§ 51. Examples of Transformer Construction

The most common form of construction for single-phase transformers is that illustrated in Fig. 81. The core consists of stampings arranged to form two upright cores connected by yokes at the top and bottom. The stampings are held together by insulated bolts, the positions of which are indicated by the small circles in the figure. In order to avoid butt joints (which introduce a short air-gap into the magnetic circuit and so increase its reluctance), the stampings may be made to overlap alternately at the corners, a layer consisting of two short core-plates and two long yoke-plates (as indicated by the full lines in the figure) being succeeded by another, in which there are two long core-plates and two short yoke-plates (dotted lines in figure). The butt joints in any two consecutive layers thus occur at different places, and the flux is free to pass from one layer to another through the lap joints (of large area, and therefore low reluctance) between different layers.* In order to economize copper as much as possible, the coils are generally made circular, the core being given a cross-section which is stepped—as shown in the figure—so as to fit into the coils. In the figure the low-voltage coil is shown next the core, the high-voltage coil being outside. Frequently, in order to reduce magnetic leakage, the high-voltage coil is wound in two sections, the low-voltage coil being sandwiched in between them. In other cases, each winding is subdivided into a number of very short coils, the sections of the high-voltage winding alternating with those of the low-voltage one on the core (the sectional coils of both windings being of the same size, and placed on the top of each other). Even with the simple arrangement shown in the figure, it is advisable, in the case of very high voltages, to wind the high-voltage coil in a number of equal sections, separated from each other by barriers of insulating material.

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* This principle of construction is not always adhered to. It has the disadvantage that the dismantling of the transformer, and the subsequent building up of the core

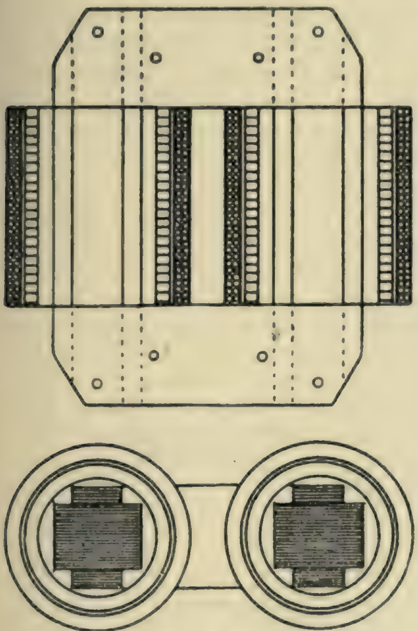

Fig. 81.—Single-phase Transformer.

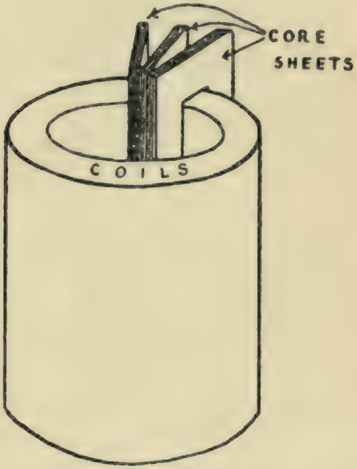

Fig. 82.—Berry Transformer.

Fig. 82 illustrates the construction of the Berry transformer. The circular coils having been wound, a number of bundles of core-sheets

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(in carrying out repairs), are rather slow processes. By adopting a butt-joint pure and simple between the core and yoke-plates, the transformer may be dismantled and built up again very quickly.

are assembled around them, these bundles being arranged in radial planes. For the sake of clearness only three of these are shown in the figure.

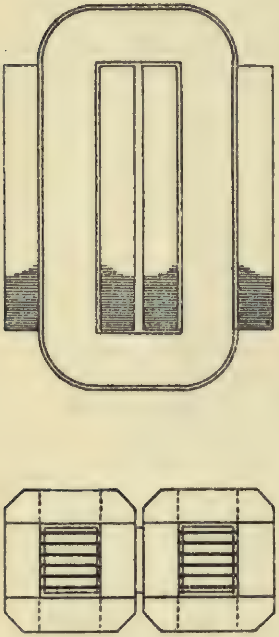

Fig. 83.—Type of Transformer Construction.

In Fig. 83 is shown another type of construction. The coils are of oblong shape, the core-sheets being built up around them. The thick lines in the lower figure indicate insulating barriers between the sections of the high-voltage and those of the low-voltage windings. The core-plates are pressed together between two heavy end-plates connected by bolts passing outside the core-plates.

For three-phase work, three single-phase transformers may be used. A cheaper form of construction, however, results by combining the cores of the transformers so as to form a single three-phase core. One method of doing this is shown in Fig. 84.[*] The coils (not indicated in figure) are placed around three upright laminated cores, connected at the top and bottom by laminated yoke-rings. The cores are pressed against the yoke-rings by means of end-plates fitted with conical rims, which bear against the chamfered ends of the cores. In this, as in all other cases where butt joints are used, thin sheets of insulating material are interposed at the joints, in order to prevent the formation there of conducting grids (by the contacts between the two sets of plates), which would cause dissipation of energy by eddy currents. The algebraical sum of the magnetic fluxes in the three cores being zero, any two of the cores will, at a given instant, form a return magnetic circuit for the flux in the third core, so that the principle of a closed magnetic circuit will be realized.

In Fig. 85 is shown another method of combining the cores of the three transformers. The coils are shown in position in the lower part of the figure. This arrangement is not quite symmetrical, the flux passing through the middle core encountering a somewhat lower reluctance—on account of the shorter average length of path—than that passing through either end-core.

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[*] This form of construction was a favourite one in the early days of three-phase currents, but is now being abandoned, mainly on account of its expense.

A very recent form of construction, applicable to either single-phase or polyphase transformers, is shown in Fig. 86. The core of a

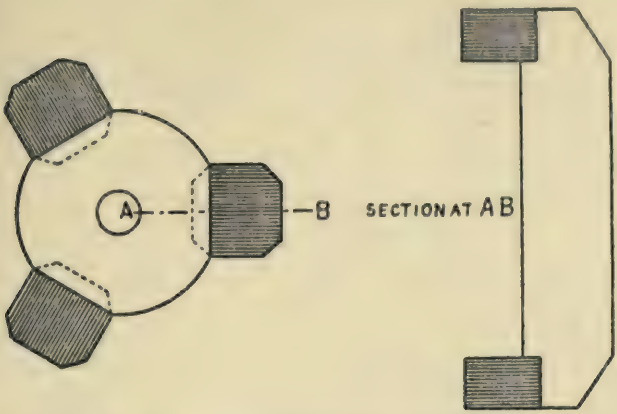

Fig. 84.—Core of Three-phase Transformer.

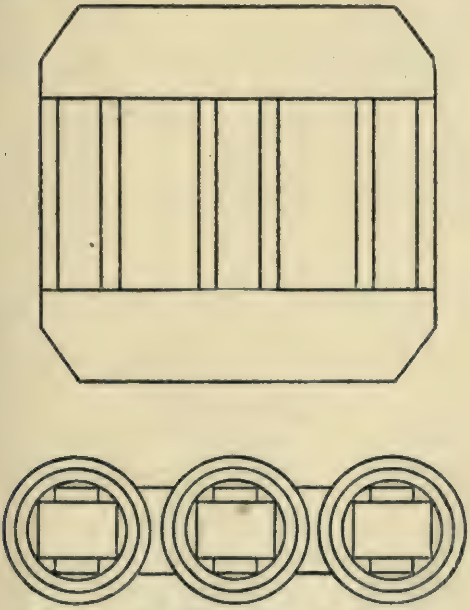

Fig. 85.—Type of Three-phase Transformer.

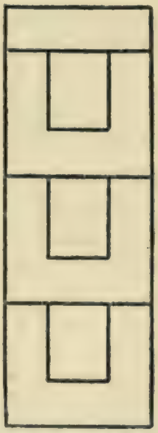

Fig. 86.—Transformer Core Construction.

single-phase transformer would have the shape corresponding to Fig. 81. In order to construct a three-phase transformer, three single-phase cores would be piled on the top of each other as in Fig. 86, only the top core being provided with a top yoke.

The thickness of the core-sheets varies from 0·014 inch to 0·020 inch. The insulation (consisting of thin paper) between them occupies some 12 per cent. of the total thickness of the core.

§ 52. Calculation of e.m.f. induced in Transformer Winding

The relation of the value of the maximum induction in the core to the e.m.f. induced in either coil at a given frequency is easily determined. Let B stand for the maximum value of the induction, a for the cross-sectional area of the core, in sq. cms., n for the frequency, and S for the number of turns in the coil. The maximum flux through each turn is aB. This changes from $+ aB$ to $- aB$ in $\frac{1}{2n}$ sec. The mean rate of change is thus $4naB$, the mean arithmetical value of the e.m.f., in volts, induced in each turn is $4naB \cdot 10^{-8}$, and the mean arithmetical value of the total e.m.f. $4naSB \cdot 10^{-8}$. Hence, if f denote the form factor of the e.m.f. wave (§ 1), we have for the r.m.s. value E of the induced e.m.f.— .

$$E = 4nafSB \cdot 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a sine wave of e.m.f., $f = 1{\cdot}11$ (§ 3), so that—

$$E = 4{\cdot}44naSB \cdot 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The usual values of B range from 2500 to 4000, the lower values being used at higher frequencies.

§ 53. Losses in Transformer. Hysteresis Loss

The losses taking place in a transformer may be divided into *core losses* and *copper losses.*

The core losses arise from hysteresis and eddy currents, and depend on the maximum value of the induction in the core. This maximum value (the amplitude of the induction wave) is frequently referred to as the induction simply.

We have seen that, owing to the smallness of the resistance drop, the e.m.f. induced in the primary is nearly equal to the primary p.d. Hence, if the latter be maintained constant, as is normally the case, the induced e.m.f. will also remain *nearly* constant. The constancy of the induced e.m.f., however, involves, in accordance with equation

(1) above, the constancy of B. Thus the core losses, which depend on B, will remain nearly constant at all loads. As a matter of fact, there will be a *very slight* decrease with increasing load, owing to the slight decrease in the counter e.m.f. brought about by the increase in the resistance drop.

According to Steinmetz's law, the hysteretic loss, in ergs per c.c. of the material per cycle, is given by $\eta B^{1\cdot6}$, where η has a value, for good transformer sheets, ranging from 0·001 to 0·0012. If we multiply the loss per c.c. per cycle by the frequency and the total volume of iron in the core we obtain the hysteresis core loss.

The hysteresis loss is found to increase considerably if the core-plates are subjected to great mechanical pressure. It is therefore advisable, in building up the core, to apply as little pressure as possible.

The core-sheets employed in some of the earlier transformers were found gradually to deteriorate, the hysteresis loss steadily increasing as time went on. This effect was found to be due to the maintenance of the core at a fairly high temperature by the losses occurring in it, and is known as the *ageing* of transformer iron. Lately, however, manufacturers of transformer sheets have succeeded in producing material which does not suffer from the "ageing" defect to any serious extent.

§ 54. Calculation of Eddy-Current Loss in Core. Discrepancy between calculated and Actual Core Losses

The eddy-current loss which occurs in sheets of a given thickness may be approximately calculated. Let in Fig. 87 AB represent part of the cross-section of a sheet, the magnetic flux being normal to the plane of the paper. The alternations in the flux give rise to eddy currents, and these will flow parallel to the boundaries of the sheet—if we except the extreme edges of the sheet, where the currents flowing along the upper half cross over to join those flowing in the lower half of the sheet. There will obviously be no flow along the central line OO′. Above this line the currents will, at a given instant, flow in one direction, below it, in the other, as indicated by the arrows. Consider a double strip, CDEF, 1 cm. long

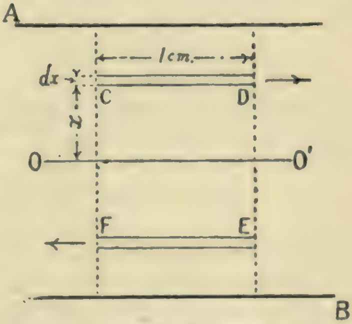

FIG. 87.—To illustrate Eddy-current Loss in Core.

in a direction parallel to OO′, and 1 cm. wide in a direction normal to the plane of the paper, of thickness dx, and at a distance x from OO′. The e.m.f. maintaining the current in this strip is that induced around the rectangle CDEF. Assuming, for the sake of simplicity, that the induction follows the simple sine law, we have for the value of the e.m.f. around CDEF, by (2) of § 52—

$$4{\cdot}44n\,.\,2x\mathrm{B}\,.\,10^{-8}$$

The resistance of the double strip is $\rho\frac{2}{dx}$, ρ being the resistivity of the material. Hence the watts lost in the double strip amount to—

$$\frac{(8{\cdot}88nx\mathrm{B}\,.\,10^{-8})^2}{2\rho}dx = \frac{39{\cdot}5}{\rho}(nx\mathrm{B} \times 10^{-8})^2dx$$

In order to find the watts lost in a strip of the entire thickness of the sheet, we have to integrate the above expression between the limits 0 and $\frac{1}{2}t$, t being the thickness of the sheet, in cms. Now—

$$\begin{aligned}\int_0^{\frac{1}{2}t}\frac{39{\cdot}5}{\rho}(nx\mathrm{B} \times 10^{-8})^2dx &= \frac{39{\cdot}5}{\rho}(n\mathrm{B}\,.\,10^{-8})^2\int_0^{\frac{1}{2}t}x^2dx\\ &= \frac{39{\cdot}5}{\rho}(n\mathrm{B}\,.\,10^{-8})^2\,.\,\tfrac{1}{24}t^3\\ &= \frac{1{\cdot}65 \times 10^{-16}}{\rho}\,.\,n^2\mathrm{B}^2t^3\end{aligned}$$

But since the volume of iron in such a strip is t, the loss per c.c. of the sheet is—

$$\frac{1{\cdot}65 \times 10^{-16}}{\rho}\,.\,n^2\mathrm{B}^2t^2$$

The resistivity of iron at 0° C. may be taken to be 10^{-5}, and the temperature coefficient 0·0045. If, then, the core be at a temperature θ° C., we have—

$$\text{eddy-current loss per c.c.} = \frac{1{\cdot}65 \times 10^{-11}}{(1 + 0{\cdot}0045\theta)}\,.\,n^2\mathrm{B}^2t^2$$

The eddy-current loss is generally from $\frac{1}{2}$ to $\frac{1}{6}$ of the hysteresis loss. The following table contains the core losses, at a frequency of 60, of four modern transformers:—

Output	0·6 k.w.	2·5 k.w.	7·5 k.w.	30 k.w.
Core loss, in watts ...	23	45	96	230
Core loss as percentage	3·85	1·8	1·3	0·77

The hysteresis and eddy-current losses as calculated by the methods explained above are always found to be less than the losses actually occurring in transformers. A little consideration will show that such a discrepancy might be expected to exist. For in calculating the losses we have made the assumption that the induction is *uniformly distributed* over the cross-section of the core. Such, however, is by no means the case, as a single glance at Fig. 88, which represents a core stamping, will show. For the length of path corresponding to ABCD is very considerably shorter than that corresponding to EFGH, with the result that the flux will be crowded towards the inner surface of the core. Now, any departure from uniformity in flux distribution results in an increase of both the hysteresis and the eddy-current loss. An upper limit to these losses may be obtained by calculating them on the assumption that the value of B is that corresponding to its value close to the inner surface (ABCD) of the core.

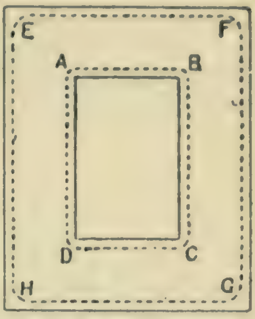

FIG. 88.—To explain Difference between Calculated and Observed Core Loss.

§ 55. Copper Losses. Best Dimensions of Core

The copper losses are easily calculated from the resistances of the coils and their full-load currents. In the case of windings consisting of heavy copper conductors, however, additional losses come into play, due to eddy currents in the conductors, and these may bring about an apparent increase of resistance, amounting, in some cases, to as much as 20 per cent. For this reason, most makers prefer to avoid the use of such heavy windings, employing instead a number of coils wound with smaller wire and connected in parallel.

The total losses at full load depend on the output of the transformer. The following table gives the efficiencies which might reasonably be demanded in modern transformers:—

Output, in k.w.	1	5	10	20	50	100	150	250
Efficiency ...	94%	95%	95·5%	96%	96·5%	97%	97·5%	98%

Let us suppose that the average length of magnetic path and the total volume of iron are given, but that it is permissible to vary the relative cross-sections of the core-plates and yoke-plates. Any departure from a uniform cross-section for the entire magnetic circuit will increase the iron losses. On the other hand, however, a reduction in the core cross-section and an increase in the yoke cross-section will reduce the mean length of a turn in each winding, and so reduce the copper losses. Now, this reduction in the copper losses may more than counterbalance the increase in the iron losses. It has been found that in transformers of ordinary construction (Fig. 81) the total loss (iron and copper) reaches a minimum value when the yoke cross-section is about 2½ times that of the core cross-section inside the coils.*

§ 56. Heating of Transformers

Transformers are generally enclosed in suitable cases, which may be either entirely of cast-iron or partly of perforated sheet metal. When the former are used, the transformer is frequently entirely immersed in oil. The oil not only improves the insulation, but facilitates the transfer of heat from the transformer to the case, and maintains the transformer at a more uniform temperature. The maximum permissible rise of temperature of the coils is about 50° C.

The difficulty of preventing an excessive rise of temperature in the case of very large transformers is a serious one. Where no oil is used, and where the natural convection currents of air are insufficient to carry off the heat rapidly enough, an air-blast—supplied by a blower driven by a small motor—is forced through ventilating ducts provided in both core and coils. With oil-insulated transformers, the following method of cooling is adopted. At the top of the oil-tank containing the transformer is fitted a coil of thin-walled brass piping, and through this cold water is circulated. The hot oil as it rises gives up its heat to the water, and sinks to the bottom of the tank, a vigorous circulation of the oil being thereby maintained.

A good deal of controversy has taken place as to the relative advantages of air-blast and oil-insulated water-cooled transformers. Difficulties have sometimes been experienced with oil insulation. These have been mainly due to the presence of moisture, either in the coils of the transformer or in the oil; to the use of an unsuitable oil, or unsuitable insulating compound in the coils, the compound being chemically attacked by the oil. It has been urged against large oil-insulated transformers that they constitute a serious fire risk. It is, no doubt, advisable to keep such transformers in separate fire-proof compartments at some distance from the generating station, and to have arrangements whereby the oil may be rapidly drained off if necessary. The oil should have a high flash-point (about

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* See A. Müller, *Zeitschrift für Elektrotechnik* (Wien), vol. xxii. p. 417 (1904); vol. xxiii. p. 243 (1905).

180° C.), and not give off any appreciable quantity of inflammable vapour.

For very high voltages—anything exceeding 30,000 volts—the oil-insulated type of transformer is the only one practicable.

§ 57. Star and Mesh Connections for Transformers. Comparison of Single-phase and Polyphase Transformers for Polyphase Circuits

The coils of transformers intended for three-phase work may be connected either mesh- or star-fashion. The mesh connection is usually preferred, for if one of the coils should burn out, causing its fuses to blow, the supply of current to the corresponding phase would still be maintained, the remaining two coils taking up the load. But with a star connection the failure of one of the coils would evidently cut off the supply to *two* phases—one of the line wires becoming entirely disconnected. Thus a breakdown in the latter case would have far more serious consequences than in the former. On the other hand, when dealing with very high voltages, the dielectric stress on each coil is reduced by adopting the star connection, since the phase p.d. then becomes only $\frac{1}{\sqrt{3}} = 0{\cdot}577$ of the line p.d. (§ 16).

The relative advantages and disadvantages of a single three-phase transformer as compared with three independent single-phase transformers have frequently been discussed. For smaller sizes (up to about 200 k.w.) a single three-phase transformer is both lighter and cheaper, for a given output, efficiency, temperature rise, and regulation,* than three single-phase transformers. For very large transformers, on the other hand, the cost is lower in the case of three single-phase transformers (the weight is, as before, greater than that of a three-phase transformer). A three-phase transformer has the advantage over three single-phase ones of maintaining better balance of the p.d. on the three phases, owing to the interlinking of the magnetic circuits.†

§ 58. Auto-transformer or Compensator

In addition to the ordinary transformers already considered, which have distinct primary and secondary windings, there is another type, known as the *auto-transformer*. In this the primary and secondary windings are represented by a single continuous winding, as shown diagrammatically in Fig. 89. The ends of the winding AB form the primary terminals, while the secondary terminals are represented by one of the primary terminals (A in Fig. 89), and a terminal C in connection with an intermediate point of the winding. We may regard the portion AC of the winding as resulting from the fusion of the primary and secondary. Now, since the primary and secondary currents are nearly in phase opposition, it follows that such a fusion of the two circuits will give rise to a current in AC which is merely the difference of the primary and secondary currents, and is thus much less than the current in the secondary of an ordinary transformer. Hence the cross-section of the portion AC of the winding may be made much less than would be necessary with a transformer of ordinary construction.

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* See Chapter X.

† Eborall, "Howard Lectures on Polyphase Electric Working," p. 40 (1902).

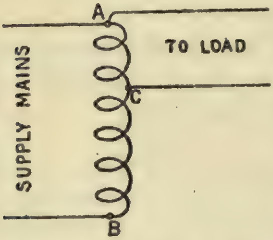

FIG. 89.—Diagram of Auto-transformer.

On account of this advantage, auto-transformers would be much more generally used than they are, were it not for the fact that in most cases it is absolutely necessary, from considerations of safety, to keep the two windings entirely distinct and heavily insulated from each other, so as to prevent all risk of the dangerous high voltage of the primary from reaching the secondary.

Auto-transformers are frequently used to supply single arc lamps from mains at a considerably higher p.d. than that taken by the lamp. They are then sometimes termed *economy coils*. Their main use is, however, in connection with the starting of various forms of alternating-current motors. When so used they are generally known as *compensators*. By means of a suitable switch, the motor is first connected across the portion AC of the winding (Fig. 89), and when it has run up to a certain speed, the switch is thrown over so as to connect it across AB. In the first or starting position, the voltage across the motor terminals is only a fraction of the normal voltage, while in the second position of the switch the motor is supplied at the full line voltage.

§ 59. Phase Transformers

Cases sometimes arise in which it is required to pass from a two-phase to a three-phase system, or *vice versâ*. Such a transformation may be effected by a suitable combination of transformers, which are then termed *phase transformers*. Various solutions of the problem of phase transformation have been devised, but the best known is that due to C. F. Scott. The principle of Scott's method is illustrated in Fig. 90. Let OA, OB, and OC in Fig. 90 (*a*) represent the primaries of three single-phase transformers which are star-connected to the neutral point O. Let A, B, and C be connected to a system of three-phase supply mains, and let the instantaneous values of the star p.d.'s be reckoned positive when acting towards O, as shown by the arrows in the diagram. In Fig. 90 (*b*) is given a vector diagram of the star p.d.'s. Now since, at a given instant, the p.d. across BC in diagram (*a*) is the algebraical *difference* of the p.d.'s across the BO and CO, we obtain the vector of this p.d. by compounding, in diagram (*b*), the vector OB with OE (which is OC *reversed*). We thus obtain the vector OD, which is at right angles to OA. Hence the p.d. across OA, Fig. 90 (*a*), differs by 90° from that across BC. In order that these p.d.'s may form a two-phase system, OD, in Fig. 90 (*b*), must be reduced in the ratio $\frac{OA}{OD} = \frac{1}{\sqrt{3}}$. This is easily accomplished by adopting the arrangement shown in Fig. 91. The primary windings

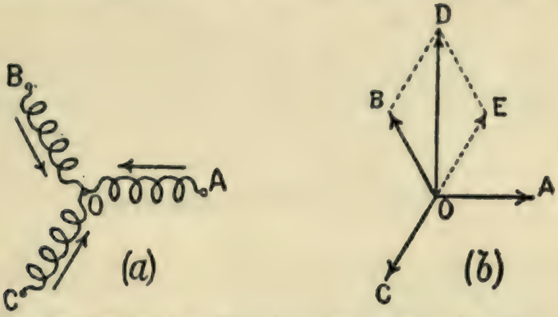

FIG. 90.—To illustrate Principle of Phase Transformation.

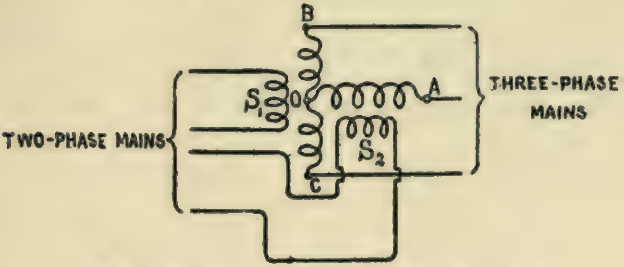

FIG. 91.—Scott's Method of Phase Transformation.

OB and OC are wound on the same core, forming a single continuous primary BC, whose middle point O is connected to the primary OA of the remaining transformer. The secondaries S_1 and S_2 of the two transformers are arranged to have the same number of turns, while the primary BC of the first transformer has $\sqrt{3}$ times the number of turns in the primary OA of the second transformer. From this it follows that the e.m.f.'s induced in the secondaries are equal, and since we have already shown that they are in quadrature (§ 6) with each other, they will form a two-phase system. It is clear that along with the phase transformation any desired voltage transformation may be effected by suitably arranging the ratios of transformation.

§ 60. Choking Coils

By a choking coil, impedance coil, or reactance coil, is meant a coil having a low resistance and high self-inductance. The primary of a transformer whose secondary is open forms a very powerful choking coil. In many cases, it is desirable to have some simple means of adjusting the reactance, and for this purpose various arrangements may be used. Thus, the coil may be provided with a sliding core, which may be clamped in any desired position relatively to the coil. Or the coil may be wound on the middle limb of a [-shaped core, and provided with a movable yoke, which may be placed at varying distances from the projecting ends of the core, thereby altering the length of the air-gaps in the magnetic circuit.

A choking coil affords a ready means of reducing the voltage of supply without any appreciable loss of power such as would occur if a resistance were used for the same purpose. This is due to the very low power factor of the coil, which enables it to take a large current at a high p.d. without the absorption of any large amount of power.

CHAPTER VIII

§ **61**. Induction motors. Squirrel-cage rotor—§ **62**. Rotor windings—§ **63**. Starting resistances for rotors. Internal short-circuiting and brush-lifting gear—§ **64**. Lewis induction motor—§ **65**. Example of induction motor. Variation of torque with position of rotor—§ **66**. Asymmetry of hemi-tropic stator winding with odd number of pole-pairs. Method of obtaining symmetry—§ **67**. General characteristics of induction motor. Air-gap length. Effect of rotor eccentricity.

§ 61. Induction Motors. Squirrel-cage Rotor

In connection with polyphase systems of distribution, a type of motor is employed which is known as the *induction motor*. The action of such a motor depends on the possibility (§ 20) of producing rotating waves of magnetic flux by means of polyphase currents.

Let a laminated stator be provided with polyphase windings (two- or three-phase) similar in every respect to the armature windings of a polyphase generator, and let the rotor consist of a solid cylinder of iron (*cf.* Fig. 27). When polyphase currents are sent through the stator windings, they give rise to rotating waves of flux—commonly spoken of as a rotating field—and as these waves sweep across the rotor they induce currents in it. The direction of these currents is, in accordance with Lenz's law, such as to oppose relative motion of the rotor and the rotating field. A driving torque is thereby exerted on the rotor, which (assuming that the resistances to its motion are not excessive) will run up to a speed only slightly below that of the rotating field. The difference between the speed of the rotating field and that of the rotor, on which depends the magnitude of the induced currents, is spoken of as the *slip*. Since with increasing load a larger driving torque, and hence larger induced currents, are required to keep the rotor running, it follows that the slip will increase with increase of load.

The rudimentary form of motor considered, in which the rotor is a solid cylinder of iron, although mechanically simple, would be unsatisfactory from the point of view of efficiency, as the loss in the rotor would be excessive. In practice, therefore, the rotor takes the form of a laminated iron cylinder having copper conductors embedded in it.

The simplest form of rotor winding is that known as the *squirrel-cage* winding. It consists of a number of copper rods or bars arranged in holes or slots around the rotor periphery, and connected at each end to a ring of copper. One example of a squirrel-cage winding is given in Fig. 92. In this, the rotor is provided with a number of equidistant circular holes (open at the top) close to its periphery, and through these holes pass copper rods. The projecting ends of the rods are slotted, and into the slots, on each side, is fitted a copper ring, which is then soldered to the rods. In Fig. 95 is shown a slightly different arrangement. The conductors are, in this case, of rectangular cross-section, and fit into rectangular semi-closed slots in the rotor core. The projecting ends of the copper bars are bolted to end-rings.

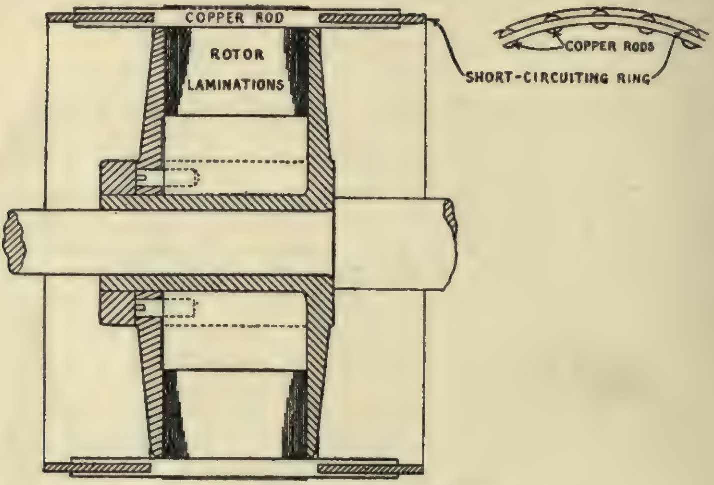

FIG. 92.—Squirrel-cage Rotor.

The squirrel-cage winding is at once the simplest and cheapest form of rotor winding, and is almost exclusively used for small motors—up to about 5 h.p. In the United States, it is used for very large motors as well. It possesses, however, the disadvantages of taking a heavy starting current and exerting a comparatively feeble starting torque. It is thus incapable of starting against a heavy load, and must be run up to full speed on a loose pulley. For starting motors with squirrel-cage rotors, auto-transformers (§ 58) are employed.

In order that an induction motor may be capable of exerting a large torque *at starting*, its rotor circuits must have a comparatively high resistance.[*] But high resistance in the rotor circuits would involve large heating loss and poor efficiency under normal running conditions. Thus, although a squirrel-cage rotor could easily be constructed to give a powerful starting torque (by the use of high-resistivity material for its conductors), its efficiency under normal running conditions would be extremely low. Large starting torque and high efficiency are, in fact, incompatible features, and the only way of securing both lies in the use of a rotor the resistance of whose windings at starting is high, and under normal running conditions, low. This variation of resistance is easily obtained by the use of an external rheostat, which may be connected in series with the rotor windings at starting, then gradually short-circuited as the motor gains speed, and finally cut out altogether. Now, a squirrel-cage winding does not readily adapt itself to the insertion of external resistances. It is for this reason that most of the larger European induction motors are provided with so-called *wound* rotors as distinguished from squirrel-cage or *short-circuited* rotors. The winding of these rotors is connected to slip-rings, which allow of the insertion of suitable starting resistances.

§ 62. Rotor Windings

Two types of three-phase windings, both of them star-connected, are commonly used in the case of rotors provided with starting resistances. One of these windings is similar to the form of armature winding for a three-phase generator already described (§ 43 and Fig. 76), while the other closely resembles the wave winding of a continuous-current armature, but differs from it in not being entirely symmetrical. This second kind of winding is frequently described as a *bar winding*, in order to distinguish it from the first type, which is spoken of as a *coil winding*. In the smaller sizes, coil windings are generally used, while the bar winding is commonly employed in large motors. In order to explain clearly how such a bar winding is carried out, we shall consider a particular case—that of a four-pole rotor having 96 conductors or bars arranged in 48 slots, so that there are 2 conductors per slot.

It will be sufficient to consider the winding of one phase, since all three phases are similarly wound. The number of conductors per phase is $\frac{96}{3} = 32$, and since the motor is a four-pole one, the number of conductors per pole per phase is $\frac{32}{4} = 8$. The winding of each phase must, therefore, clearly consist of four equally spaced groups of eight conductors, and these conductors have to be connected in series in such a manner that two conductors in immediate connection belong to two neighbouring groups. Let us suppose all the conductors to be numbered, and let conductor 1 be connected at one end to the neutral point of the winding. The four groups of conductors forming the winding of the first phase are then as follows:—

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[*] The reason for this will be understood later, when we come to study the theory of such motors in detail (Chapter XII.).

Group I: 1, 2, 3, 4, 5, 6, 7, 8
„ II: 25, 26, 27, 28, 29, 30, 31, 32
„ III: 49, 50, 51, 52, 53, 54, 55, 56
„ IV: 73, 74, 75, 76, 77, 78, 79, 80

Between Groups I and II are the sixteen conductors belonging to the other two phases, and there are similar intervals between the remaining groups.

In arranging the connections, we select conductors which are as nearly as possible separated by a distance corresponding to the pole-pitch. Now, the pole-pitch is, in terms of the number of conductors comprised in it, equal to $\frac{96}{4} = 24$. But this value could not be adopted for the pitch of the winding, since with a double layer of conductors the pitch must correspond to an *odd* number.* Hence we select a *double* pitch for our winding, namely, 23 and 25, using these numbers alternately; the mean pitch of the winding is thereby made to correspond to the pole-pitch.

We must commence with a pitch of 25, since a pitch of 23 would lead us to conductor $1 + 23 = 24$, which does not belong to our phase at all. Hence our winding proceeds thus—

1—26—49—74

A difficulty occurs at this point. We have travelled once round the rotor periphery, and the next step (assuming that we go on using the pitches 25 and 23 alternately) would bring us to conductor $74 + 23 = 97$, which is conductor 1. But this would close the winding. Hence we select the odd conductor in Group I, which is nearest conductor 1. This is obviously conductor 3, and the step from 74 to 3 (or 99) corresponds to a pitch of 25. We thus find it necessary to break the alternate sequence of the two pitches at the end of the first round, and to use the same pitch twice in succession. The same difficulty occurs, and is similarly overcome, at the end of each revolution. Bearing this in mind, we obtain the following for the first half of the winding table, each horizontal line representing one complete revolution:—

---

* Otherwise, starting from conductor No. 1, which is in the top layer, we should be using up odd conductors, all of which are in the top layer; this would not allow of the compact arrangement of end connections, which becomes possible when odd and even conductors (*i.e.* conductors in the top and bottom layer) are connected alternately.

1—26—49—74
3—28—51—76
5—30—53—78
7—32—55—80

We have now used up half the conductors of the phase, and at this stage a fresh difficulty arises. All the odd conductors in Group I have been used up, so that we cannot proceed from conductor 80 (which is the last *even* conductor in Group IV) to an *odd* conductor in Group I. A pitch of 25 gives conductor $80 + 25 = 105$, or 9, which does not belong to our phase at all. The nearest odd conductor in Group I is 7, but this has already been made use of. If we try going back to Group III, we again find that all the odd conductors have been used up. Two possible courses are open—to step forward to Group I, using the even conductor 2 of this group, or to go back to Group III, using the nearest even conductor corresponding to a pitch of 24 (*i.e.* conductor $80 - 24 = 56$). In either case, the even conductor 80 will have to be connected to another even conductor. It will be found, however, that if we go forward to Group I, we shall involve ourselves in still further difficulties, necessitating further irregularities in the winding; whereas by going back to Group III, thereby reversing the original direction of travel around the rotor circumference, and continuing to step round in this reverse direction, we encounter no further difficulties, the winding proceeding exactly as before, the only difference being in the negative sign of the pitches. We thus complete the winding as follows:—

56—31—8—79
54—29—6—77
52—27—4—75
50—25—2—73—slip-ring

The peculiarities exhibited by the above example are characteristic of every rotor bar winding of the wave type in which the conductors are arranged in two layers. In each case, we have to (1) break the alternate sequence of the pitches after the completion of each revolution; and (2) take a *backward* step, represented by the mean pitch,—thereby reversing the direction of travel—after the completion of half the winding, when the odd and even conductors in alternate groups respectively have been used up.

If, however, the conductors are arranged in a single layer, with evolute end connectors (instead of the "straight-out" or "barrel" type), there is no reason why an even pitch should not be used. In that case, after each revolution, the normal pitch must be increased by unity. Taking the example just considered, but assuming the number of slots to be doubled, so as to bring all the conductors into the same layer, we have the following winding table :—

1—25—49—73
2—26—50—74
3—27—51—75
4—28—52—76
5—29—53—77
6—30—54—78
7—31—55—79
8—32—56—80

The voltage for which a rotor is wound is immaterial * so far as the behaviour of the motor is concerned, and is merely a matter of convenience from a manufacturing point of view. It generally lies between 100 and 400 volts across the slip-rings when the rotor is at rest and the slip-rings are open-circuited.

§ 63. Starting Resistances for Rotors. Internal Short-circuiting and Brush-lifting Gear

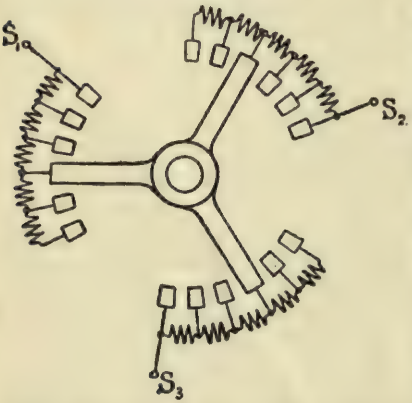

Fig. 93.—Starting Resistance for Wound Rotor.

The usual form of starting resistance for a three-phase rotor is shown diagrammatically in Fig. 93. It consists of a three-armed switch provided with a triple set of contacts and resistances. The movable arms of the switch form the neutral point of the starting resistance. The terminals marked S_1, S_2, and S_3 are in connection with the slip-rings. As the switch is moved over the contacts, the resistances are gradually cut out.

In the case of large motors, it becomes necessary to provide a considerable number of contacts, in order to avoid violent fluctuations in the current (and consequent disturbance of the voltage of the supply system). This increases the cost of the starting switch. The same end may, however, be attained with a comparatively small number of contacts by means of the arrangement

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* Assuming the same amount of copper to be used in each case.

shown in Fig. 94, which is due to Kahlenberg, and is used by Siemens and Halske. It will be noticed that the neutral point of the resistance is represented by a single movable arm, whose contact is of sufficient width to cover three segments. As the arm moves over the contacts, it cuts out a section of the resistance in one phase only at a time; this destroys the balance of the rotor windings, which, however, is not an important point.

Contact with the slip-rings is maintained by means of carbon brushes. In order to do away with the heating which occurs at the contact surfaces when once the motor has been run up to full speed, it is usual to provide an internal short-circuiting arrangement. This consists essentially of a sleeve capable of sliding along the shaft. By means of a lever the sleeve is forced into contact with three springs, which form the ends of the rotor windings, thereby short-circuiting the rotor independently of the brushes. The brushes are now no longer required, and may be lifted off the slip-rings; arrangements for doing this easily are frequently provided.

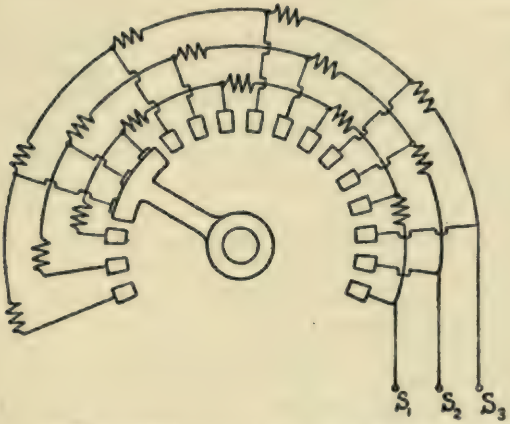

Fig. 94.—Kahlenberg's Starting Resistance.

§ 64. Lewis Induction Motor

The use of wound rotors with slip-rings introduces numerous complications—an external starting resistance, and the somewhat intricate internal short-circuiting and brush-lifting devices. Attempts have, therefore, been made to design motors which combine the advantage of simplicity possessed by a rotor having no slip-rings with the high starting torque and moderate starting current characteristic of the wound rotor designed for use with an external starting resistance. One of the most ingenious solutions of this problem is to be found in the motor recently patented by Mr. F. Lewis, and manufactured by the Electric Construction Co., of Wolverhampton. The construction of this motor is based on the fact that whereas a squirrel-cage rotor may be used in connection with any rotating field, quite independently of the number and sequence of the magnetic poles, a wave-wound rotor will only develop a torque in a field having the correct number and sequence of magnetic poles. In the Lewis motor, the rotor is provided with a double winding—a low-resistance wave winding, which is permanently short-circuited on itself instead of being connected to slip-rings, and a *high-resistance* squirrel-cage winding. At starting, the low-resistance winding is rendered inoperative by reversing the normal polarity of one half of the field. Thus, considering the case of an eight-pole motor, in which the normal sequence of poles is NSNSNSNS, at starting the sequence is arranged to be NSNSSNSN. The rotor, therefore, behaves as if it were provided with the *high-resistance* squirrel-cage winding alone, and starts with a powerful torque. If it were allowed to run on this winding, not only would an excessive temperature rise take place, but the efficiency would also be poor. The normal polarity is therefore restored as soon as the speed has reached a certain limit, and the low-resistance wave winding now comes into play. The speed rises, the slip decreasing, and with it also the currents in the squirrel-cage. Although under normal running conditions the squirrel-cage conductors are still traversed by feeble currents and do part of the driving, yet the bulk of the torque is due to the wave winding, the presence of the squirrel-cage being simply equivalent to a slight increase of cross-section in the conductors of the wave winding. The squirrel-cage consists of conductors of small cross-section arranged in the upper parts of the slots containing the large conductors which form the wave winding.

§ 65. Example of Induction Motor. Variation of Torque with Position of Rotor

In Fig. 95 is shown a 5-h.p. three-phase induction motor, having a short-circuited rotor.* The stator is provided with a three-phase coil winding embedded in open slots, the coils being wound on formers and then fitted into position in the slots. The stator stampings or laminations are supported by a hollow cylindrical

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* The author is indebted to the Electric Construction Co., Ltd., of Wolverhampton, for drawings from which Fig. 95 has been prepared.

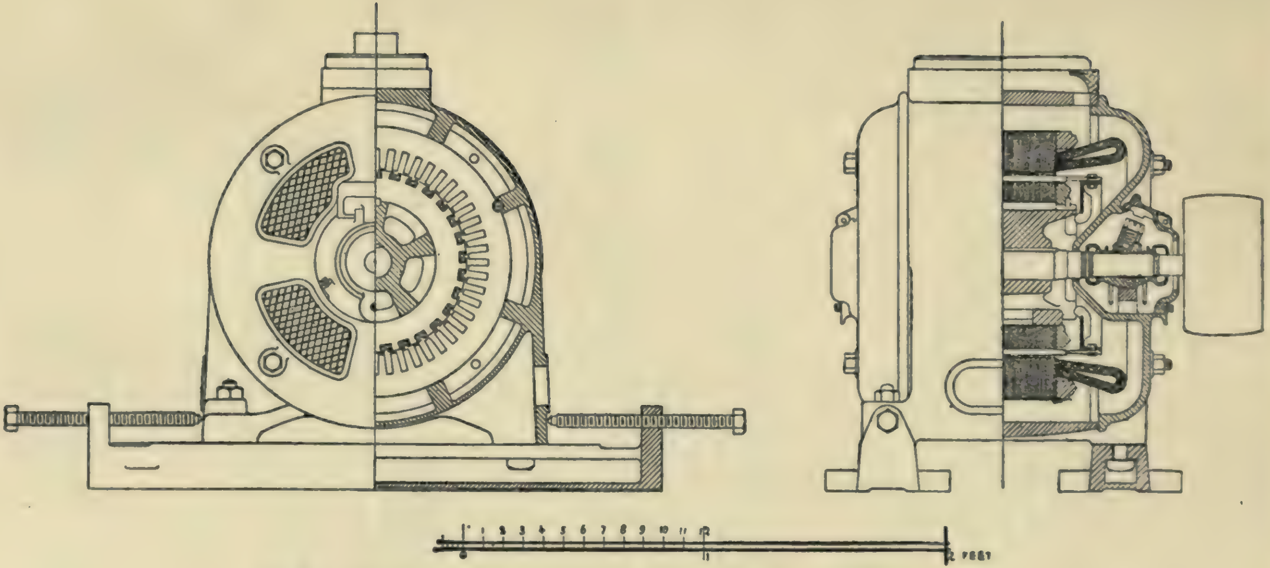

FIG. 95.—Five Horse-power Induction Motor (The Electric Construction Co.).

casting, which carries two end-shields containing the bearings. In order to improve the ventilation, each end-shield has four windows, closed by expanded metal gratings. The bearings are of the usual self-oiling type, each having two oiling rings, which ride loose on the shaft and dip into oil-wells. The rotor core-plates are mounted on a spider, to which they are secured by means of a key. The rotor conductors are bolted to the short-circuiting rings. It will be noticed that whereas there are forty-eight slots in the stator, the number of rotor slots is twenty-nine, so that the two numbers have no common factor. This arrangement is invariably adopted in the case of motors having squirrel-cage rotors, and its object is to prevent variations in the torque with varying position of the rotor; such variations occur to a marked extent when the number of stator and that of rotor slots have a common factor. These variations are due to the fact that in certain positions of the rotor the reluctance of the magnetic circuit is less than in others, and the rotor will always tend to pass from a position of higher to one of lower reluctance. If, *e.g.*, the stator and rotor have the same number of slots, then on exciting the stator the (open-circuited) rotor will tend to move so as to bring about coincidence of the teeth and slots in the two cores.

With a *wound* three-phase motor, however, it is impossible to prevent the stator and rotor slots from having a common factor. For since there must be at least three slots per pole per phase (corresponding to the three phases), the stator and rotor slot numbers must both be multiples of six. It is found, accordingly, that the starting torque of such motors has a larger value in certain positions of the rotor than in others.

§ 66. Asymmetry of Hemi-tropic Stator Winding with Odd Number of Pole-pairs. Method of obtaining Symmetry

When the hemi-tropic type of winding (§ 43) with alternately straight and bent coils (Fig. 77) is adopted for the stator, a slight asymmetry is introduced into the arrangement of the coils in cases where there is an *odd* number of pairs of poles. This will be readily seen by considering a six-pole motor. With a hemi-tropic winding there will be three coils per phase (one coil per *pair* of poles), or a total of nine coils. If we proceed to arrange the coils on the stator core, making them alternately straight and bent, we are left with an odd coil, and in order to fit this in between a bent coil on one side and a straight coil on the other, one half of it must be "straight" and the other "bent." The shape of the projecting ends of the odd coil will thus be somewhat irregular, and will differ from that of the ends of the remaining two sets of coils. This asymmetry may, however, be entirely done away with by adopting the arrangement shown in Fig. 96. Instead of being made alternately straight and bent, the coil ends are here all of the same type, being bent obliquely so as to clear each other.

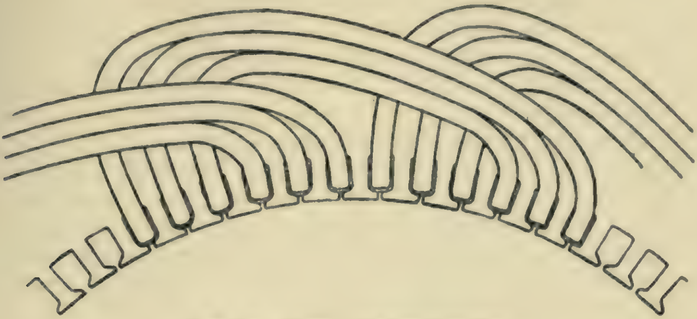

FIG. 96.—Type of Stator Winding.

§ 67. General Characteristics of Induction Motor. Air-gap Length. Effect of Rotor Eccentricity

In its general characteristics the polyphase induction motor resembles the ordinary shunt-wound continuous-current motor. It is essentially a constant-speed motor, the drop of speed from no load to full load being very slight. One of the disadvantages of this type of motor is its somewhat low average power factor, on account of which the current drawn from the mains is larger than it need be for the amount of power developed by the motor. The power factor of an induction motor which is running light is very low, being generally less than 0·2. It increases, however, very rapidly with increase of load. The slip at full load, when expressed as a percentage of the speed of the stator field, varies from about 10 per cent. for very small motors to below 2 per cent. for very large ones. The following table may be taken as representing the average performance of modern induction motors:—

Horse-power	1	5	20	300	1000
Efficiency at full load	80	85	88	93	95
Full-load power factor	0·80	0·84	0·87	0·91	0·94
Slip as percentage of field speed	10	6	4	2	1·2

The power factor of an induction motor depends very largely on the length of air-gap, and it is important to keep the air-gap as small as mechanical considerations will allow. The extreme shortness of the air-gap of induction motors is an important constructional feature, which forms a striking contrast to the relatively very long air-gap of the continuous-current type of motor. Thus, taking the case of a railway motor, the air-gap of the continuous-current type will generally lie between 3 and 7 mm.; whereas an induction motor will have a gap ranging from 1 to 3 mm. Even a 250-h.p. modern induction motor may have a gap as small as 1·5 mm.

Owing to the shortness of the air-gap, exact centering of the rotor relatively to the stator is a matter of great importance, as a slight amount of eccentricity may result in a strong side-pull, tending to bend the shaft. The bearings must be of very ample proportions, so that the wear is inappreciable. The effect of a slight amount of eccentricity in producing a side-pull has formed the subject of several recent investigations. The following very simple formula is given by J. K. Sumec:[*]—

$$P = \frac{B^2}{8\pi} \cdot S\frac{\varepsilon}{\delta} \cdot \frac{1}{\left\{1 - \left(\frac{\varepsilon}{\delta}\right)^2\right\}^{\frac{3}{2}}}$$

where P = total side-pull, in *dynes;* B^2 = mean square of magnetic induction around the rotor periphery, on the assumption that the rotor is exactly coaxial with the stator; S = total cylindrical surface of rotor, in sq. cms.; δ = (single) air-gap; ε = eccentricity (distance between axes of stator and rotor).

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[*] *Zeitschrift für Elektrotechnik* (Wien), vol. xxii. p. 727 (1904).

CHAPTER IX

§ 68. Alternator used as motor. Synchronism—§ 69. Stability of synchronous motor—§ 70. Magnitude and phase of current for various conditions of load. Overload capacity—§ 71. V curves of synchronous motor—§ 72. Condenser action of over-excited synchronous motor. Use of synchronous motor as compensator—§ 73. Hunting of synchronous motor—§ 74. Prevention of hunting—§ 75. Starting of synchronous motor—§ 76. Paralleling or synchronization of alternators—§ 77. Everett-Edgcumbe rotary synchronizer—§ 78. Siemens and Halske three-phase synchronizer—§ 79. Parallel running of alternators. Starting of new machines

§ 68. Alternator used as Motor. Synchronism

A SINGLE-PHASE alternator is, like a continuous-current dynamo, a *reversible* machine—*i.e.* it is capable of being *driven* as a motor when supplied with alternating currents. The possibility of using a single-phase alternator as a motor is immediately obvious. For, during the rotation, each armature conductor comes alternately under cover of poles of north and south polarity. If, then, we send current impulses through the armature winding so timed that they always give rise to a driving torque, a series of impulses will be communicated to the rotor, and the effect will be the same as that of a steady driving torque whose value is equal to the mean value of the fluctuating torque due to the current impulses. Since these current impulses must obviously alternate in direction, a reversal of current taking place in an armature conductor as it passes from a field of one polarity into a field of opposite polarity, it is evident that they will constitute an alternating current. We thus see the possibility of communicating a driving torque to the machine by sending an alternating current of suitable frequency through its armature. The frequency of this current must obviously be the same as that of the e.m.f. generated in the armature coils. Hence the frequency of the alternating current which drives the machine is equal to $P \times \frac{m}{60}$, where P = number of pairs of poles and m = revs. per min. The corresponding speed of the alternator is known as the *synchronous* speed, or speed of *synchronism*, and it is evident that an alternator is only capable of running as a motor at this particular speed.

A polyphase alternator may also be used as a motor. This is at once evident from the fact that the polyphase alternating currents flowing in the armature windings give rise to a rotating field, and if we imagine the magnet wheel to be rotating at the same speed—that of *synchronism*—and to be suitably placed relatively to the field due to the armature currents, a driving torque will be exerted on it. As in the case of a single-phase machine, a polyphase synchronous motor is only capable of running at one particular speed—that of synchronism. For otherwise we should get an irregular succession of driving and retarding impulses, whose mean algebraic value is zero.

Another way of regarding such a polyphase motor is to consider each armature phase as acting independently, and giving rise to a fluctuating driving torque, such as we get in a single-phase machine. The fluctuating torques due to all the phases become fused into a single steady driving torque.

On account of the fact that alternators, whether single- or polyphase, when used as motors are only capable of running at one particular speed—that of synchronism—they are termed *synchronous motors.*

§ 69. Stability of Synchronous Motor

We have assumed the alternating current to be so adjusted as to give rise to the necessary value of the driving torque required to overcome all the resistances to the motion. We shall now show that when a motor is so running, with a definite p.d. across its terminals, it is in a condition of *stability—i.e.* any tendency on the part of the motor to run faster or slower, due to a decrease or increase of load, is *automatically* checked by a suitable change in the magnitude and phase of the current.

It is obvious that in order to provide the necessary driving torque, the succession of impulses contributed by the current in any one phase need not necessarily be of the same sign; we may, for example, have each large driving impulse succeeded by a smaller retarding impulse, so that there is, on the whole, a preponderance of driving impulses, and a resultant mean driving torque. Such a succession of alternate impulses will occur if the reversal of current does not take place at the precise moment when an armature conductor is passing from a field of one polarity into a field of opposite polarity; and corresponding to each wave of current there will be *four* impulses—two large driving ones and two small retarding ones.

Let us suppose that the p.d. across the armature of the synchronous motor and the exciting current supplied to its field are maintained constant. Let V = p.d., E = open-circuit e.m.f. corresponding to given exciting current, and I = armature current per phase. In what follows, we shall consider the action of one phase only, so that the reasoning will apply to both single- and polyphase motors.

Let, in Fig. 97, $OV = V$ denote the p.d., and $OE = E$ the e.m.f. induced in the armature winding. The vector resultant of these two, OR, gives the e.m.f. available for overcoming the armature impedance. The current is represented by $OI = I$, and lags behind OR by an angle whose tangent $= \frac{\text{armature reactance}}{\text{armature resistance}}$.[*] If OI' = projection of OI on OV, and OI'' = projection of OI on OE produced backwards, then $V \times OI' (= VI \cos \underline{/IOV})$ represents the total electrical power supplied to the motor (excitation not included, of course), while $E \times OI'' (= EI \cos \phi)$ represents that portion of the total power which undergoes conversion into mechanical power.

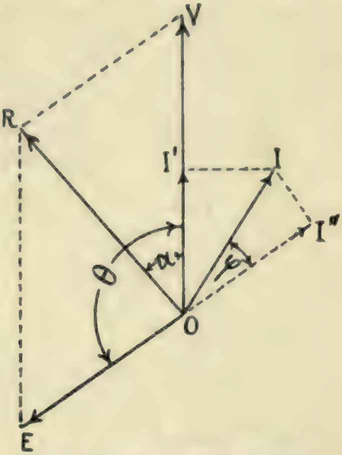

FIG. 97.—Vector Diagram of Synchronous Motor.

If the load is suddenly decreased, acceleration begins to take place, and the vector OE swings forward (*i.e.* in a counter-clockwise direction), gradually gaining in phase on OV. The effect of this is to reduce OR, and so to reduce the current OI in the same ratio. The driving power[†] $E \times OI''$ is thereby decreased, and this decrease will go on until the driving power becomes equal to that required to deal with the decreased load.

The opposite effect takes place with a sudden increase of load. Thus the current taken by the motor automatically adjusts itself, both as regards magnitude and phase, to the exact value required; hence the motor is running under stable conditions.

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[*] It is here assumed that the armature possesses a definite constant self-inductance—an assumption which is by no means legitimate. It must, therefore, be understood clearly that the theory given is only *approximate*.

[†] By the "driving power" is here meant the total power which actually undergoes conversion into mechanical power; the "driving power" therefore includes the power lost in overcoming frictional resistances.

§ 70. Magnitude and Phase of Current for Various Conditions of Load. Overload Capacity

By assuming various positions for OE relatively to OV, we can determine the load with which the motor is capable of dealing for each position of OE, and the magnitudes and phase relations of OR and OI relatively to OV. This might be done graphically, but more accurate results may be obtained by calculation, as follows:—

If θ = angle by which the motor e.m.f. is in advance of the p.d., r = resistance, and pL = reactance of armature, then we have—

$$\mathrm{OR} = (\mathrm{V}^2 + \mathrm{E}^2 + 2\mathrm{VE}\cos\theta)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{I} = \frac{\mathrm{OR}}{\sqrt{r^2 + p^2\mathrm{L}^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\sin\alpha = \frac{\mathrm{E}}{\mathrm{OR}}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\phi = \pi - \left(\theta + \tan^{-1}\frac{\mathrm{L}p}{r} - \alpha\right) = \pi - \tan^{-1}\frac{\mathrm{L}p}{r} - (\theta - \alpha) \quad . \ (4)$$

$$\text{driving power} = \mathrm{EI}\cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The above five equations enable us to find the values of I and the driving power for various values of θ; V and E being maintained constant.

For given values of V and E, the relations connecting θ and I, and θ and driving power, may be graphically exhibited by means of curves. Such curves have been plotted in Figs. 98 and 99, for the case of a motor whose armature has a resistance of 0·2 ohm, and a reactance of 2 ohms, the p.d. being maintained constant at 1000 volts throughout. The different curves relate to different values of the motor e.m.f., *i.e.* to different excitations. The ascending portions (shown dotted) of the power curves in Fig. 99 correspond to a condition of instability. For, let us suppose that the motor is running under conditions corresponding to a point on the dotted branch of one of the power curves, and let there be a slight increase of load. This will cause a retardation of the rotor, *i.e.* a decrease of θ—the angle by which the motor e.m.f. is in advance of the p.d. Now, along the dotted branch of a curve a decrease of θ will be accompanied by a *decrease* of driving power, so that the retardation will go on, and the motor will drop out of step. On the other hand, if we suppose a slight decrease of load to take place, acceleration will result, θ increasing, and with it also the driving power, so that the acceleration will go on until the top of the curve is passed, and some point on the stable descending branch is reached. Along any descending branch, the condition of stability is satisfied, a momentary retardation resulting in an increase of driving power, and a momentary acceleration in a decrease. The conditions represented by the dotted branches of the power curves could therefore exist only momentarily—as, *e.g.*,

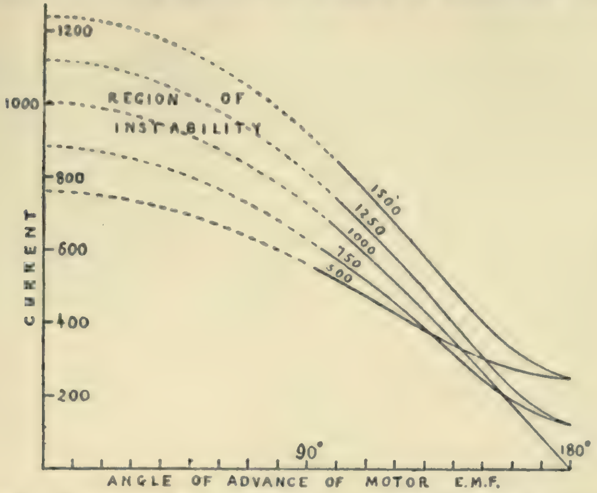

Fig. 98.—Variation of Current with Angle of Advance of Motor e.m.f.

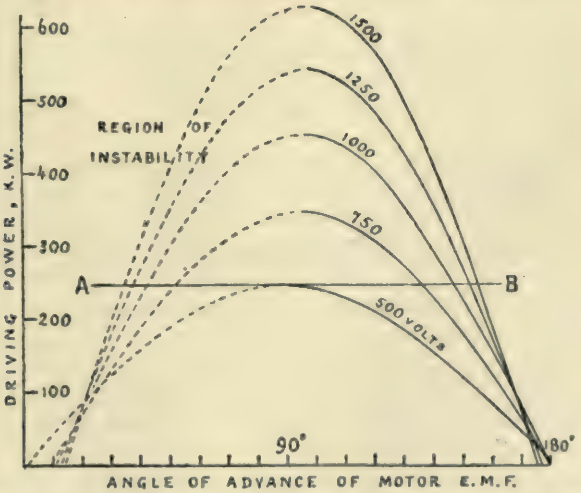

Fig. 99.—Relation connecting Driving Power with Angle of Advance of Motor e.m.f.

when, owing to an extremely heavy overload, the motor is caused to drop out of step. The values of the armature current corresponding to unstable conditions of running are also shown dotted in the current curves of Fig. 98.

The power curves of Fig. 99 show very clearly that corresponding to each excitation (or each value of the motor e.m.f.) there is a certain maximum load—represented by the top of the curve, where it becomes horizontal, and where the stable descending branch passes into the unstable ascending one—beyond which the motor will refuse to run. This maximum load is seen to increase with the excitation, and the relation connecting the maximum load with the motor e.m.f. is represented graphically in Fig. 100. From this it will be seen that if a large overload capacity is desired, the motor e.m.f. should have a relatively large value.*

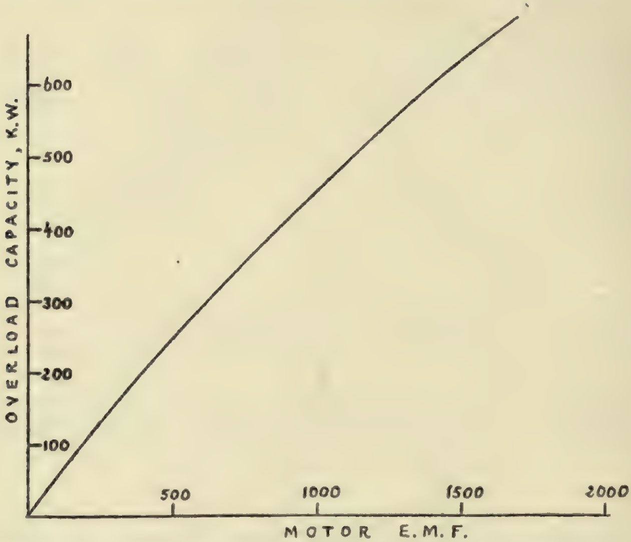

FIG. 100.—Connection between Excitation and Overload Capacity.

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* If the excitation is increased indefinitely, then beyond a certain value the overload capacity *decreases* with further increase of excitation. The value of the excitation corresponding to this is, however, so great as to be practically unrealizable under ordinary conditions of working.

§ 71. V-Curves of Synchronous Motor

By assuming any constant value for the load, and drawing a straight line such as AB in Fig. 99 corresponding to this load, we can determine, from the intersections of this line with the consecutive power curves, the values of θ corresponding to various motor e.m.f.'s. By then referring to the curves of Fig. 98, we can find the values of the armature current corresponding to the different values of θ, and so obtain the relation connecting the armature current with the e.m.f. when the load is maintained constant. One such curve, corresponding to a load of 250 k.w., is shown in Fig. 101. It will be noticed that with this load the motor will not run at all unless its excitation is such that the e.m.f. exceeds about 500 volts. As the exciting current increases, the armature current decreases to a minimum value, and for still higher excitations again increases. This variation of the armature current with varying excitation is an important characteristic of the synchronous motor, first pointed out by Mr. Mordey.* The variations in the armature current for a given range of variation in the excitation are greater at lighter loads, and the curve obtained when the motor is running light is more nearly V-shaped than that

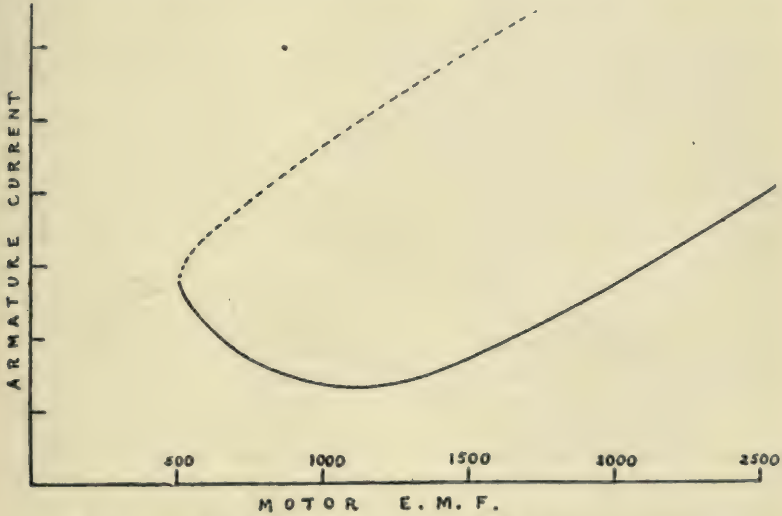

FIG. 101.—Relation connecting Excitation and Armature Current.

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* *Journal of the Institution of Electrical Engineers*, vol. xxii. p. 128.

shown in Fig. 101. On account of their shape, these curves of a synchronous motor are frequently spoken of as its V-curves. The dotted branch represents conditions of instability.

§ 72. Condenser Action of Over-excited Synchronous Motor. Use of Synchronous Motor as Compensator

A reference to the vector diagram of Fig. 97 shows at once that when θ and ϕ are known, we can find the angle VOI $= \psi$, since $\psi = \pi - (\theta + \phi)$. Now, ψ is the angle by which the armature current lags behind the p.d., so that cos ψ gives us the power factor. By once more making use of the power curves of Fig. 99, and equations (1), (3) and (4) of § 70, we can determine the relation connecting motor e.m.f. with the angle ψ for a constant value of the load. We then find that for small values of the e.m.f., ψ is positive—*i.e.* the current lags behind the p.d.; as the excitation is increased, ψ decreases, reaching a zero value (corresponding to a power factor of unity) when the armature current is at its minimum, and beyond this point it assumes a negative value, corresponding to a *leading* current. A strongly excited synchronous motor thus behaves as if it possessed capacity.

The fact that by sufficiently increasing the excitation of a synchronous motor it may be made to take a large *leading* current, has been practically applied in a number of instances, notably in the United States. In cases of power transmission over long distances, the load has not unfrequently a comparatively low power factor, so that a large lagging current has to be transmitted along the line. Not only does this reduce the efficiency of transmission, but—a more important matter—it causes a large drop along the line, rendering satisfactory regulation very difficult. The regulation might be improved by using larger generators at the generating station. But a cheaper solution of the problem has in some cases been found by installing a synchronous motor at the receiving end of the line, the excitation of the motor being adjusted so that the leading current which is taken by it exactly balances the lagging current taken by the load. The wattless current is thereby entirely confined to the local circuit formed by the synchronous motor and the load, while the line current is in phase with the p.d. at the receiving end of the line. The synchronous motor here plays the part of a compensator for the wattless current of the load.

§ 73. Hunting of Synchronous Motor

A trouble which sometimes arises in connection with synchronous motors is that of *hunting*, or *phase-swinging*. By this are meant the persistent periodic fluctuations in the speed and armature current of the motor which are observed under certain conditions. It is not difficult to see how such fluctuations may be started. Let us suppose that the generator supplying the p.d. runs at an absolutely uniform speed, so that the vector OV of Fig. 97 has a perfectly constant speed. Let the motor be also running at a steady speed under a constant load, the angle θ remaining constant. Suppose now that a sudden change of load takes place—say an increase. A momentary retardation of the motor results, and this retardation will go on until the increase in the driving torque (or the driving power), consequent on the decrease of θ (see power curves, Fig. 99), becomes equal to the increase in the resisting torque due to the larger load. When the retardation * ceases, however, the motor is running at a lower speed than that of synchronism. Hence θ will *go on* decreasing; the driving torque will accordingly increase above the value required to overcome the resisting torque, acceleration will take place, and the speed of the motor will increase; when the speed of synchronism is reached, θ will cease to decrease,† and will then begin to increase, since acceleration is still taking place. The vector OE (Fig. 97) now gains on OV. As θ increases, the driving torque decreases, and at a certain stage becomes equal to the resisting torque. Acceleration now ceases; but at this point the motor is running at a speed above synchronism, so that θ will *go on* increasing, and the driving torque decreasing: retardation takes place and synchronous speed is reached, when θ becomes a maximum. Further retardation now takes place, θ begins to decrease, and so on.

We see, then, that any sudden change of load will cause the speed of the motor and the current taken by it to undergo fluctuations. Such fluctuations would be indicated by an ammeter in the armature circuit. The irregular motion of rotation which causes the fluctuations may be regarded as consisting of a uniform motion of rotation at synchronous speed, combined with a to-and-fro or pendular motion. The pendular motion is, in the vector diagram of Fig. 97, represented by the swaying to and fro of the vector OE relatively to the vector OV. Under ordinary circumstances, the pendular motion will die out after a time, owing to the resistances encountered by it; these resistances are due to friction, hysteresis, and eddies.*

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* Retardation = rate of decrease of angular velocity.

† OV and OE (Fig. 97) now having the same angular velocity—that of synchronism—and hence θ becoming momentarily constant.

A sudden change of load is thus seen to start oscillations, which become superposed on the uniform rotation of the motor. There are, however, other ways in which such oscillations may be started. Let, for instance, the load remain quite constant, but let the speed of the generator supplying the p.d. undergo a sudden increase. This corresponds, in the vector diagram of Fig. 97, to a sudden advance of OV towards OE (*i.e.* to a decrease of θ), and it is evident, from what has already been said, that this will start oscillations.

Again, a sudden change in the exciting current of either generator or motor will have a similar effect.

It must be clearly understood that the oscillations under consideration are very *slow* in comparison with the frequency of the alternating current which drives the motor. The vector diagram of Fig. 97 has to revolve a considerable number of times before a single to-and-fro oscillation of OE relatively to OV is completed. Hence it is that such oscillations are readily observed on the ammeter, whose pointer sways to and fro in time with the oscillations.

As already mentioned, oscillations started by any sudden disturbance, such as those we have considered, will gradually subside, their energy becoming dissipated by friction, hysteresis, and eddy currents. If, however, before the oscillations have been damped out a fresh disturbance arises, of such a nature as to reinforce the already existing oscillations, then their amplitude may be considerably increased. With a rapidly and suddenly fluctuating load on the motor, or a generator whose speed fluctuates regularly during each revolution, the disturbances will be repeated at intervals, a fresh disturbance starting new oscillations before those due to the previous disturbance have been damped out. Now, in general, the interval between two disturbances will not bear any definite relation to the natural period of the oscillations; as a result, the disturbances will act in such a way that sometimes they re-inforce the existing oscillations, while at other times they weaken them. An ammeter in the circuit will show this effect very clearly; its pointer sometimes swinging violently—showing that the disturbances are re-inforcing the natural oscillations—while at other times it remains nearly stationary.

A particularly troublesome condition arises when the disturbances have a definite period not differing greatly from the natural period of the oscillations. The time during which a series of reinforcing disturbances is received may then become so considerable, and the amplitude of oscillation of θ may increase to such an extent, as to ultimately carry it beyond the top of the power curve (Fig. 99), into the region of instability, when the motor will "topple over," dropping out of synchronism.

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* The frictional, hysteresis, and eddy-current losses here referred to are merely the *additional* losses brought about by the *oscillations* of the rotor, and must be carefully distinguished from the *ordinary* losses of this nature which are due to the rotation at constant angular speed. Any change in θ causes a shifting of the flux across the pole-face, so that as θ oscillates the magnetic flux sways to and fro across the pole-pieces, giving rise to *additional* hysteresis and eddies, over and above the normal losses occurring when θ has a constant value.

One of the most troublesome consequences of the pendular motion we have just considered, which is variously known as "hunting," "pumping," "surging," or "phase-swinging," are the comparatively large fluctuations in the generator p.d. caused by the current fluctuations. If the generator supplying the motor is also used for feeding incandescent lamps, the hunting of the motor will cause the brightness of the lamps to vary periodically, rendering the lighting extremely unsatisfactory.

§ 74. Prevention of Hunting

One method of reducing the amplitude of the oscillations is immediately suggested by an inspection of the power curves of Fig. 99. From these power curves it is evident that a given change of torque will be obtained with a smaller change of θ when the field is strong than when it is weak. Thus the use of strong fields is favourable in checking hunting.

A highly effective and largely used method of preventing hunting and its troublesome accompaniments is that in which special devices, known as "damping coils" or "dampers," are employed to dissipate the energy of the oscillations.

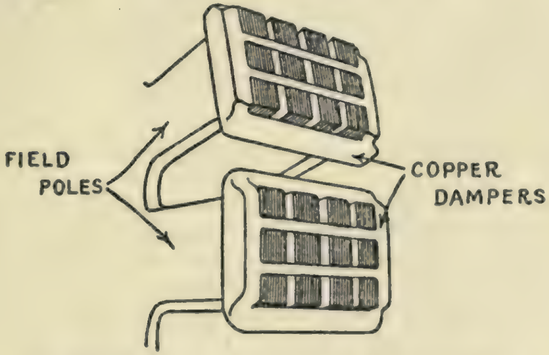

FIG. 102.—Damping Grids fitted to Alternator Poles.

One of the earliest forms of damper consisted of a solid band of copper closely surrounding the top of the pole-piece. This device was not very effective, as it is merely capable of resisting a change in the total flux—*i.e.* fluctuations in the demagnetizing effect of the armature current—but offers no resistance to the periodic distortion of the field, or the swaying of the flux inside the pole-piece (corresponding to fluctuations in the cross-magnetizing effect of the armature current).

A more effective form of damper is that illustrated in Fig. 102. It consists of a regular grid of copper embedded in the pole-piece, the outer bars forming a closed band around the pole-piece.

Leblanc's damper ("amortisseur") is shown in Fig. 103. A series of thick rods of copper is embedded in each pole-shoe, and these rods are connected at each end by heavy rings of copper. The system practically forms a squirrel-cage rotor winding like that used in induction motors, and any oscillations of the flux relatively to the field-poles are damped so powerfully that the arrangement is capable of preventing hunting even in the most troublesome cases.

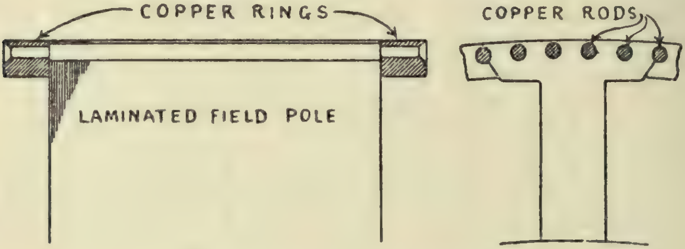

FIG. 103.—Leblanc's "Amortisseur."

§ 75. Starting of Synchronous Motor

In starting a single-phase synchronous motor, it is necessary to run the machine up to the speed of synchronism—*i.e.* up to the speed at which the frequency of the motor e.m.f. equals that of the supply p.d.—and to adjust the phase of the motor e.m.f. so that it is in opposition to the p.d. The excitation is conveniently adjusted to such a value as to make the e.m.f. equal to the p.d. At the instant of closing the armature switch, no current will in that case pass through the armature. If the supply of power to the auxiliary motor used in starting up the synchronous motor be now cut off, the motor will begin to undergo retardation, and receive power as already explained (§ 69). The load may now be put on, and the excitation adjusted to correspond to minimum armature current—*i.e.* to maximum power factor.

The auxiliary motor employed in starting up the synchronous motor may be either an induction motor having a smaller number of poles than the synchronous motor, so that it is capable of running up to a speed slightly above the synchronous speed of the main motor, or it may be the exciter used as a continuous-current motor, and supplied with continuous current from any available source.

The operation of running the motor up to synchronous speed and phase opposition of its e.m.f. relatively to the p.d. is termed *synchronization*. The operation of synchronizing will be considered in detail in connection with the parallel running of alternators (§ 76).

Polyphase synchronous motors are best started in the same way as single-phase motors—by means of a suitable auxiliary motor. There is, however, a method of starting such motors which, although not requiring an auxiliary motor, is not to be recommended, as it is liable to throw severe strains on the motor and to cause serious fluctuations of the generator voltage. The method consists in closing the armature switch, the field circuit being open. In order to limit the current, a transformer with a variable number of turns is used, so that the voltage may be gradually increased as the motor gains speed. The polyphase armature current gives rise to a rotating magnetic field, and the motor starts by virtue of the resistance to relative motion of the rotating field and the field-magnet system caused by hysteresis and eddy currents. It ultimately runs up to synchronous speed. The field current is then switched on.

With this method of starting, the starting current is generally very large, and seriously disturbs the regulation of the system; the ammeters and fuses must be short-circuited to protect them against the excessive starting current. Since at the moment of starting the field coils will act as the secondaries of transformers of which the primaries are represented by the armature winding, a very large e.m.f. will be induced in the field winding if the coils are left connected in series. It therefore becomes necessary to open the field winding at a number of points, the connections being re-established when synchronous speed is reached.

There is a further difficulty connected with this method of starting. When the motor has run up to synchronous speed, the magnetization of its field-poles is due entirely to armature reaction. Now, it may happen that when the field current is switched on, it will reverse the polarity due to armature reaction, thereby also reversing the phase of the motor e.m.f. The motor will be pulled into proper phase relation, but only at the expense of an enormous momentary rush of current and a heavy mechanical strain on the field.*

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* This difficulty may be overcome by providing a suitable arrangement for indicating the polarity of the field due to armature reaction.

§ 76. Paralleling or Synchronization of Alternators

Alternators are capable of being run in parallel like continuous-current generators. In order to throw an additional alternator into parallel with a number of others, it is necessary to adjust its excitation, speed, and phase so that its e.m.f. is about equal to the 'bus bar p.d., is of the same frequency, and in phase opposition to the p.d. This process of adjustment is known as *paralleling* or *synchronization*, and the instrument or collection of apparatus by means of which the proper phase relation is ascertained is termed a *synchronizer*.

The arrangement of synchronizer connections suitable for a single-phase machine is shown in Fig. 104. The synchronizer consists of two small transformers and an incandescent lamp or voltmeter. The primary P_1 of one transformer is across the 'bus bars, while that of the other—P_2—is across the terminals of the incoming machine. The secondaries S_1 and S_2 are connected in series with each other and with a lamp L (or voltmeter), the connections being so arranged that when the e.m.f. of the incoming machine is in phase opposition to the 'bus bar p.d., the e.m.f.'s in the secondaries are added, and the lamp L glows brightly.* The e.m.f. of the incoming machine is adjusted to the desired value by means of the voltmeter V. When the machine

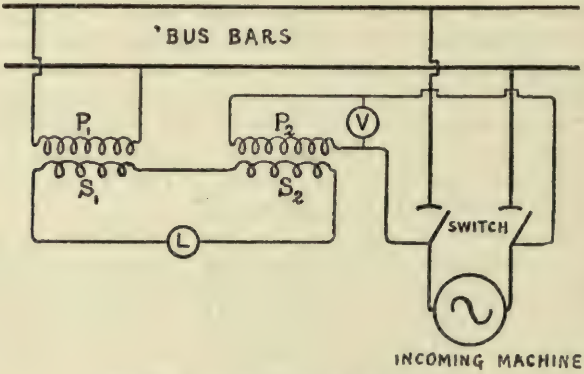

FIG. 104.—Connections of Synchronizer.

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* In some instances, the connections are arranged so that with phase opposition of the 'bus bar p.d. and the machine e.m.f. the secondary e.m.f.'s oppose each other and the lamp L is dark. Since, however, a brightly glowing lamp is far more sensitive to small changes of voltage than a dark one, this arrangement is not to be recommended.

is first started, the e.m.f. in the secondary S_2 is very feeble and of low frequency, and the lamp L is barely aglow. As the amount and frequency of the e.m.f. in S_2 gradually increase, very rapid fluctuations in the light of the lamp become noticeable, and as synchronous speed is approached, the frequency of the fluctuations is reduced, the lamp slowly passing from a brilliant glow to complete darkness, and then slowly lighting up again. These fluctuations are easily explained by the fact that when the frequency of the e.m.f.'s in S_1 and S_2 is nearly, but not quite, the same, one of the e.m.f. vectors (in the vector diagram of e.m.f.'s) slowly gains on the other, gradually passing from coincidence of phase to phase opposition, and then again to coincidence of phase, and so on. The resultant e.m.f., represented by the diagonal of the parallelogram constructed on the two e.m.f. vectors as sides, will thus slowly fluctuate between a maximum and a zero value. When the pulsations are as slow as it is practicable to make them, and the lamp is at its maximum brilliancy, the main switch is closed.

The lamp L forms a convenient visual signal for the engine-driver. In order to enable the switchboard attendant to ascertain more accurately the correct moment of closing the switch, a synchronizing voltmeter is frequently provided in addition to the lamp, the two being connected in parallel; the switch is closed when the voltmeter gives its maximum reading.

In Fig. 104 the primary P_1 of the synchronizing transformer is shown permanently connected across the terminals of the incoming machine. In a large generating station it would not, of course, be necessary to provide a synchronizing transformer for each machine, as the same synchronizer could be used for several machines in succession as required, the transformer being switched off as soon as the main switch has been closed. In order to enable the same synchronizing transformer to be used for all the generators, a set of "synchronizer 'bus-bars" is frequently provided.

It is evident that instead of using two distinct transformers as in Fig. 104, the primaries P_1 and P_2 may be wound on a common core, the secondaries S_1 and S_2 becoming a single secondary winding. The arrangement is thereby rendered cheaper and more compact. This is the *usual practice* when the simple form of synchronizer just described is employed.

Such a synchronizer might also be used in connection with three-phase machines, the primary P_1 of the synchronizing transformer being connected across two of the 'bus bars, and the primary P_2 across the two corresponding terminals of the incoming machine.

A synchronizer of this form, although indicating any difference of frequency or phase, is incapable of showing whether the incoming alternator is running above or below the speed of synchronism.

Hence, the more recent types of synchronizer are provided with additional visual signals, in the form of lamps, which indicate to the engine-driver whether the speed is too low or too high.

§ 77. Everett-Edgcumbe Rotary Synchronizer

The principle underlying the action of this instrument is as follows. Imagine a small induction motor, whose stator and rotor are both provided with polyphase (two- or three-phase) windings, the rotor winding being connected to slip-rings. If polyphase currents of the *same frequency* be supplied to stator and rotor, the magnetic fields due to them will revolve at the same speed, the rotor taking up a position which gives coincidence of the two fields, and remaining *stationary*. It is evident that any relative displacement of the fields will result in a couple tending to turn the rotor one way or the other, according to the direction of displacement. If, therefore, we suppose the frequency of the rotor currents to be reduced below that of the stator currents, the rotor field will tend to lag behind the stator field, and the rotor will be pulled round by the stator *in the direction of rotation* of the stator field, so that the two fields will keep in step as before, the speed of rotation of the rotor corresponding to the difference of the speeds of the stator and rotor fields. Similarly, if the frequency of the rotor currents be increased above that of the stator currents, the rotor field will tend to gain on the stator field, and the rotor will be *pulled back* by the stator field, so as to make both fields keep in step as before. The arrangement, therefore, furnishes an extremely accurate method of comparing two nearly equal frequencies, the rotor rotating one way or the other according as the frequency of the rotor currents is below or above that of the stator currents, and its speed of rotation being directly proportional to the frequency difference.

The stator and rotor cores of the Everett-Edgcumbe rotary synchronizer are shown at the top of Fig. 105. The two-phase currents for the stator and rotor are obtained by adopting the arrangement of connections shown in Fig. 106. It will be seen that the two phases are connected in parallel across the mains,* the necessary phase difference being obtained by the insertion of a high non-inductive resistance (an incandescent lamp) in series with one phase, and a high reactance (a choking coil) in series with the other. The core of the choking coil is shown in the middle of Fig. 105. The rotor runs in ball bearings, and a pointer attached to it, and arranged

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*** The connections are, however, made through two synchronizing transformers, one for the 'bus bars and the other for the incoming machine.**

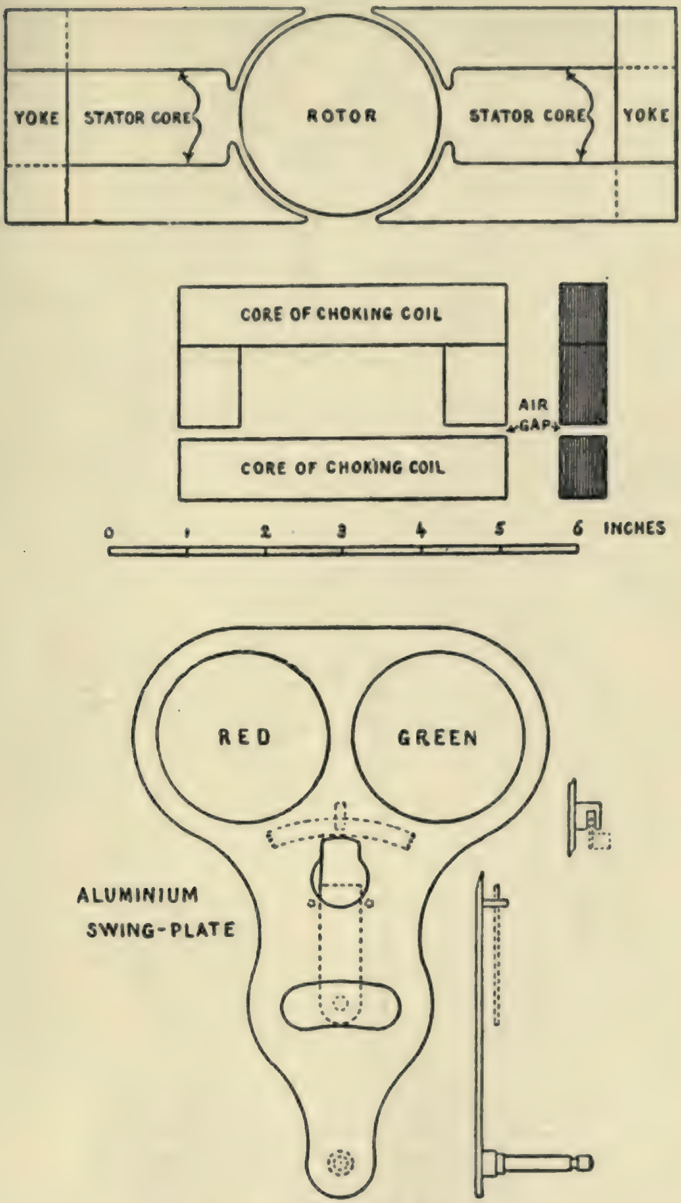

Fig. 105.—Details of Rotary Synchronizer.

to move over the face of the dial-plate, as shown in Fig. 107, indicates by the direction and speed of its rotation whether, and by how much,

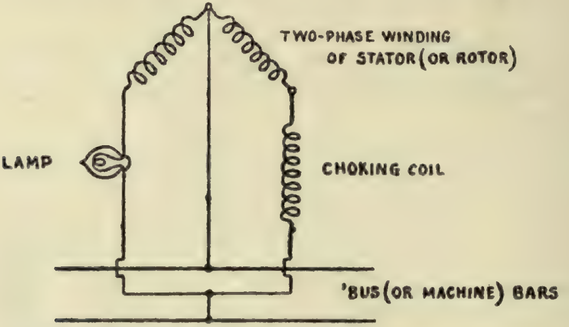

FIG. 106.—Connections of Rotary Synchronizer.

the incoming machine is running fast or slow. The proper time of closing the main switch is when the pointer is vertical and stationary, or moving with extreme slowness.

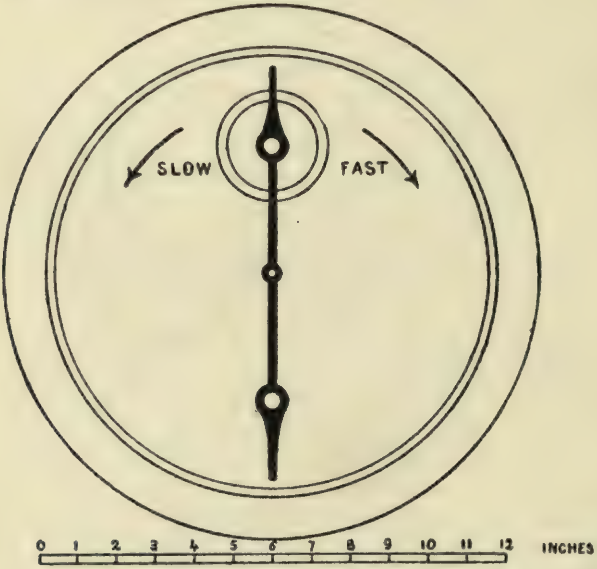

FIG. 107.—Dial-plate and Pointer of Rotary Synchronizer.

A very useful feature of this instrument is the ingenious visual engine-driver's signal. In the upper part of the dial-plate, Fig. 107, is a circular opening, behind which is placed an incandescent lamp. In front of this lamp, and immediately behind the dial-plate, is a pivoted vertical aluminium swing-plate of the shape shown in the lower part of Fig. 105. This plate is fitted with two coloured transparent screens, a red and a green one, and is free to swing to one side or the other, so as to place one or other of the screens in front of the

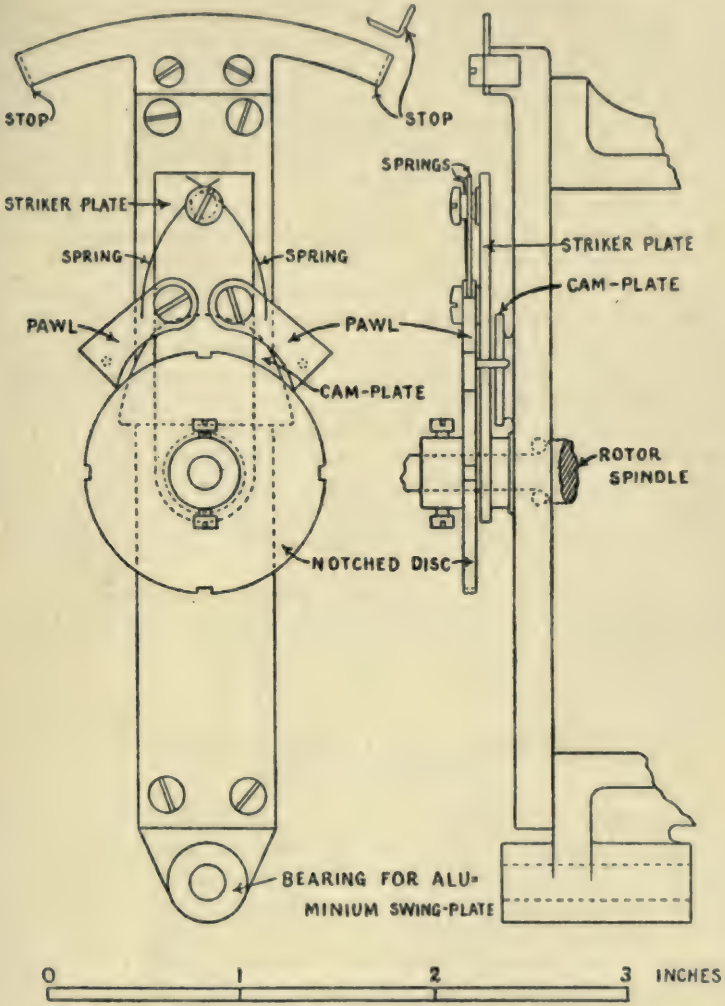

FIG. 108.—Mechanism of Visual Signal.

lamp. A red light indicates to the engine-driver that his engine is running too fast, a green light that it is running too slowly. The motion of the plate is limited by means of a Π-shaped block of brass attached to the back of the plate (Fig. 105), which embraces a circular strip of brass (shown dotted in Fig. 105), whose ends are bent so as to form two stops. This strip is more clearly shown to a larger scale in Fig. 108; it is screwed to the heavy brass plate which contains the front ball bearing of the rotor. The mechanism which throws the swing-plate against one or other of its stops will be understood by reference to Fig. 108. Mounted *loosely* on the rotor spindle is a striker-plate (this is also shown dotted in the lower part of Fig. 105), which is placed between two pins projecting from the back of the swing-plate (Fig. 105). The swing-plate is thereby carried round to one side or the other, according to the motion of the striker-plate. The latter is fitted with two pawls, which, by the action of two light phosphor-bronze springs fixed into them, press against the edge of a notched disc rigidly mounted on the rotor spindle (Fig. 108). Each pawl is provided with a pin, which, in the extreme position of the striker-plate, comes into contact with a cam-plate, thereby lifting the pawl out of the notch in the disc and allowing the disc to revolve without carrying the striker-plate beyond that position. In Fig. 108 the striker-plate is shown in the halfway position; but as soon as the rotor begins to move, the striker-plate will be thrown to the right or left, according to the direction of rotation, and will communicate its motion to the swing-plate.

When this synchronizer is used on a three-phase circuit, its stator is connected across two of the 'bus bars, and its rotor across two of the terminals of the incoming machine.

§ 78. Siemens and Halske Three-phase Synchronizer

Messrs. Siemens and Halske some time ago introduced a form of synchronizer for three-phase machines which has largely come into use. The principle of this synchronizer will be understood by reference to Fig. 109. Let OA, OB, OC be the secondary windings of a transformer whose primaries are across the 'bus bars, and OA′, OB′, OC′ the secondary windings of a transformer whose primaries are across the terminals of the incoming alternator; the connections being such that when the machine is ready to be switched in, the e.m.f.'s in OA and O′A′, OB and O′B′, and OC and O′C′ respectively, are in phase with each other. In that case, it is evident, there will be no p.d. between A and A′, B and B′, and C and C′, so that three lamps connected between these pairs of points would all remain dark. If the incoming machine were running at a speed either slightly below or slightly above synchronism, the lamps would periodically and *simultaneously* brighten up and grow dark.

Let us next suppose that one lamp is connected between A and A′, another between B and C′, and a third between C and B′, as shown in Fig. 109. For the sake of simplicity, we may suppose the neutral points O and O′ of the two secondaries connected together. Let in each case the instantaneous value of the e.m.f. be reckoned positive when acting away from the neutral point. Then for any phase relation of the secondary e.m.f.'s, the e.m.f. acting around the entire circuit which contains any lamp is clearly the vectorial difference of the e.m.f.'s in the two phases of the star-connected windings on either side of the lamp; thus, the e.m.f. acting around OAA′O′O, the circuit which contains the lamp L_1, is the vectorial difference of the e.m.f.'s in OA and O′A′; similarly for the remaining lamps L_2 and L_3. Consider now the instant at which all the e.m.f.'s are in phase, as shown by the vector diagrams (*a*) and (*b*) in Fig. 110. The vector difference of OA and O′A′ is clearly zero, so that L_1 is dark. The vectorial difference of OB and O′C′ is OL_2, as shown in the vector diagram (*c*), and the vectorial difference of OC and O′B′ is OL_3 in diagram (*d*). It is thus evident that since $OL_2 = OL_3$, the lamps L_2 and L_3 will be partly incandesced. Thus at the proper instant for closing the switch L_1 is dark, while L_2 and L_3 are equally bright, but duller than when fully incandesced.

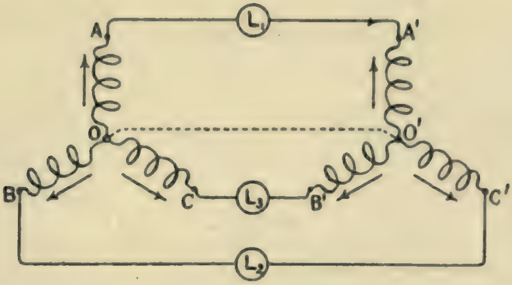

FIG 109.—Three-phase Synchronizer.

Let us now consider the changes which take place when the incoming machine is running below the speed of synchronism, so that the three vectors of the diagram (*b*), Fig. 110, gradually lag behind those of diagram (*a*). It is evident that as the lag increases* the vector difference of OA and OA′ increases, the lamp L_1 beginning to glow. At the same time, the vector difference of OB and O′C′ decreases, owing to the increase of the angle BOC′ in diagram (*c*); the lamp L_2 therefore grows duller. Lastly, the vector difference of OC and O′B′ increases, owing to the decrease in the angle COB′ in diagram (*d*), hence the lamp L_3 grows brighter. Thus L_1 begins to glow, L_2 approaches maximum brightness, while L_3 grows less bright.

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* The vectors are supposed, as usual, to revolve *counter-clockwise*, so that a lag of (*b*) relatively to (*a*) implies a *clockwise* rotation of (*b*) *relatively to* (*a*).

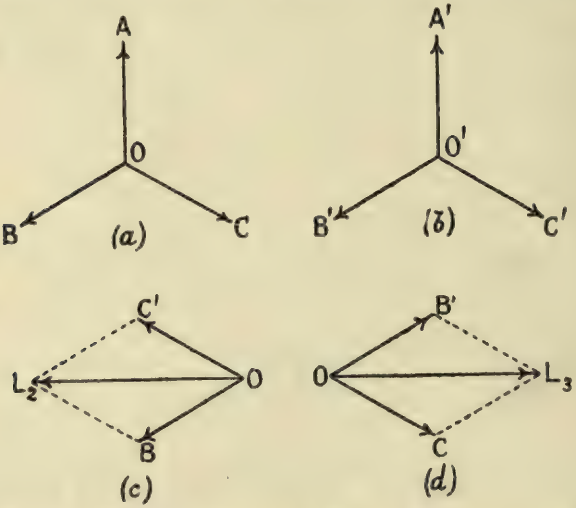

FIG. 110.—To explain Principle of Three-phase Synchronizer.

This goes on until the vector O′B′ in (*b*) comes into phase opposition with OC in (*a*), corresponding to a lag of 60°; at this instant, L_2 is at its maximum brightness, while L_1 and L_3 are both equally bright, but much below maximum brightness. After another increase of 60° in the lag, O′C′ comes into phase coincidence with OB, and L_2 is dark. A further increase of 120° in the lag will bring O′B′ into phase coincidence with OC, and L_3 will become dark.

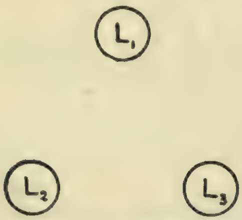

FIG. 111.—Showing Arrangement of Lamps of Three-phase Synchronizer.

We thus see that when the machine is running too slowly, the lamps darken and brighten in the order L_1, L_2, L_3. Similarly, it may be shown that when it is running too fast, the order of brightening or extinction is L_1, L_3, L_2.

Hence, by noticing the order in which the lamps go out we can ascertain whether the speed is too low or too high. In order to make this observation as easy as possible, the lamps are in practice mounted at the corners of an equilateral triangle, as shown in Fig. 111. The light will appear to travel round the triangle, counter-clockwise if the speed is too low, clockwise if it is too high.

In addition to the lamps, two voltmeters are generally provided: one for adjusting the e.m.f. of the incoming machine to the required value, and the other for ascertaining the exact instant of phase opposition with greater accuracy than is afforded by the darkening of the top lamp.

§ 79. Parallel Running of Alternators. Starting of New Machines

Let us suppose that an alternator has been duly synchronized and switched into parallel with other machines. By increasing the supply of power to the engine driving the alternator, it may be made to take part of the load. The phase relation of the alternator e.m.f. to the current and the 'bus bar p.d. is determined by (1) the mechanical power supplied to the alternator, and (2) the excitation.

In Fig. 112 is given a vector diagram—similar to that of Fig. 97—showing the connection between the e.m.f., p.d., and current, and their phase relations. OV, as before, denotes the p.d., V; OE the e.m.f., E; OR the impedance drop in the alternator armature, and OI the current, which lags behind OR by an angle $\tan^{-1}\frac{Lp}{r}$. The total electrical power developed by the alternator is given by E × OI″, while the useful power transmitted to the 'bus bars is V × OI′.

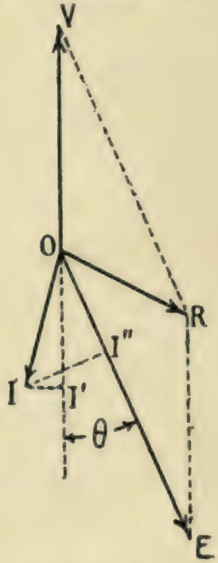

FIG. 112.—Vector Diagram of Parallel-connected Alternator.

By assuming different values for the angle θ, and supposing E to be constant, we can find the relation connecting θ with the useful or the total electrical power. This relation may be plotted in the form of a curve. Corresponding to each value of the exciting current, or to each value of E, we can find such a power curve. By this means a series of power curves similar to those shown in Fig. 99 for a synchronous motor may be obtained. From these curves we see that, within the limits of stability, an increase of θ results in an increase of power. If the torque exerted by the engine fluctuates during each revolution, it will tend to set up oscillations such as those we have already considered in connection with the running of synchronous motors. Any tendency towards hunting or phase-swinging may be counteracted by the use of damping circuits or of sufficiently heavy fly-wheels.

In order to test the connections of the synchronizer and those of the armature coils of a newly installed alternator, a good plan is to couple this alternator in parallel with another which is not running, and to start both machines very slowly and simultaneously. Any error in the coupling of the armature circuits of the new machine will cause the circuit-breakers between the machines to open, while an error in the synchronizer connections will cause this instrument to give wrong indications when the machines are running smoothly in parallel.

CHAPTER X

§ **80**. Regulation of alternators and transformers—§ **81**. Behn-Eschenburg and Kapp's method—§ **82**. Kapp's diagram—§ **83**. Algebraical method of determining drop—§ **84**. Rothert's ampere-turn method—§ **85**. Analysis of armature self-inductance into two components—§ **86**. Experimental determination of two components of armature reactance—§ **87**. Potier's method of predetermining the regulation of an alternator.

§ 80. Regulation of Alternators and Transformers

ONE of the most important problems connected with the behaviour of an alternator or transformer is that of *regulation*. By this is meant the ability of the apparatus to maintain a more or less constant terminal p.d. with varying load. In the case of an alternator, the exciting current is supposed to be constant; in that of a transformer, the primary p.d. is supposed constant. The regulation is frequently expressed in terms of the percentage drop of voltage from no load to full load. This drop depends very largely, as we shall see, on the nature of the load, *i.e.* on its power factor.

The regulation of either an alternator or transformer may be determined by direct experiment if a suitable load and a sufficient amount of power be available for carrying out the test. In many cases, however, the power available is insufficient. Some indirect mode of arriving at the regulation thus becomes desirable. Again, the indirect method of determining the regulation leads to much more accurate results in the case of transformers than could be obtained, under ordinary testing conditions, by a direct test. Numerous attempts have been made to devise such indirect methods in connection with alternators, which present much more difficulty than transformers.

§ 81. Behn-Eschenburg and Kapp's Method

The earliest of these methods is one which is equally applicable to alternators and transformers. Its application to alternators we owe to Behn-Eschenburg,* while Kapp † devised a similar method for transformers. In this method it is assumed that the apparatus under test behaves as if it possessed a definite constant resistance, and a definite constant self-inductance, under all possible conditions of load. These assumptions are practically quite correct in the case of transformers, but by no means so in the case of alternators.

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* *The Electrician*, vol. xxxv. p. 424 (1895).

† *Elektrotechnische Zeitschrift*, vol. xvi. p. 260 (1895).

The values of the resistance and self-inductance are determined by the *short-circuit test.* This consists in short-circuiting the apparatus —the armature of the alternator in the one case, the secondary of the transformer in the other—through an ammeter of negligible resistance and self-inductance, and adjusting the exciting current in the one case, and the primary p.d. in the other, until the full-load current is obtained.* A second test, the *open-circuit test*, is then carried out. The apparatus is open-circuited, and a voltmeter connected across the armature in the one case, the transformer secondary in the other. The reading of this voltmeter is noted when the exciting current in the one case, and the primary p.d. in the other, has the same value as it had in the short-circuit test. Then the ratio $\dfrac{\text{voltmeter reading in open-circuit test}}{\text{ammeter reading in short-circuit test}}$ gives the impedance of the apparatus, and in the method about to be explained this impedance is assumed to remain constant under all conditions of load.

We have next to analyze the impedance into its resistance and reactance components. The resistance of the alternator armature will be approximately equal to its resistance as measured by means of continuous currents (owing to eddy currents, an apparent increase of resistance may be produced amounting, perhaps, to 20 per cent. in extreme cases). In order to find the equivalent resistance r of the transformer,† due to the resistances r_1 and r_2 of its primary and secondary respectively, we notice that, since for a given winding space and given mean length of turn, the resistance varies as the *square* of the number of turns, a resistance r_1 in the primary is equivalent to a resistance $r_1\left(\frac{S_2}{S_1}\right)^2$ in the secondary, S_1 and S_2 denoting the primary and secondary turns respectively. Thus the total equivalent resistance component of the transformer impedance is $r_2 + r_1\left(\frac{S_2}{S_1}\right)^2$.
The reactance component is given by $\sqrt{(\text{impedance})^2 - (\text{resistance})^2}$. Since the resistance component seldom exceeds $\frac{1}{10}$th of the reactance component, its exact determination—involving the allowance to be made for eddy currents—is not a matter of very great importance.‡

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* See, however, § 90.

† Referred to its secondary circuit.

‡ It may be noted that the *alternator* reactance here considered is practically the *total* reactance of the armature (not merely its *leakage* reactance, § 85), *i.e.* it is practically identical with the value of the reactance which would be obtained if the

Having determined the (equivalent) resistance and reactance of our apparatus as explained above, we may construct the vector diagram connecting the constant open-circuit p.d. corresponding to any constant excitation (*i.e.* any constant value of the exciting current in the case of an alternator, and any constant primary p.d. in the case of a transformer) with the terminal p.d. for a given load current of known power factor. Let us provisionally assume the terminal p.d. to be known, and to be represented by OA in Fig. 113. Let ϕ be the angle of lag ($\cos \phi =$ power factor of load) of the current behind the terminal p.d., so that OI gives the direction of the current vector. We obtain the open-circuit p.d. (e.m.f. in case of alternator) by adding *vectorially* to the terminal p.d. OA the resistance drop $r\mathrm{I} = \mathrm{AB}$ (which is parallel to the current vector OI) and the reactance drop $p\mathrm{LI} = \mathrm{BC}$ (this is perpendicular to OI).* We thus get the open-circuit p.d. OC. Instead of considering the resistance and reactance drops separately, we may consider the impedance drop AC, which makes an angle $\tan^{-1} \frac{\mathrm{L}p}{r}$ with the current vector. By reversing the construction, we can easily obtain the terminal p.d.'s corresponding to various load currents, as follows. With O as centre, and radius = constant open-circuit p.d., describe an arc. Draw OA to represent the direction of the terminal p.d. vector, and OI to represent the direction of the current vector. From O draw a line making an angle $\tan^{-1} \frac{\mathrm{L}p}{r}$ with OI, and along it lay off OD to

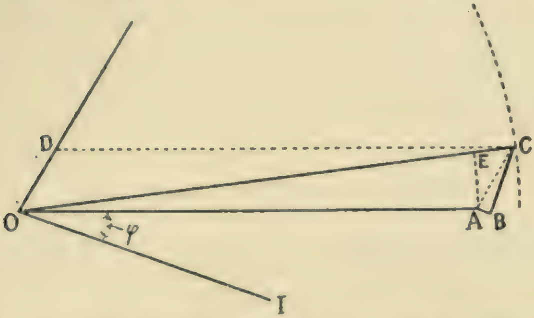

FIG. 113.—Vector Diagram of Loaded Alternator or Transformer.

(synchronously rotating) field were open-circuited and an alternating p.d. applied to the armature from some external source; whereas, in the case of the *transformer*, the reactance here considered is merely its *leakage reactance*, which is enormously smaller than the total reactance (or the reactance obtained by open-circuiting the primary and applying an alternating p.d. to the secondary).

* r = equivalent resistance, L = equivalent inductance, I = load current.

represent the impedance drop corresponding to the given current. Through D draw DC parallel to OA, and through C draw CA parallel to DO. Then OA = OE gives the required terminal p.d., while EC represents the drop corresponding to the given current (the *percentage* drop is $100 \frac{EC}{OC}$). It is evident from the diagram that the value of EC will, for a given load current, depend very largely on ϕ, and that when ϕ assumes a sufficiently large *negative* value—*i.e.* when the current becomes a *leading* one, such as might be obtained with a condenser or over-excited synchronous motor (§ 72) in circuit—EC may become negative, the drop becoming *added* to the open-circuit p.d. to give the terminal p.d. This happens when the angle which OD makes with OA becomes sufficiently obtuse to cause CA to intersect OA *outside* the circle.

§ 82. Kapp's Diagram

The variation of the drop with varying power factor for a given constant value of the load current is best studied by the aid of a very elegant construction due to Kapp.*

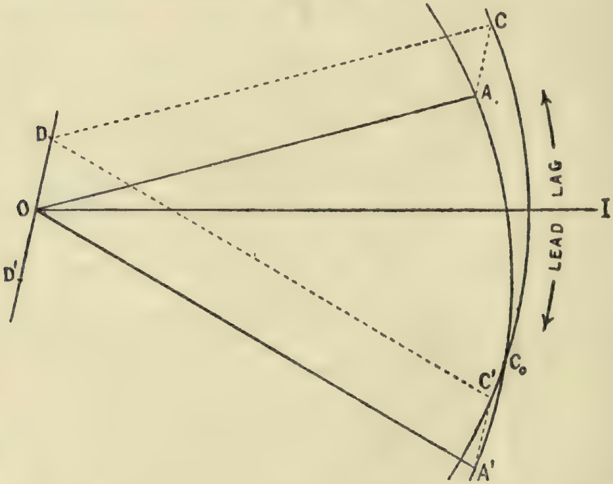

Fig. 114.—Kapp's Construction for Variation of Drop with Power Factor of Load.

If the value of the load current remains constant, the vector OD in Fig. 113 will remain of constant length. Let us assume the directions of OI and OD as the fixed lines of reference in our diagram. As the power factor varies, the point C moves along the circumference of a circle having its centre at O. Now, the position of A is obtained by displacing C through a distance CA equal and parallel to DO. But since DO remains fixed in magnitude and direction, it follows that the locus of A may be derived from that of C by displacing the latter through a distance equal and parallel to DO. It is thus evident that the locus of A is a circle whose centre is at D′, as shown in Fig. 114, where OD′ = DO. The angle IOA is the angle of lag. The diagram shows very clearly the rapid increase in the drop with increasing angle of lag, the drop being represented by the *intercept between the two circles* of the radius vector drawn from O to A.* If the current is a *leading* one, *i.e.* if OA falls *below* OI, the drop steadily decreases with increasing angle of lead, vanishes at the intersection C_0 of the two circles, and beyond this point changes sign, becoming a *rise* instead of a drop.

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* *Loc. cit.*

§ 83. Algebraical Method of determining Drop

Although the graphical methods explained above are useful in exhibiting the relations connecting the various quantities in a manner which enables them to be more easily remembered than would be the case with a purely algebraical method, yet for the purpose of actually determining the drop a purely algebraical method is preferable. Purely graphical methods are, in fact, incapable of giving very accurate results, unless the diagrams are drawn to an inconveniently large scale, and with extreme care. It may be mentioned in this connection that the drop in Figs. 113 and 114 is greatly exaggerated for the purpose of making the diagram clear, and that in actual practice the angle AOC, in the case of transformers, may

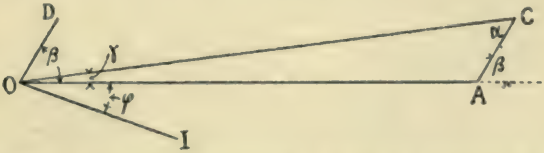

FIG. 115.—To illustrate Algebraical Method of determining Drop.

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* When speaking of the drop in connection with voltage regulation, we mean simply the *arithmetical* difference between the open-circuit p.d. and the terminal p.d. of the loaded apparatus; we do *not* mean the vectorial drop AC, which represents the voltage required to overcome the impedance of the apparatus.

be less than 1°. The value of the terminal p.d. OA is easily calculated from the known values of OC and AC, as follows:—

In the triangle OAC, reproduced for the sake of clearness in Fig. 115, we have—

$$\beta = \tan^{-1}\frac{Lp}{r} - \phi$$

$$\sin\gamma = \frac{AC}{OC}\sin\beta$$

and—

$$OA = \frac{\sin\alpha}{\sin\beta}.OC = \frac{\sin(\beta-\gamma)}{\sin\beta}.OC$$

These three equations enable us to find OA, and so to determine the percentage drop $100\,\frac{OC - OA}{OC}$.

§ 84. Rothert's Ampere-turn Method

The method just explained yields very satisfactory results with transformers, but in the case of alternators it is unsatisfactory, and can only be depended on in cases where the alternator field is well below saturation. With a strongly saturated field, it yields excessive values for the drop, so that it is, at all events, a *safe* method, and has, for this reason, been termed the pessimistic method by Behrend. A different method, involving the vectorial composition of *ampere-turns* instead of e.m.f.'s, has been proposed by Rothert.* For the purpose of applying this method, two curves, the *open-circuit* and the *short-circuit* curves or characteristics, are required. The open-circuit curve is the curve connecting the field current (or field ampere-turns) with the e.m.f. or open-circuit p.d., while the short-circuit curve is the curve connecting the field current with the short-circuit current. In the construction, it is immaterial whether we use field current or field ampere-turns, since the latter are proportional to the former. The armature ampere-turns corresponding to a given current are taken to denote that value of the field ampere-turns which is necessary in order to produce a short-circuit current equal to the given armature current; the *armature ampere-turns are thus obtainable from the short-circuit curve.*

When the armature current is in phase with the e.m.f., armature reaction consists of a cross-magnetizing or distorting effect pure and simple; a lagging current gives, in addition to the cross-magnetizing effect, a demagnetizing one, while a leading current gives a magnetizing effect (§ 47). Hence Rothert, regarding the actual or effective ampere-turns (to which the terminal p.d. is regarded as due) as the vectorial resultant of the field ampere-turns and the armature ampere-turns, compounds these two vectorially, making the angle between the resultant and the armature ampere-turns 90° for zero phase difference, and $90° \pm \phi$ for a phase difference of ϕ degrees, the + sign being taken for a lagging current and the − sign for a leading one.

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* *Elektrotechnische Zeitschrift*, vol. xx. p. 619 (1899).

In this construction, ϕ should, strictly speaking, be taken as the angle of phase difference between the current and the *e.m.f.* But since this angle is not known, we may, as an approximation, assume it to be equal to the angle of phase difference between the current and the p.d. The actual diagram is then constructed as follows. Lay off OR (Fig. 116) to represent the *direction* of the resultant or effective ampere-turns, and draw OA, making an angle $90° + \phi$ with OR, $\cos \phi$ being the given power factor of the load (the current is assumed to lag; for a leading current, the angle AOR would be acute, and equal to $90° - \phi$). Find from the short-circuit curve the value of the armature ampere-turns corresponding to the given load current, and lay off a length OA in the diagram to represent these ampere-turns. With A as centre, and radius AR equal to the given field ampere-turns, describe an arc cutting OR at R. Then OR is taken to correspond to the resultant ampere-turns. Now refer to the open-circuit curve, and find the e.m.f. corresponding to these resultant ampere-turns; this e.m.f. is taken to be equal to the required terminal p.d. corresponding to the given load current, power factor, and excitation.

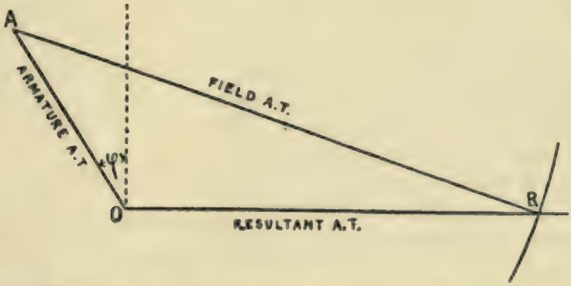

FIG. 116.—Rothert's Ampere-turn Method of determining Drop.

Rothert's method is only capable, like Behn-Eschenburg's, of giving approximately correct results so long as the field is well below saturation. Beyond a certain value of the exciting current, it yields values of the p.d. which are far too high. In this respect, it is an extremely unsafe method, unlike Behn-Eschenburg's, which yields wrong values, but on the safe side. Behrend has termed Rothert's method the *optimistic* method.

§ 85. Analysis of Armature Self-inductance into Two Components

Practical experience having shown that neither of the two methods described yields reliable results, numerous attempts have been made to establish some more satisfactory method. In many of these, the total armature self-inductance is regarded as made up of (1) that portion of the flux due to the armature current which penetrates the field and produces a weakening of it; and (2) the remaining portion, which is made up of armature leakage lines, *i.e.* lines linked with the armature windings, but not passing into the field cores, and hence incapable of affecting the main field strength. Where such a splitting up of the total armature self-inductance is adopted, the first portion of the self-inductance is generally spoken of as *armature reaction*, while the second is termed *armature self-inductance* (in reality, it is only the *leakage* self-inductance).

Looking at the matter from this point of view, and considering the special case in which the armature current lags 90° behind the e.m.f.*—when the drop will have its greatest possible value—it is obvious that the leakage self-inductance e.m.f. will be simply subtracted arithmetically from the open-circuit e.m.f. (the two being in direct opposition of phase), while armature reaction will be equivalent in its effect to a definite reduction in the field ampere-turns. So long as the armature current is constant, both these effects will remain independent of the exciting current. Hence it follows that if we plot, on the same sheet of paper—as in Fig. 117—the open-circuit curve OP and the curve LP′ connecting the terminal p.d. with the exciting current when the wattless armature current is maintained constant (by suitably adjusting the reactance which constitutes the load), the two curves will be so related that any point P′ on the latter may be obtained from a certain point P on the former by the subtraction of a constant amount PR (representing leakage self-inductance drop) from the e.m.f., and the addition of a constant amount RP′ to the field ampere-turns or exciting current (RP′ representing the amount required to balance armature reaction). Now, this is equivalent to a bodily displacement of the open-circuit curve through a distance PP′. Thus LP′ should simply represent OP displaced through this distance. Potier[*] found experimentally that such is the case.

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* This, of course, is an ideal case, as we can never get rid of resistance in the circuit. But with a load of very low resistance and large self-inductance it may be closely approached in practice.

If I be taken to represent the constant wattless current corresponding to the curve LP′, then we may write—

$$PR = E_s I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and—

$$RP' = A_r I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where E_s and A_r are two constants.[†] In order to find the values of these constants, we determine the curve LP′[‡] for any convenient value of I, and, having made a tracing of LP′ and the axis of exciting current, and marked the position of a convenient point P′ on LP′, we displace the tracing, keeping the axis of exciting current parallel to its original direction, until a position is found in which the tracing of LP′ exactly fits over OP. We then prick the position of P′ through the tracing, thus obtaining the corresponding position of P on OP. Through P and P′ lines are drawn parallel to the two axes, and the lengths PR and P′R thus obtained enable us, by using equations (1) and (2), to find E_s and A_r.

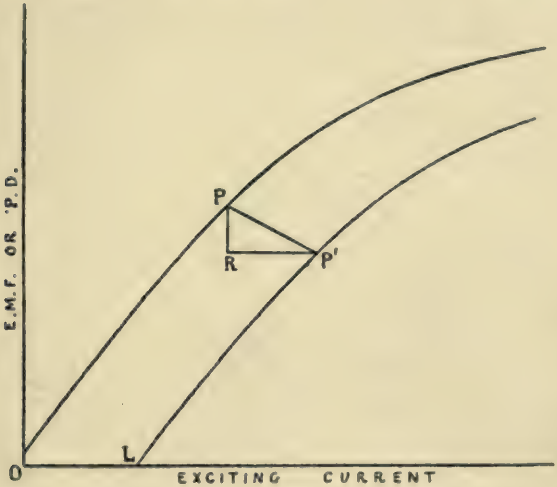

Fig. 117.—Analysis of Armature Reactance into Two Components.

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[*] *Éclairage Électrique*, vol. xxiv. p. 133 (1900).

[†] E_s = reactance corresponding to leakage self-inductance, A = armature reaction constant.

[‡] LP′ is frequently termed the *load characteristic* corresponding to a given armature current and power factor.

§ 86. Experimental Determination of Two Components of Armature Reactance

The method just explained of finding E_s and A_r involves the complete determination of the "load curve" LP′. Blondel * and Fischer-Hinnen † have, however, indicated methods by means of which E_s and A_r may be found from a single point on LP′, involving the taking of a single reading instead of the determination of the entire curve.

Let in Fig. 118 PBA be the open-circuit curve, P′A giving the value of the e.m.f. which corresponds to the excitation OP′, and let us provisionally assume that we know the position of P, where P is that point on the open-circuit curve which corresponds to P′ on the load curve for a wattless armature current equal to the armature short-circuit current at excitation OP′. If the position of P were actually known, E_s and A_r could be found at once.‡ Let I_s = short-circuit armature current at excitation OP′; this may be found from the short-circuit curve. Further, let V = the experimentally determined p.d. corresponding to excitation OP′ and any convenient wattless armature current I (this is the single additional reading required for determining E_s and A_r, besides the open and short-circuit curves). Corresponding to a wattless armature current I, the reduction in the ampere-turns is $P'S = \frac{I}{I_s} \cdot P'R$, and the leakage self-inductance voltage drop is

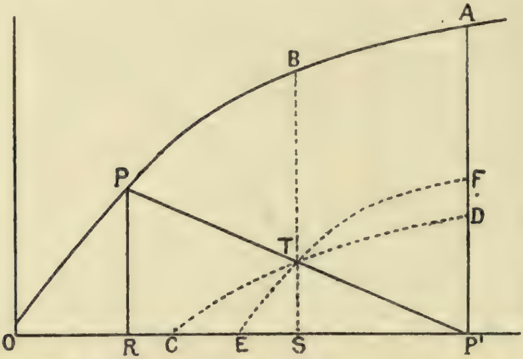

FIG. 118.—Blondel's Determination of Two Components of Armature Reactance.

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* *Elektrotechnische Zeitschrift*, vol. xxii. p. 474 (1901).

† *Ibid.*, vol. xxii. p. 1061 (1901).

‡ $E_s = \frac{PR}{I_s}$, and $A_r = \frac{RP'}{I_s}$, by equations (1) and (2) of § 85; I_s being the short-circuit current at excitation OP′.

$ST = \frac{I}{I_s} \cdot RP$. Hence the p.d. is equal to $SB - ST = TB$, so that the point T obviously lies on a curve CTD, obtained by subtracting from each of the ordinates of the open-circuit curve the constant value $TB = V$. Again, since $P'T = \frac{I}{I_s} \cdot P'P$, it follows that T also lies on a curve ETF, similar to PBA, the centre of similarity being at P′, and the radii drawn from P′ to ETF being in the constant ratio $\frac{I}{I_s}$ to those drawn to PBA. It is now obvious that the position of T may be obtained by the following construction. Draw the curve CTD, by lowering the open-circuit curve through a vertical distance = V. Next, draw ETF similar to the open-circuit curve, the centre of similarity being at P′, and the ratio of the radii vectores drawn from P′ to the two curves being $\frac{I}{I_s}$. The intersection of CTD and ETF gives T, and we then find $E_s = \frac{ST}{I}$, $A_r = \frac{SP'}{I}$.

The method given by Fischer-Hinnen is as follows. In Fig. 119, let $P'V = V$ be the p.d. corresponding to a wattless current I at excitation OP′. Lay off OB to represent the excitation (obtained from the short-circuit curve) required to produce a short-circuit current I. Join BV, and displace the open-circuit curve (by using a tracing) parallel to itself along BV until B coincides with V. The intersection L of the original and the displaced curve enables us to find the direction

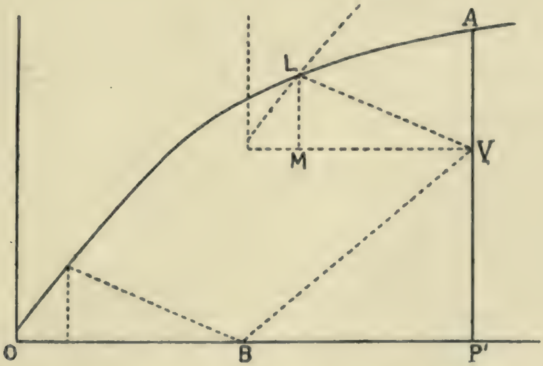

FIG. 119.—Fischer-Hinnen's Method of determining the Two Components of Armature Reactance.

LV (corresponding to PP′ in Fig. 117), and we have $E_s = \frac{LM}{I}$, $A_r = \frac{MV}{I}$.*

§ 87. Potier's Method of Predetermining the Regulation of an Alternator

Making use of the above results, Potier † determines the p.d. corresponding to given excitation, armature current, and external power factor cos ϕ as follows. Rothert's method (Fig. 116) is first used to determine the effective excitation, and from the open-circuit curve the e.m.f. corresponding to this excitation is found. From this e.m.f. is next subtracted vectorially the combined drop corresponding to armature resistance and *leakage* self-inductance, as in Behn-Eschenburg's method.‡ This gives a rough value for the p.d., and also for the angle of phase difference θ between the *e.m.f.* and the current. If θ differs appreciably from ϕ, the entire construction is repeated, using θ in place of ϕ, and a more accurate value of the p.d. is obtained.

Potier's method has been found to give much better results than either Behn-Eschenburg's or Rothert's. This is due to the fact that the variation in the reluctance of the magnetic circuit is taken into account. §

Where accuracy is required, it is advisable to calculate the drop, making use of the diagram only for the purpose of deducing the necessary simple formulæ.

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* In order to secure accuracy, the point A must (in either method) be chosen well above the knee of the open-circuit curve, otherwise the intersection (T in Fig. 118, and L in Fig. 119) will take place at a very acute angle, and accuracy will be impossible.

† *Elektrotechnische Zeitschrift*, vol. xxii. p. 1061 (1901).

‡ Using, however, the *leakage* self-inductance drop instead of the *total* self-inductance drop.

§ Behn-Eschenburg has recently (*The Electrical Engineer*, vol. xxxiii. p. 979 (1904)) published a more exact and elaborate method than that originally proposed by him, and an almost identical method was published, practically at the same time, by Torda-Heymann (*The Electrician*, vol. liii. p. 6 (1904)). This method involves the application of Kirchhoff's well-known network laws to the magnetic circuits of the alternator. As, however, the theory of the method is much more complicated than that of Potier's elegant construction, and as the accuracy of the results does not appear to be any greater, we do not propose to consider this method here in detail.

A very elaborate and exhaustive investigation of the voltage drop in alternators has recently been published by Messrs. Henderson and Nicholson, *Journal of the Institution of Electrical Engineers*, vol. xxxiv. p. 465 (1905).

CHAPTER XI

§ **88**. Resistance measurements of alternating-current machinery—§ **89**. Insulation tests—§ **90**. Experimental determination of equivalent impedance of transformer—§ **91**. Transformer efficiency tests—§ **92**. Separation of core losses—§ **93**. Heating test of transformer—§ **94**. Ageing of transformer core—§ **95**. Determination of open-circuit and short-circuit characteristics of alternator—§ **96**. Hopkinson efficiency test—§ **97**. Determination of no-load losses—§ **98**. Determination of total losses—§ **99**. Retardation method of determining losses—§ **100**. Heating test of alternator. Mordey's and Behrend's methods—§ **101**. Goldschmidt's method—§ **102**. Hobart and Punga's method.

§ 88. Resistance Measurements of Alternating-current Machinery

Some of the tests which have to be carried out in connection with alternators and transformers are common to almost every class of electrical machinery. Thus we have to determine the resistances of the windings and the insulation resistances, and to test for dielectric strength. We have, further, to find the maximum temperature rise under normal working conditions.

The conductor resistances are most conveniently determined by sending a suitable current from a battery of secondary cells through the coil or coils under test, and measuring the drop of potential. In the case of three-phase apparatus, we may have either a mesh or a star connection of the windings. With the former arrangement of circuits, the resistance of each phase (*i.e.* of each side of the triangle formed by the windings) is clearly equal to 1½ *times* the measured resistance between two terminals, while with a star connection the resistance of each phase is *one-half* of the measured resistance between two terminals.*

The voltage absorbed by the resistance of the windings if expressed as a percentage of the total voltage may, in the case of a *balanced*

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* Let x = resistance of each phase of a Δ system. The resistance between any two terminals is then the joint resistance of one phase connected in parallel with the other two joined in series. It is, therefore, $\frac{x \cdot 2x}{x + 2x} = \frac{2}{3}x$, so that $x = \frac{3}{2} \times$ measured resistance. In the case of a star-connected system, it is obvious that between any two terminals there are two phases in series with each other, so that the measured resistance is twice that of one phase.

system, be calculated from the measured resistance between two terminals and the current in each line wire, without any knowledge of how the windings are connected. For if V = voltage between line wires, I = current in each line wire, and r = measured resistance between any two terminals, then in a mesh-connected system we have for the resistance per phase $\frac{3}{2}r$, for the current per phase $\frac{\mathrm{I}}{\sqrt{3}}$, and for the voltage per phase V, so that the percentage drop is $100\frac{\frac{3}{2}r\frac{\mathrm{I}}{\sqrt{3}}}{\mathrm{V}} = 100 \cdot \frac{\sqrt{3}}{2\mathrm{V}}r\mathrm{I}$. In a star-connected system, the resistance per phase is $\frac{1}{2}r$, the current per phase is I, and the voltage per phase is $\frac{\mathrm{V}}{\sqrt{3}}$, the percentage drop amounting to $100\frac{\frac{1}{2}r\mathrm{I}}{\frac{\mathrm{V}}{\sqrt{3}}} = 100 \cdot \frac{\sqrt{3}}{2\mathrm{V}}r\mathrm{I}$—an expression identical with that obtained for a mesh-connected system.

It must be remembered, in dealing with alternating-current machinery, that the resistance as measured by means of a continuous current is always *less* than the true resistance corresponding to an alternating current, on account of eddy-current loss. With conductors of small cross-section, the difference is inappreciable; but with large conductors the eddy-current loss may be considerable, amounting to as much as 20 per cent. of the loss calculated from the resistance as determined by means of continuous currents.

§ 89. Insulation Tests

A considerable amount of importance was formerly attributed to insulation resistance tests. Such tests are now, however, regarded as of little value. The insulation resistance is an extremely variable quantity, and measurements of it are liable to be misleading, both when the results are very high and somewhat low.* A much more important test is that for *dielectric strength.* By means of a suitable testing transformer, a voltage considerably in excess of the normal working voltage is applied to the winding for a definite period, and the test may be regarded as satisfactory if no breakdown takes place during that time. The dielectric strength test is applied to the insulation between the winding and the core, and also to the insulation between different windings if there are several on the same core. The test should be applied to the machine when *hot*, after a load run of several hours' duration.

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* It is, in any case, unreasonable to expect the insulation resistance to exceed 1 megohm.

The value of the testing voltage is determined by the normal working voltage of the winding under test. Now, since the factor of safety in the case of low-voltage windings is much higher than that which it is possible to attain with high-voltage windings, the ratio $\frac{\text{testing voltage}}{\text{working voltage}}$ should clearly be less for the higher voltages, as otherwise there may be serious danger of damaging the winding. Again, the application of a moderately high voltage over a prolonged period must be regarded as furnishing a more satisfactory—and less dangerous—test of dielectric strength than that of a very much higher voltage during a shorter period.

For voltages up to 5000 volts, the testing voltage may be double the working voltage. Between 5000 and 10,000, the testing voltage may be 5000 volts in excess of the working voltage, while for any voltage exceeding 10,000 the testing voltage should not exceed 1½ times the working voltage. The testing voltage should be applied for ½ hour.*

§ 90. Experimental Determination of Equivalent Impedance of Transformer

For the purpose of determining the regulation † of a transformer, we have to find its equivalent *reactance*. This may be done by short-circuiting the secondary, and applying a p.d. to the primary sufficient to produce the full-load currents in the windings. In order to eliminate any error due to the resistance and reactance of the ammeter, it is preferable to place this in the primary circuit. If V = primary p.d. required to produce the full-load current I_1 in the primary of the short-circuited transformer, then $\frac{V}{I_1}$ gives us the equivalent impedance of the short-circuited transformer, the equivalent resistance being $r_1 + \left(\frac{S_1}{S_2}\right)^2 r_2$, where r_1, r_2 are the resistances, and S_1, S_2 the turns in the primary and secondary respectively. The transformer behaves as if there were no magnetic leakage and no resistance in its coils, but as if connected in series with its primary there were an impedance $\frac{V_s}{I_1}$, the resistance component of this impedance being $r_1 + \left(\frac{S_1}{S_2}\right)^2 r_2$. Or, if we prefer to refer everything to the secondary circuit,* the transformer behaves as if magnetic leakage and coil resistance were absent, and as if in series with the secondary there were connected an impedance $\left(\frac{S_2}{S_1}\right)^2 \frac{V_s}{I_1}$, whose resistance component is $r_2 + \left(\frac{S_2}{S_1}\right)^2 r_1$.

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* The above rules are those adopted by the German Elektrotechnischer Verein.

† See §§ 80–83.

When the transformer is short-circuited, the induction in its core is very small, and hence the magnetic leakage is not quite the same as that occurring at full load. In order to overcome this objection to the method of measuring leakage reactance just explained, Berry recommends † measuring the impedance which corresponds to normal working conditions, the arrangement of connections being as shown in Fig. 120. The method can only be used if *two* similar transformers are available. The secondaries S_1 and S_2 are connected in series with each other, the connections being so arranged that the e.m.f.'s induced in them *oppose* each other. One of the primaries, P_2, is connected directly across mains between which the normal working primary p.d. is maintained. If the primary P_1 of the other transformer were similarly connected, the secondary e.m.f.'s would exactly balance each other, there would be no secondary current, and each primary would take only its small magnetizing current; the transformers would, in fact, behave as if their secondaries were open-circuited. In order to cause the full-load currents to flow in the transformer windings, a small auxiliary transformer is used, whose primary P is connected in series with an adjustable resistance R, while its secondary S is connected in series with P_1, the e.m.f.

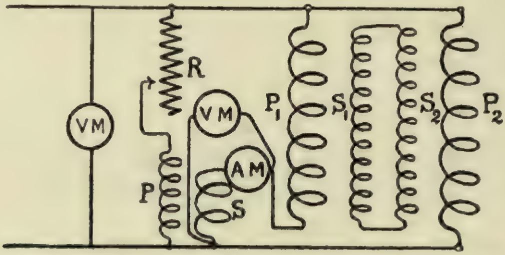

Fig. 120.—Berry's Method of determining the Equivalent Impedance of a Transformer.

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* As was done in § 81.

† "Modern Electric Practice," vol. ii. p. 86.

induced in S being added to the p.d. which exists across the mains. *Half* the reading of the voltmeter across S gives the impedance drop in *one* transformer.

§ 91. Transformer Efficiency Tests

Although the efficiency of a transformer might be determined by measuring the useful power across its secondary terminals, and the total power supplied to the primary, and dividing the former by the latter, this method is neither so exact nor so convenient as methods in which the *lost power* is measured directly. A very simple method of carrying out the efficiency test, originally suggested by Dr. Sumpner, consists in finding, by means of a wattmeter, (1) the core loss and (2) the copper loss at full load. Since the core loss remains sensibly constant at all loads,* its value at full load may be taken to be the same as on open circuit. Hence the core loss is equal to the power supplied to the primary when the secondary is open-circuited.† In order to measure the copper loss, the secondary is short-circuited (as in the regulation test), and the primary supplied at a p.d. sufficient to produce the full-load currents in the windings. The core induction in this case is so small that the hysteresis and eddy-current losses in the core may be neglected, and the power supplied to the primary—measured, as in the first test, by means of a wattmeter—taken to represent the copper loss. The sum of the core and copper losses being known, the efficiency is easily calculated.

Another method of determining transformer efficiency, also due to Dr. Sumpner, consists in a modification of the well-known Hopkinson test for dynamos. Two similar transformers are required for this test. Let us suppose the primaries of the transformers connected across the mains, and the secondaries connected so that their e.m.f.'s oppose each other, and let, as shown in Fig. 121, a wattmeter W_1 be connected up to measure the power supplied to the two transformers, r_1 denoting the non-inductive resistance in series with the fine-wire

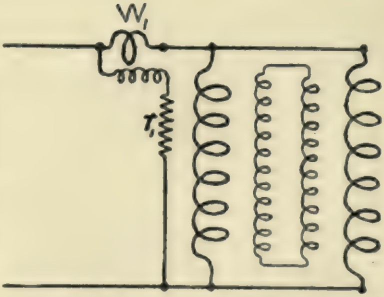

FIG. 121.—Measurement of Core Losses of Two Similar Transformers.

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* See § 53.

† Strictly speaking, this power also includes the (extremely small) copper loss in the primary due to the magnetizing or no-load current.

coil of the wattmeter. It is evident that W_1 measures the sum of the iron or core losses in the transformers. Let next the balance of the secondary e.m.f.'s be disturbed by means of an auxiliary transformer, as shown in Fig. 122, the resistance R in series with the primary P of the auxiliary transformer being so adjusted as to produce the desired current in the primary P_1 of the first transformer, the current being measured by the ammeter AM. The wattmeter W_2 measures the power supplied by the auxiliary transformer. This power corresponds to the copper losses occurring in the transformers—as will be evident from Fig. 123, where the two transformers under test are shown entirely disconnected from the mains. Since the secondary e.m.f.'s are now added together instead of opposing each other, it is evident that each transformer behaves as if its secondary were independently short-circuited, and the wattmeter W_2 measures the sum of the copper losses. Now, the mere connection of the primaries of the transformers across the mains—as in Fig. 122—simply introduces two equal and opposite p.d.'s into the *local* circuit formed by P_1, P_2, and S, and in no way affects the *local* current, *i.e.* the current passing through AM and the thick-wire wattmeter coil. Thus the

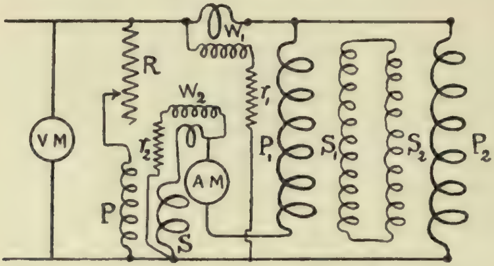

Fig. 122.—Sumpner's Transformer Efficiency Test.

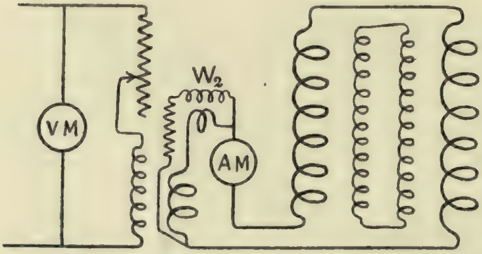

Fig. 123.—Measurement of Copper Losses of Two Similar Transformers.

reading of W_2 will remain unaltered, and be the same in Fig. 122 as in Fig. 123. The sum of the readings of W_1 and W_2* gives the total loss of power in the two transformers, and half this sum represents the loss in each transformer. From the loss and the load (as determined by the reading of AM) the efficiency is easily determined.†

One of the great advantages of this method lies in the fact that it enables a full-load test of two large transformers to be carried out with the expenditure of only a small amount of energy—the power drawn from the mains simply representing the losses in the transformers.

The efficiency test just described may be conveniently combined with the leakage reactance test explained in § 90.

§ 92. Separation of Core Losses

In many cases, it is interesting to separate the core losses into their hysteresis and eddy-current components. Such a separation may be effected by a method due to Carhart.‡ The principle of the method is based on the fact that, for a given constant value of the maximum induction B, the eddy-current loss varies as the square of the frequency (see § 54), while the hysteresis loss varies in simple proportion to the frequency. Hence the total core loss at any frequency n may be written—

$$w = \mathrm{E}n^2 + \mathrm{H}n$$

w denoting the total watts lost in the core, $\mathrm{E}n^2$ the eddy-current watts, and $\mathrm{H}n$ the hysteresis watts; E and H being two constants for a constant value of B.

If, then, we carefully determine w at two different frequencies, n_1 and n_2, using the same value of B, the two equations—

$$w_1 = \mathrm{E}n_1^2 + \mathrm{H}n_1$$
$$w_2 = \mathrm{E}n_2^2 + \mathrm{H}n_2$$

enable us to find E and H, and thus to calculate separately the hysteresis and eddy-current losses occurring at any frequency—always assuming B to remain constant.

Instead of determining only two values—w_1 and w_2—of w at two different frequencies, it is advisable to take an entire series of readings connecting w and n, to plot the results on squared paper, and select two well-defined points on the curve for the purpose of finding E and H.

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* Corrected, if necessary, for any loss of power in the connecting wires and instruments.

† Efficiency $= \dfrac{\text{load}}{\text{load} + \text{loss}}$. The load may be practically taken to be VA, where V = p.d. of mains and A = reading of the ammeter AM.

‡ *Electrical World*, vol. xxxi. p. 306 (1898).

A practically constant value of B with varying frequency is easily secured by keeping the exciting current of the alternator used in the experiment constant, and running the machine at different speeds, so as to obtain different frequencies.

§ 93. Heating Test of Transformer

An important test—which applies to all classes of electrical machinery—is the heating test. This is carried out for the purpose of ascertaining the temperature to which the windings of a transformer will ultimately rise when the transformer is kept fully loaded. The importance of this test is due to the fact that when the temperature exceeds a certain limit, the insulation of the windings undergoes rapid deterioration, and frequent renewals will be necessary. The safe temperature limit for the windings may be taken as 90° C. Since the temperature of the surroundings may at times be as high as 35°, it follows that 55° C. must be regarded as the maximum permissible rise of temperature for any portion of the windings. But since the temperature of the coil is not uniform, it is advisable to fix the limit for the mean temperature rise of the coil at 50° C.

The most reliable method of determining the temperature of a coil is the increase of resistance method. The resistance is measured before commencing the test, and is then periodically measured as the test proceeds. The temperature coefficient of copper may be taken as 0·004, so that if $r_c =$ resistance of coil, measured at a *known* temperature—say t—just before the commencement of the test, and $r_0 =$ resistance of the same coil at 0^0 C., then—

$$r_0 = \frac{r_c}{1 + 0{\cdot}004t_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If after a few hours' run the resistance is found to have reached the practically steady value r, the corresponding mean temperature of the coil is given by—

$$t = 250\frac{r - r_0}{r_0}$$

and since r_0 is known, being given by (1), t may be calculated, and so the rise of temperature found by subtracting from t the temperature of the surroundings at the end of the test.

In Fig. 124 are shown the results of a heating test for an oil-cooled transformer, the upper or full-line curve giving the temperature rise of the windings as determined by the increase of resistance method, and the dotted curve giving the temperature rise of the oil as measured by a thermometer.

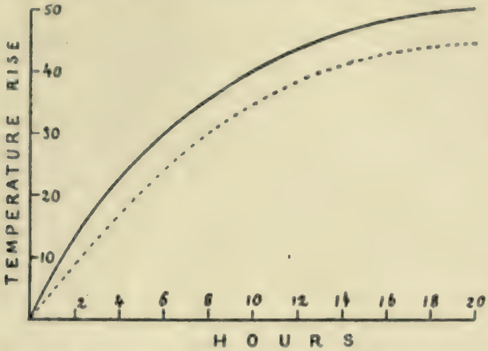

Fig. 124.—Showing Temperature Rise in Transformer.

In carrying out this test, the most convenient and economical method is to employ the arrangement of connections used in the Sumpner efficiency test, and shown in Fig. 122.

§ 94. Ageing of Transformer Core

The magnetic ageing test is one of great importance in connection with transformers. It was originally noticed by Mr. Partridge that the core loss of certain transformers gradually increased in course of time. The effect has since been dealt with by a large number of investigators.* The slow increase in the core loss, or the *ageing* of the core stampings, has been shown to be due to the maintenance of the core at a moderately high temperature by the losses taking place in it, and a precisely similar effect, it was found, could be brought about by simply baking the core in an oven maintained at the proper temperature.

Manufacturers of transformer sheets have now succeeded in producing a *non-ageing* material, *i.e.* one whose hysteresis loss is unaffected by long-continued baking. The only method of applying the ageing test is to go on measuring the core loss periodically for some time after the transformer has been installed. If after a few months' use no appreciable increase has taken place in the core loss, the transformer may be regarded as fulfilling the non-ageing requirement.

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* The most complete investigation of the ageing effect is that carried out by Roget, see *The Electrician*, vol. xli. p. 182 (1898), also *Roy. Soc. Proc.*, vol. lxiv. p. 150 (1899).

§ 95. Determination of Open-circuit and Short-circuit Characteristics of Alternator

The determination of the open-circuit and short-circuit characteristics of an alternator presents no difficulty, and involves the use of only a small fraction of the power required to drive the machine when fully loaded. In finding the short-circuit characteristic, the armature short-circuit current may be increased up to three or even more times the normal full-load value of the current, provided care is taken not to maintain it for too long a period. As the readings for both characteristics are slightly different with ascending and descending values of the exciting current, it is advisable to determine both branches of the curve, and to use the mean in calculations regarding the regulation of the alternator. It will, of course, be understood that in obtaining the short-circuit curve of a polyphase generator *all* the phases must be simultaneously short-circuited.

Potier's method (see §§ 86 and 87) requires the determination, in addition to the short-circuit and open-circuit curves, of at least one point on the load curve corresponding to a highly inductive load. The load being highly inductive, it follows that this test also can be carried out with a small expenditure of power, suitable choking coils being used as the load.

§ 96. Hopkinson Efficiency Test

As in the case of dynamos and transformers, the efficiency test of an alternator is most economically and conveniently carried out by the use of an indirect method, the power lost in the alternator being measured directly.

Where two similar alternators are available, the Hopkinson test may be used. The shafts of the alternators are coupled to each other and to a continuous-current motor, which is large enough to supply the power corresponding to the losses. This motor is first carefully tested for efficiency, so that the exact amount of mechanical power transmitted by it to the alternators is known for any given values of the p.d. and current supplied to the motor. By weakening the field of one of the alternators, any desired current may be made to circulate in the local circuit of the two armatures, which are short-circuited on each other through an ammeter; the machine with the stronger field acting as a generator, while that with the weaker field acts as a motor. If the relative positions of the alternator armatures are such that their e.m.f.'s are in direct opposition, the power factor will be practically unity. A test at any other power factor may be carried out by altering the coupling so that there is a corresponding phase difference between the alternator e.m.f.'s.

§ 97. Determination of No-load Losses

The losses due to hysteresis, friction, and eddy currents may be measured by coupling the alternator to a standardized motor (*i.e.* a motor of known efficiency), and finding the power required to drive the alternator on open circuit when excited with the normal exciting current. If to this is added the armature copper loss, which may be calculated from the known values of the armature resistance and current, the total loss taking place in the machine is determined, and the efficiency may be calculated.

Another simple method of finding the open-circuit or no-load losses consists in running the alternator as a synchronous motor, the normal exciting current being used, and the p.d. across the armature terminals being adjusted to correspond to minimum armature current. The power supplied to the alternator when corrected for the small armature copper loss gives the power lost in hysteresis, friction, and eddy currents.

§ 98. Determination of Total Losses

It must be remembered, however, that owing to field distortion the actual hysteresis and eddy-current losses under full-load running conditions will in general be somewhat greater than those determined by either of the two methods just explained. If two similar alternators are available, the total losses under full-load running conditions may be approximately determined as follows. One of the alternators is coupled to a standardized motor, and is driven as a generator, supplying current to the other alternator, which runs as a synchronous motor. By sufficiently increasing the exciting current of the generator, and reducing that of the motor, the full-load current may be made to circulate between the two machines. The power supplied by the standardized motor then corresponds approximately to twice the total full-load loss in either machine (exclusive of excitation).

Another variety of this method of determining the total full-load loss consists in running one of the alternators as a synchronous motor at normal excitation, the armature p.d. being adjusted by varying the generator excitation so as to cause the current to assume its full-load value. The power supplied to the motor armature, which corresponds to the total full-load loss, is measured by means of a wattmeter.

§ 99. Retardation Method of determining Losses

A method equally applicable to continuous and alternating-current generators is that originally suggested by J. R. Ashworth,* and first fully described by Routin.† It depends on the determination of the retardation which takes place in the rotating member or rotor of a machine when the driving torque is removed. Let the alternator under test be run up, on open circuit and without any excitation, to a speed somewhat above its normal speed, and let the supply of power driving the machine be then cut off (this may be done by throwing off the belt if a belt drive is used, by switching off the current supplied to the direct-coupled exciter if this is used for running the alternator up to the desired speed, or—if the machine is driven electrically as a synchronous motor—by switching off the armature current, and immediately afterwards the field current). The rotor begins to slow down, and readings of its speed are taken at definite intervals; this may be done by means of a tachometer or a dead-beat voltmeter, which forms a very good speed indicator. The curve connecting the angular velocity ω (radians per sec.) with the time is then plotted, and from it the value of $\frac{d\omega}{dt}$, the angular retardation, is found for that value of ω which represents the normal speed of the alternator. If $\mathrm{I} =$ moment of inertia of rotor, in C.G.S. units, then the retarding couple—due to frictional resistances—is $\mathrm{I}\frac{d\omega}{dt}$, and the power lost by friction is $\mathrm{I}\omega\frac{d\omega}{dt} \times 10^{-7}$ watts. If I were known, we could thus find the frictional losses. In order to determine I, the experiment is repeated, an additional *known* retarding couple C being, however, applied to the rotor (by means of a suitable rope or band brake). If $\left(\frac{d\omega}{dt}\right)_1$ stand for the new value of the angular acceleration at the normal speed, then we must have $\mathrm{C} = \mathrm{I}\left[\left(\frac{d\omega}{dt}\right)_1 - \frac{d\omega}{dt}\right]$, so that I may be determined. By next repeating the first experiment, but with the field fully excited, we find for the frictional and iron losses $\mathrm{I}\omega\left(\frac{d\omega}{dt}\right)_2 \times 10^{-7}$, where $\left(\frac{d\omega}{dt}\right)_2$ is the new value of the angular retardation.

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* *The Electrician*, vol. xxx. p. 459 (1893).
† *Éclairage Électrique*, October 24, 1896.

§ 100. Heating Test of Alternator. Mordey's and Behrend's Methods

In order to determine the temperature rise of an alternator, the machine must be kept running under normal full-load conditions until the temperature approaches a steady value. Since the period during which an alternator of large output would have to be run for this purpose is very considerable, such a heat test would prove very expensive if the machine were actually run fully loaded. Numerous methods have, therefore, been devised by means of which the heat test may be carried out with the expenditure of only a relatively small amount of energy.

If two similar machines are available, the arrangement used in connection with the Hopkinson test (§ 96) furnishes a ready solution of the problem. Or the method of driving one of the machines as an under-excited generator, and running the other as a synchronous motor (§ 98) may be used.

Where only a single alternator is available for the test, the method originally suggested by Mordey,[*] and recently improved by Behrend,[†] may be resorted to.

In Mordey's method, the armature winding is divided into two unequal portions, which are coupled in series so that their e.m.f.'s oppose each other, the entire winding being then short-circuited through an ammeter. The short-circuit current so obtained is due to the difference between the e.m.f.'s in the two sections of the winding, and its value may be adjusted by means of a suitable choking coil included in the armature circuit. Instead of driving the alternator mechanically, it may be driven electrically, by supplying to it an alternating current at a much lower p.d. than the normal p.d. of the alternator, the auxiliary source of alternating e.m.f. (a small alternator or transformer) being introduced into the local circuit of the two unequal sections of the armature winding, which are connected so as to be in opposition to each other. The stronger section (*i.e.* the one having the higher e.m.f.) then acts as a *motor*, the weaker one acting as a *generator*.

Instead of dividing the armature winding into two unequal sections, the field winding may be similarly divided, the connections being arranged so that two opposing unequal e.m.f.'s are induced in the armature winding.

Neither of these arrangements can be successfully employed in connection with a modern revolving field alternator, on account of the excessive mechanical vibration which is set up. This is due to the powerful magnetic side-pull brought into play by the strengthening of the field-poles corresponding to the motor section, and the weakening of those corresponding to the generator section of the armature winding, by armature reaction. In order to adapt the method to modern types of alternators, Behrend * divides the field coils into two equal sections, and varies the exciting currents of these independently so as to obtain magnetic balance. With this modification, it is possible to overcome the vibration trouble to a considerable extent.

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[*] *Journal of the Institution of Electrical Engineers*, vol. xxii. p. 116 (1893).

[†] *Electrical World and Engineer*, vol. xlii. p. 715 (1903).

§ 101. Goldschmidt's Method

The following method of carrying out the heat test of a large alternator has been devised by Goldschmidt.* The alternator is run at normal full-load excitation, so that the iron and field copper losses are the same as those occurring at full-load. The armature copper loss is supplied by sending a *continuous* current from a battery or dynamo around the armature winding, the continuous current being made equal to the r.m.s. value of the full-load armature current. It need hardly be pointed out that the connections must be arranged so as to prevent the high e.m.f. of the alternator armature from reaching the source of continuous current. Various ways of securing this result are available. In the case of a single-phase machine, the armature coils may be divided into two equal groups, connected so as to oppose each other; the resultant alternating e.m.f. will then be zero, and the armature circuit will form a simple resistance so far as the source of continuous current is concerned. In a two-phase machine, each phase may be similarly treated, a *separate* source of continuous e.m.f. being used for each phase. With three-phase machines having a Δ-connected armature, the Δ may be opened at one corner to allow of the introduction of the continuous current. If the windings are star-connected, they may be temporarily altered to a Δ connection for the purposes of the test. Another method may also, however, be used. A suitable star-connected three-phase transformer is joined across the alternator terminals, and the continuous current led into the system at the neutral point of the transformer windings, and out of it at the neutral point of the armature winding—each phase carrying in this case a current equal to $\frac{1}{3}$ of the continuous current. In order to balance the magnetic effect of the continuous current on the transformer core, a continuous current producing an equal and opposite number of ampere-turns must be sent through the secondary windings of the transformer.

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* *Elektrotechnische Zeitschrift*, vol. xxii. p. 682 (1901).

§ 102. Hobart and Punga's Method

What promises to be an extremely convenient method of carrying out the heat test of a large generator has recently been described by Hobart and Punga.* The method consists essentially in running the machine alternately on open circuit and on short circuit, the periods of the two sets of runs and the values of the exciting current corresponding to them being so chosen that the mean values of the various losses during the entire time of the test are equal to the actual values of these losses at full load.

The losses may be regarded as made up of (1) mechanical friction losses; (2) armature copper loss, w_a; (3) iron loss, w_i; (4) field copper loss, or excitation loss w_e.

The friction losses are independent of the excitation, and their value may be determined by driving the unexcited alternator at its normal speed by means of an auxiliary standardized motor. The armature copper loss is easily calculated, as is also the field copper loss, for a given value of the current. The iron loss is found by driving the alternator on open circuit by means of a standardized motor, the exciting current having its normal full-load value; if from the total power employed in driving the alternator we subtract the friction losses as previously determined, we obtain the iron loss. For the purposes of the heat test under consideration, it is necessary to know the relation connecting the iron loss with the excitation loss, so that a set of readings must be obtained connecting these two, and the results plotted in the form of a curve as in Fig. 125.†

Let the alternator be run for t_1 minutes on open circuit with a certain excitation, then for t_2 minutes on short circuit with a different excitation, then again for t_1 minutes on open circuit, for t_2 minutes on short circuit, and so on, the open-circuit and short-circuit runs alternating with each other. During the open-circuit runs, there are only frictional, excitation, and iron losses taking place in the machine, while during the short-circuit runs we have only frictional, excitation, and armature copper losses, the iron loss being negligible. The machine running at its normal speed, it is evident that the frictional loss is identical, all the time, with the full-load frictional loss. The other losses will depend on the excitation, and the problem is to so choose the values of the exciting current corresponding to the open-circuit and short-circuit runs, and the relative values of t_1 and t_2, as to make the *average* values of all the losses equal to those which occur at full load.

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* *Electrical World and Engineer*, vol. xlv. p. 759 (1905).

† On account of armature reaction, which produces a distortion of the field, the full-load iron loss w_i will generally be in excess of the open-circuit iron loss with the same exciting current. We have already indicated (§ 98) how this increase in the iron loss may be determined.

The short-circuit curve of the alternator being supposed known, and being assumed to be a straight line, we can find from it the short-circuit current which would be obtained with the normal full-load excitation. Let the ratio of this short-circuit current to the normal full-load current be denoted by a. If we reduce the exciting current to $\frac{1}{a}$ of its full-load value, then (the short-circuit curve being a straight line) the short-circuit current will become equal to the normal full-load current. Thus in order to produce a short-circuit current equal to the normal full-load current, the exciting current must be made equal to $\frac{1}{a}$ times its full-load value.

Let $y = \frac{t_1}{t_1 + t_2} = \frac{\text{duration of open-circuit runs}}{\text{total time of test}}$, so that $1 - y = \frac{\text{duration of short-circuit runs}}{\text{total time of test}}$, then it is evident that if the average armature copper watts during the test are to be equal to the actual full-load armature copper watts, the short-circuit current during the short-circuit runs must equal $\frac{1}{\sqrt{1-y}}$ times the full-load current. Hence the exciting current corresponding to this short-circuit current must be $\frac{1}{\sqrt{1-y}} \cdot \frac{1}{a}$ times the full-load exciting current, so that the exciting watts $= \frac{1}{1-y} \cdot \frac{1}{a^2}$ times the full-load exciting watts $= \frac{1}{1-y} \cdot \frac{1}{a^2} w_e$. If T = total duration of test, in hours, then the watt-hours dissipated in the field coils during the short-circuit runs are—

$$\frac{1}{1-y} \cdot \frac{1}{a^2} \cdot w_e \cdot (1-y)\mathrm{T} = \frac{w_e}{a^2} \cdot \mathrm{T}$$

But we must make the *total* watt-hours dissipated in the field coils equal to those which would be dissipated under full-load conditions, *i.e.* to $w_e\mathrm{T}$. Thus the difference

$$w_e\mathrm{T} - \frac{w_e}{a^2}\mathrm{T} = w_e\left(1 - \frac{1}{a^2}\right)\mathrm{T}$$

must correspond to the watt-hours dissipated in the field coils during the open-circuit runs, and since the duration of these runs is $y\mathrm{T}$ hours, the field watts during these runs must be—

$$\frac{1}{y} w_e\left(1 - \frac{1}{a^2}\right)$$

Again, the full-load iron losses amounting to w_i, it follows that since the duration of the open-circuit runs is yT, the iron losses during these runs must be $\frac{w_i}{y}$ if the average iron losses during the test are to be equal to the normal iron losses at full load. We thus find, considering the *open-circuit* runs—

$$\frac{\text{excitation or field copper loss}}{\text{iron loss}} = \frac{\frac{1}{y}w_e\left(1 - \frac{1}{a^2}\right)}{\frac{1}{y}w_i} = \left(1 - \frac{1}{a^2}\right)\frac{w_e}{w_i}$$

Now a, w_e, and w_i being known, the right-hand member of the above equation represents a known quantity. In order, therefore, to find the excitation and iron losses during the open-circuit runs, we take the curve connecting these losses (Fig. 125), and from the origin draw a line making an angle with the horizontal axis (the axis of open-circuit iron loss) whose tangent is equal to $\left(1 - \frac{1}{a_2}\right)\frac{w_e}{w^{i}}$. It is obvious that the intersection A of this straight line with the curve of losses gives us a point on the curve the ratio of whose ordinate to its abscissa satisfies the above equation. Thus we find OB = iron loss during open-circuit runs, BA = excitation loss during open-circuit runs, $y = \frac{w_i}{\text{OB}}$, and the other quantities immediately follow.

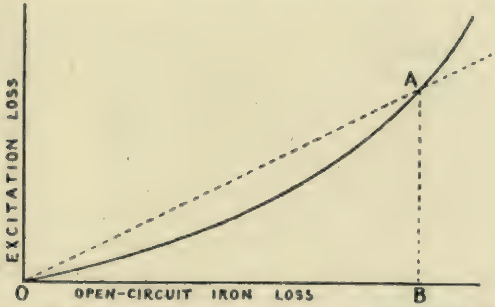

FIG. 125.—Relation connecting Iron with Excitation Loss.

As regards the actual values of t_1 and t_2, we may make $t_1 + t_2$ anything between 10 and 20 minutes, say. If, for example, we find $y = 0{\cdot}6$, and we make $t_1 + t_2 = 15$, the duration of each open-circuit run will be 9 minutes, and that of each short-circuit run 6 minutes.

As in other heat tests, the most reliable method of ascertaining the rise of temperature is the increase of resistance method (§ 93).

CHAPTER XII

§ **103**. Assumptions underlying approximate theory of induction motors—§ **104**. Leakage coefficients—§ **105**. Behaviour of motor independent of number of turns in rotor winding. Rotor slip—§ **106**. Induction motor replaced by equivalent transformer—§ **107**. Torque of induction motor—§ **108**. Study of transformer equivalent of induction motor. Vector diagram—§ **109**. Transformation of vector diagram. Deduction of circle diagram—§ **110**. Construction for slip and torque—§ **111**. Case of negligible primary resistance. Heyland's circle diagram—§ **112**. Effect of resistance in (1) stator and (2) rotor—§ **113**. Use of starting resistances—§ **114**. Effect of stator core loss. Efficiency of motor.

§ 103. Assumptions underlying Approximate Theory of Induction Motors

In dealing with the theory of induction motors, it is convenient to make certain assumptions which, though not corresponding exactly to the actually existing conditions, lead to results sufficiently near the truth for all practical purposes. We shall assume the stator and rotor windings to be identical in every respect, each consisting of the same number of conductors arranged in the same number of similar slots.* We shall, further, assume that the reluctance of the iron path is negligible in comparison with that of the air-gap; this assumption is equivalent to supposing the flux to be proportional to the current. As in § 20, we shall assume the rotating waves of magnetic flux produced by the stator and rotor currents to be distributed in space according to the simple sine law, and we may also conveniently assume all the p.d.'s, e.m.f.'s, and currents to follow the same law with respect to time.

Since (apart from the question of phase difference) the phases of an induction motor may be regarded as practically identical, we shall confine our attention to one phase, and study the changes taking place in the current, power factor, etc., of that phase as the load changes. A precisely similar series of changes will take place in the remaining phases.

When an induction motor is running, we may regard the systems of polyphase currents in the stator and rotor windings as each giving rise to its own wave of magnetic flux, the actually existing wave being obtained by the superposition of the two hypothetical waves corresponding to the stator and rotor currents. In considering the theory of induction motors, we may either deal with the hypothetical waves which the currents in each winding would produce if those in the other winding were non-existent, or we may take as the basis of our investigation the actually existing flux due to the superposition of the hypothetical fluxes. Both modes of investigation have been employed, and according to the method adopted different writers on the subject have defined and used various so-called "leakage factors" or "leakage coefficients," whose meaning and mutual relationship we now proceed to explain.

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* In practice, the number of slots in the stator and that in the rotor are chosen so as to have the least possible common factor; otherwise the torque corresponding to different relative positions of stator and rotor will fluctuate very considerably. See § 65.

§ 104. Leakage Coefficients

Let the secondary winding be open, and let polyphase currents be supplied to the primary, the maximum value of the current in each phase being I_1. Let F_1 stand for the maximum value of the total flux linked with the windings of each phase. Then, according to our assumption, F_1 is proportional to I_1, so that we may write—

$$F_1 = L_1 I_1$$

and in addition the instantaneous values of these two quantities are in phase with each other. We may term L_1 the *virtual self-inductance* of each phase of the winding, since it represents the maximum value of the total flux linked with one phase of the winding when *all* the phases are traversed by currents of unit amplitude.*

Similarly, if we suppose the primary open-circuited, and polyphase currents supplied to the secondary, we may write—

$$F_2 = L_2 I_2$$

where F_2 is the maximum value of the flux linked with one phase of the secondary when all its phases are supplied with (polyphase) currents of amplitude I_2. The constant L_2 we may term the virtual self-inductance of one phase of the secondary.

In dealing, in § 20, with the stationary or alternating flux waves produced around the rotor periphery by the alternating currents in the various phases—waves whose superposition gives rise, as we have seen (§ 20), to a rotating wave of magnetic flux in the air-gap—we considered only those lines of induction which actually cross the gap and penetrate into the rotor core, there becoming linked with the rotor conductors. In addition to such effective or useful lines, however, there will be others which simply become linked with the stator windings, without undergoing any linkage with the rotor windings. All such lines, which do not contribute anything towards the magnetic link connecting the two sets of windings, are spoken of as leakage lines, and the corresponding flux as leakage flux. Out of the total maximum flux L_1I_1 linked with the primary, a certain proportion, MI_1, will become linked with the secondary, and the remainder will form the primary leakage flux. We may term M the *virtual mutual inductance* of the two windings.

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* In the case of a two-phase winding, L_1 is identical with the true self-inductance of either phase, *i.e.* with the flux which becomes linked with that phase when supplied with unit current; for, as has been shown in § 20, the amplitude of the resultant rotating wave is the same as that of the oscillating wave due to one phase only; but in a three-phase motor L_1 is greater than the true self-inductance, and includes, in addition to the true self-inductance, the effect of the mutual inductance of the neighbouring phases (§ 20).

The currents circulating in the secondary similarly give rise to a flux L_2I_2 in each phase, an amount MI_2 becoming linked with the primary, and the remainder forming the secondary leakage flux.*

The ratios $v_1 = \frac{L_1}{M}$ and $v_2 = \frac{L_2}{M}$ are termed the Hopkinson leakage coefficients. They were first introduced by A. Blondel.†

The reciprocals of the Hopkinson leakage coefficients, viz. $\nu_1 = \frac{1}{v_1} = \frac{M}{L_1}$ and $\nu_2 = \frac{1}{v_2} = \frac{M}{L_2}$, are known as Behrend's leakage coefficients.

The ratios $\tau_1 = \frac{L_1 - M}{M}$ and $\tau_2 = \frac{L_2 - M}{M}$ of the leakage flux to the useful flux in the two windings are known as Heyland's leakage coefficients.

Lastly, a very important coefficient, which has been used by Heyland, Behrend, Blondel, Hobart, and many other writers, and which is known as the *dispersion coefficient*, is the ratio—

$$\sigma = \frac{L_1L_2 - M^2}{M^2} = \tau_1 + \tau_2 + \tau_1\tau_2 = v_1v_2 - 1 = \frac{1}{\nu_1\nu_2} - 1 \ddagger$$

In the special case under consideration, in which the stator and rotor windings are supposed to be identical, it is evident that $L_1 = L_2$, $\tau_1 = \tau_2$, $v_1 = v_2$, and $\nu_1 = \nu_2$, so that—

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* In the special case under consideration, in which the primary and secondary windings are supposed to be identical in every respect, $L_1 = L_2$; this, however, would not hold generally.

† It must be clearly understood that the definitions $v_1 = \frac{L_1}{M}$ and $v_2 = \frac{L_2}{M}$ are based on the assumption of *equal numbers of turns* in the stator and rotor windings. If the turns are different, we must imagine the rotor winding replaced by one having the same number of turns as the stator winding.

‡ Behn-Eschenburg uses σ in a slightly different sense, viz. $\sigma = \frac{L_1L_2 - M}{L_1L_2}$ (*Journal of the Institution of Electrical Engineers*, vol. xxxiii. p. 239).

$$\sigma = \frac{L^2 - M^2}{M^2} = 2\tau + \tau^2 = v^2 - 1 = \frac{1}{\nu^2} - 1$$

Since in a well-designed motor the difference between L and M would always be small, we may write, approximately, $L + M = 2M$, so that—

$$\sigma = 2\frac{L - M}{M} = 2\tau = 2(v - 1) = 2\left(\frac{1}{\nu} - 1\right)$$

§ 105. Behaviour of Motor Independent of Number of Turns in Rotor Winding. Rotor Slip

We shall next show that, so long as the number and shape of the slots in the rotor core, and the total cross-section of copper in each slot, remain unaltered, no difference whatever is produced as regards the behaviour of the motor by altering the number of turns in the rotor winding.

For, let us suppose that the turns have been increased m-fold. This will have the effect of an m-fold increase in the e.m.f. induced in the secondary winding. But the resistance and self-inductance of this winding having both been increased m^2 times, the impedance will be increased in the same ratio, while the angle of lag, whose tangent is given by $\frac{\text{reactance}}{\text{resistance}}$, will remain unaltered. Thus the current will be reduced in the ratio $\frac{1}{m}$, but its phase will remain unaltered. The ampere-turns on the rotor, and the rotating field due to them, will obviously be the same as before, for while the current has been reduced to $\frac{1}{m}$th of its original value, the turns have been increased m-fold; the phase of the current having undergone no change.

On account of the rotation of the rotor, the frequency of the rotor currents is much less, under normal running conditions, than that of the stator currents. Let n_1 be the primary frequency, and n_2 the secondary. The slip s (§ 61) is defined by the equation—

$$s = \frac{n_2}{n_1}$$

and is frequently expressed as a percentage, in the form $100\frac{n_2}{n_1}$.

§ 106. Induction Motor replaced by Equivalent Transformer

Let us now suppose that the motor is running with a slip s, exerting a definite torque, and that while the torque remains unaltered, resistance is introduced into the rotor windings, the slip increasing until ultimately the rotor comes to rest—still exerting its original torque. Let r_2 be the resistance of each phase of the rotor winding, and let r be the additional resistance which must be introduced into each phase in order to reduce the rotor to rest. Then the total resistance R_2 of each phase, when the rotor has been reduced to rest, is $R_2 = r_2 + r$. If n_2 denote the original frequency of the rotor currents, then it is clear that by reducing the rotor to rest we have increased the e.m.f. induced in its windings (by the hypothetical field due to the stator currents) in the ratio $\frac{n_1}{n_2}$; the frequency and reactance of the rotor windings have each been increased in the same ratio. It is, therefore, clear that in order that the secondary current and its phase relatively to the induced secondary e.m.f. may remain unaltered, the total resistance of the rotor windings must also be increased in the same ratio $\frac{n_1}{n_2}$, *i.e.* we must have—

$$R_2 = \frac{n_1}{n_2} r_2 = \frac{r_2}{s}$$

or—

$$r = \frac{1 - s}{s} r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The primary and secondary currents, their phase relations, and the total power supplied to the primary, remain unaltered. But the rotor is now at rest, and although still exerting its original torque, does not develop any mechanical power. It is evident that the power which formerly corresponded to mechanical power is now employed in producing heat in the additional or external resistance r, and is represented by rI_2^2 per phase, I_2 being the secondary current in each phase.

We have thus reduced our induction motor to an equivalent three-phase transformer, the total mechanical power developed by the motor corresponding to the power absorbed by the resistances r external to the rotor windings, and the slip s of the motor corresponding, according to equation (1), to—

$$s = \frac{r_2}{r + r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

§ 107. Torque of Induction Motor

If T = torque exerted by motor,* in *lb.-feet*, and P = number of pairs of poles in motor, then corresponding to a slip s the speed of the rotor is $\frac{(1-s)n_1}{P}$ revs. per sec., and the total mechanical power is—

$$\frac{2\pi(1-s)n_1}{P}\cdot\frac{T}{550}\text{ horse-power, or }\frac{2\pi(1-s)n_1}{P}\cdot\frac{T}{550}\times 746\text{ watts}$$

But if N = number of phases in rotor winding, we must have—

$$\frac{2\pi(1-s)n_1}{P}\cdot\frac{746}{550}T = NrI_2^2$$

whence—

$$T = 0{\cdot}737\frac{P}{(1-s)p}\cdot NrI_2^2$$

or, using (1)—

$$T = 0{\cdot}737\frac{PNr_2}{p}\cdot\frac{I_2^2}{s}\quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $p = 2\pi\times$ primary frequency.

§ 108. Study of Transformer Equivalent of Induction Motor. Vector Diagram

We may now study the behaviour of the motor by substituting for it the equivalent transformer shown in Fig. 126, which replaces each phase of the winding; λ_1 and λ_2 denoting those portions of the primary and secondary self-inductances which do not contribute anything towards the mutual inductance.† The non-inductive resistance h, shown as a shunt across T′T″, is intended to represent a load equivalent to the hysteresis and eddy-current loss in the stator core. This loss will not be quite constant, for with increasing load the p.d. across T′T″ will decrease, and with it also the hysteresis and eddy-current loss. As an approximation, however, we may suppose this loss to remain constant at all loads, which is equivalent to supposing h connected across TT′.

Provisionally, however, we shall neglect the hysteresis loss entirely,

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[*] T stands for the *total* torque developed by the rotor, and not merely for the *useful* torque available at the pulley.

[†] λ_1 and λ_2 may be termed the *leakage self-inductances* of the windings; by many writers they are, incorrectly, called the self-inductances of the windings simply.

and investigate the law according to which the primary and secondary currents, the power factor, etc., vary with increasing load when the primary p.d. across the terminals of the transformer is kept constant.

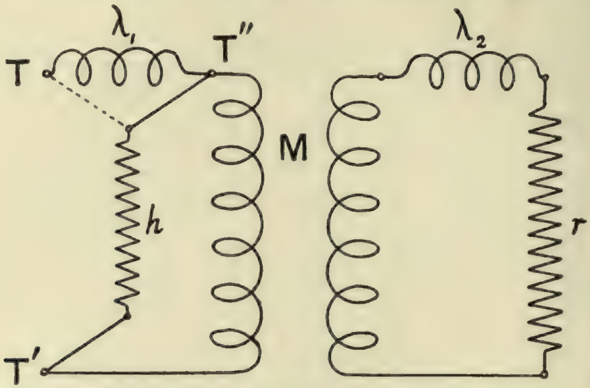

Fig. 126.—Transformer Equivalent of Induction Motor.

The method which we shall follow is one originally given by F. Bedell,* and recently applied to the induction motor by J. Bethenod.†

Let I_1 be the (r.m.s.) primary current, and r_1 the primary resistance (per phase). The primary phase p.d.,‡ which we shall denote by V, may be regarded as made up of the following components :—

(1) The resistance component, r_1I_1, in phase with I_1.

(2) The primary self-inductance component, pL_1I_1, 90° ahead of I_1.

(3) The component corresponding to the mutual inductance of the two circuits. In order to find the value of this, let us assume the instantaneous value of the secondary current to be $i_2 = I_m \sin pt$. The hypothetical flux through the primary, due to this secondary current, is Mi_2, and the e.m.f. induced by it in the primary is given by—

$$-\frac{d}{dt}(Mi_2) = -M\frac{di_2}{dt} = -pMI_m \cos pt$$

In order to balance this, the primary phase p.d. must provide a component $+pMI_m \cos pt$, *i.e.* a component whose r.m.s. value is, say, pMI_2, and which is 90° ahead of I_2.

Considering next the secondary circuit, we have in it—

(1) The e.m.f. induced by the hypothetical field through the

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* *Proceedings of the Physical Society of London*, vol. xiv. p. 327 (1896).

† *L'Éclairage Électrique*, vol. xl. p. 253 (1904).

‡ *I.e.* the p.d. per phase, or $\frac{1}{\sqrt{3}}$ × line p.d.

secondary due to the current I_1 in the primary. The magnitude of this e.m.f. is pMI_1, and it is 90° behind the primary current. We may regard it as an e.m.f. impressed on the secondary, and as made

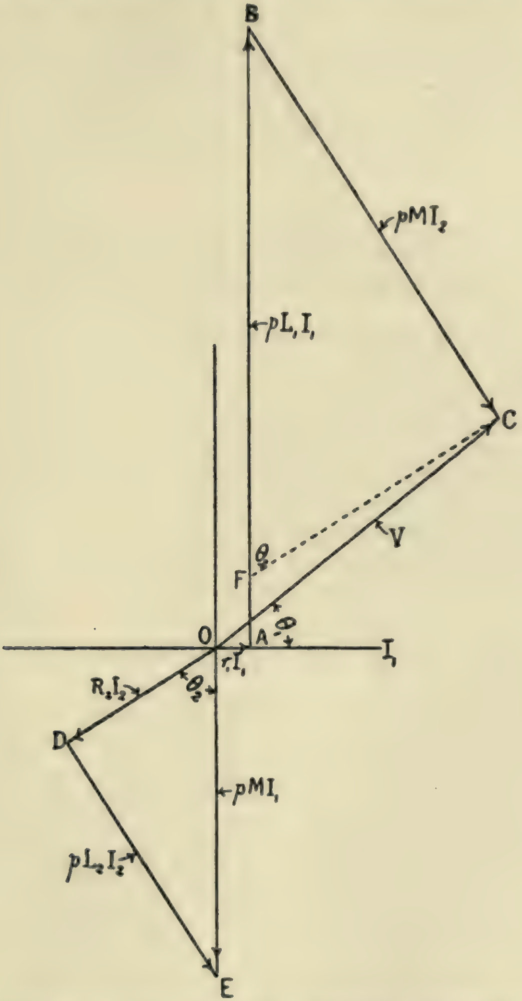

FIG. 127.—Vector Diagram of Equivalent Transformer.

up of (2) the R_2I_2 component, in phase with I_2, and (3) the pL_2I_2 component, 90° ahead of I_2.

These relations are graphically exhibited in Fig. 127, in which we take the primary current I_1 as our vector of reference in a horizontal direction. The resultant of $OA = r_1I_1$, $AB = pL_1I_1$, and $BC = pMI_2$ in the primary circuit gives us V, the primary phase p.d., the angle $AOC = \theta_1$ being the angle of lag of the primary current behind the p.d., so that $\cos \theta_1 =$ power factor of motor. In the secondary, the resultant of $OD = R_2I_2$ and $DE = pL_2I_2$ gives us the e.m.f. pMI_1, while the angle $EOD = \theta_2$ is the angle by which I_2 lags behind pMI_1. From C let CF be drawn parallel to OD. Then angle $BFC = \theta_2$, while BCF is a *right angle.*

§ 109. Transformation of Vector Diagram. Deduction of Circle Diagram

Let us now transform our vector diagram by dividing the length of each vector in it by I_1. We then obtain the diagram of Fig. 128. A, B, and E now become *fixed points* ($OA = r_1$, $AB = pL_1$, and $OE = pM$ being constant). Hence, since, as the load varies, D describes a semicircle on OE as diameter, and since BC is parallel and proportional to ED, it follows that C *is constrained to move along a circle described on* BF *as diameter.*

Since O is fixed with respect to the circle, it follows by a well-known geometrical proposition that $OP \times OC =$ constant for all possible positions of C. Hence—

$$OP = \frac{\text{constant}}{OC} = \frac{\text{constant}}{V} I_1 = \text{constant} \times I_1$$

since V is by supposition maintained constant. OP is, therefore, proportional to the primary current, and we have the important result that as the load changes *the extremity of the primary current vector moves along a circle.* Since the angle which this vector makes with the horizontal is θ_1, we may, in our new diagram, take the horizontal direction to be that of the primary phase p.d. vector.

Let the line OB be drawn, intersecting the circle at Q. Then we have $OQ \,.\, OB = OP \,.\, OC$, or $\frac{OQ}{OP} = \frac{OC}{OB}$. Hence, the triangles OQP and OCB are similar, and $\frac{QP}{OP} = \frac{BC}{OB}$, or—

$$QP = \frac{OP}{OB} \,.\, BC = \text{constant} \times I_2 \quad . \quad . \quad . \quad . \quad . \quad (4),$$

since OB, p, and M are all constant. Thus, QP is proportional to the secondary current.

We may now write—

$$I_1 = K_1 . OP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$I_2 = K_2 . QP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

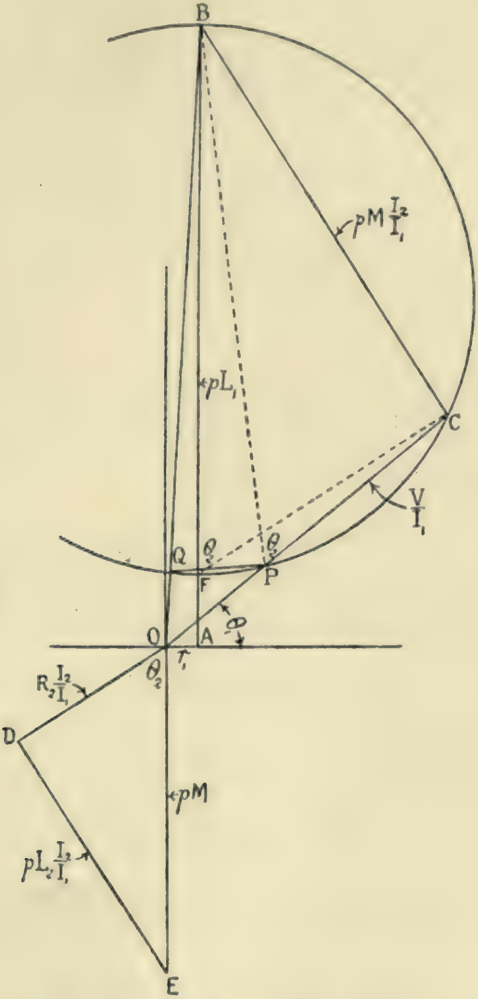

Fig. 128.—Transformed Vector Diagram.

where K_1 and K_2 are constants. We proceed to determine their values.

The value of r_1 being, in any well-designed motor, extremely small in comparison with pL_1, we may assume that O is practically at the same distance from the centre of the circle as A, so that $OP \,.\, OC = AF \,.\, AB$.

Now—

$$FB = \frac{BC}{\sin\theta_2} = pM\frac{I_2}{I_1} \,.\, \frac{I_1 \,.\, pM}{pL_2I_2} = \frac{pM^2}{L_2}$$

and—

$$AF = AB - FB = p\left(L_1 - \frac{M^2}{L_2}\right) = \frac{p}{L_2}(L_1L_2 - M^2)$$

Thus—

$$AF \,.\, AB = p^2\frac{L_1}{L_2}(L_1L_2 - M^2)$$

and—

$$OP = \frac{AF \,.\, AB}{OC} = p^2\frac{L_1}{L_2} \,.\, \frac{L_1L_2 - M^2}{V} \,.\, I_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

On comparing this with (5), we find—

$$K_1 = \frac{V \,.\, L_2}{p^2L_1(L_1L_2 - M^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Considering next equation (4), and putting, approximately, $OB = pL_1$, and utilizing (7), we find—

$$QP = \frac{p^2M}{L_2} \,.\, \frac{L_1L_2 - M^2}{V} \,.\, I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

and on comparing this with (6) we get—

$$K_2 = \frac{VL_2}{p^2M(L_1L_2 - M^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

The scales according to which the two currents are to be measured are thus different, the ratio of the corresponding constants being—

$$\frac{K_1}{K_2} = \frac{M}{L_1}\text{*}$$

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* Always supposing the secondary winding to contain the same number of turns as the primary.

§ 110. Construction for Slip and Torque

We shall now show that the *slip* and *torque* may also be represented by a very simple graphical construction, shown in Fig. 129.

If, in Fig. 128, we join BP, then the angle BPC, which stands on the same arc as BFC, is equal to this latter, *i.e.* to θ_2. Thus, the angle between the primary current vector and the line PB is equal to θ_2.

In Fig. 129 part of the diagram of Fig. 128 has been reproduced.

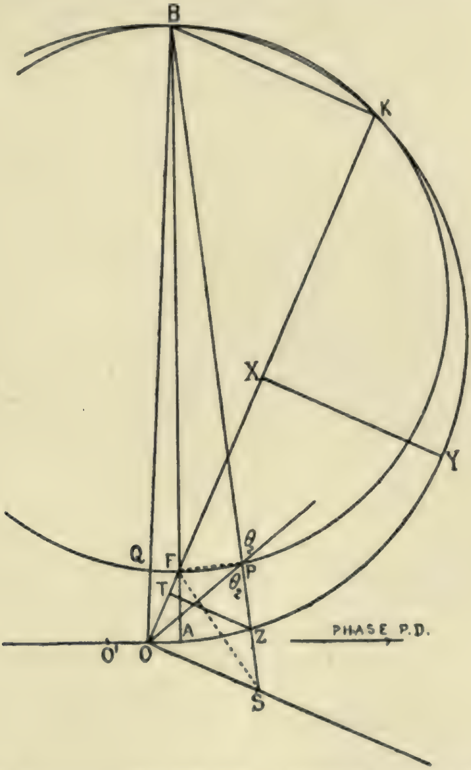

Fig. 129.—Construction for Slip and Torque of Induction Motor.

Referring once more to Fig. 128, we see that $\tan\theta_2 = \frac{pL_2}{R_2}$. But since (§ 106) $R_2 = \frac{r_2}{s}$, we have—

$$\tan \theta_2 = \frac{spL_2}{r_2}$$

or—

$$s = \frac{r_2}{pL_2} . \tan \theta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

so that the slip is proportional to $\tan \theta_2$.

In Fig. 129, join OF, and draw $OS \perp OF$. Join BP, and produce it until it intersects OS at S. Since the angle FPS is a right angle, both the triangles FSP and FSO will be right-angled, and hence a circle described on FS as diameter will pass through P and O. Since the angle OFS stands on the same arc of this circle as OPS, it follows that $\underline{/OFS} = \theta_2$, hence $\tan \theta_2 = \frac{OS}{OF}$, and by (11)—

$$s = \frac{r_2}{pL_2} . \frac{1}{OF} . OS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

but OF being constant, it follows that the *slip is proportional to* OS.

We may notice that the point K where OF intersects the circle corresponds to an infinite value of the slip, BK being parallel to OS, *i.e.* intersecting it at an infinite distance.

We shall next explain the graphical construction for the *torque.*

On OB as diameter, describe a semicircle intersecting BS in Z, and from Z let fall the perpendicular ZT on OK.

Equation (3), § 107, given above may be written—

$$T = K_3 \frac{I_2^2}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

K_3 being a constant whose value (if the torque is expressed in lb.-feet) is given by—

$$K_3 = 0{\cdot}737 \frac{PNr_2}{p}$$

Now in the triangle QPB—

$$\frac{QP}{QB} = \frac{\sin QBP}{\sin QPB}$$

so that—

$$I_2 = K_2 . QP = K_2 \frac{QB}{\sin QPB} . \sin OBZ$$

or—

$$I_2 = K_2 \frac{QB}{OB} \cdot \frac{1}{\sin QPB} \cdot OZ\text{*}$$
$$= K_4 . OZ$$

where K_4 is a constant whose value is $K_4 = K_2 \frac{QB}{OB \sin QPB}$.†

Hence—

$$I_2^2 = K_4^2 . OZ^2$$

Again, by (12), $s = K_5 . OS$, where $K_5 = \frac{r_2}{pL_2 . OF}$. Substituting these values for I_2^2 and s in (13), we find—

$$T = \frac{K_3 . K_4^2}{K_5} \cdot \frac{OZ^2}{OS} = K_6 \frac{OZ^2}{OS}$$

where K_6 is the constant $\frac{K_3 . K_4^2}{K_5}$. Let ZT be drawn ∥ SO. Then since $\frac{OZ^2}{OS} = OZ \times \frac{OZ}{OS} = OZ \cos ZOS = OZ \cos OZT = ZT$, we have—

$$T = K_6 . ZT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

i.e. the torque is *proportional to the length of the perpendicular let fall from Z on* OK.

It is evident that the maximum torque which the motor is capable of exerting is given by the length XY of the perpendicular erected at the middle point X of OK.

Since (Fig. 128) $FB = \frac{BC}{\sin \theta_2} = \frac{pM^2}{L_2}$, and $AF = AB - FB = p\left(L_1 - \frac{M^2}{L_2}\right)$, we see that—

$$\frac{AF}{FB} = \frac{L_1L_2 - M^2}{M^2} = \sigma, \text{ by § 104} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and—

$$\frac{AB}{AF} = \frac{1 + \sigma}{\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

So long, therefore, as L_1, L_2, and M remain unaltered, AF and FB, and the circle described on FB as diameter, remain unchanged. We may now investigate—assuming L_1, L_2, and M to remain constant—what effect is produced by a change in the primary (or stator) resistance.

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* The straight line joining O and Z is, for the sake of clearness, not shown in the diagram.

† The angle QPB remains constant, because QB is a fixed chord of the circle described by P.

§ 111. Case of Negligible Primary Resistance. Heyland's Circle Diagram

We shall in the first place consider the ideal case of a motor whose stator winding is of absolutely negligible resistance. The point O (Fig. 129) in this case moves up to A, K moves up to B, the diameters BF and BO of the two circles coincide, and the diagram assumes the simple form shown in Fig. 130, U and X being the centres of the two circles. We can easily express the maximum value of the power factor in this ideal case in terms of the dispersion coefficient σ. The maximum power factor $\cos\theta_0$ is obtained when the primary current vector OP becomes tangential to the smaller circle. We have in this case—

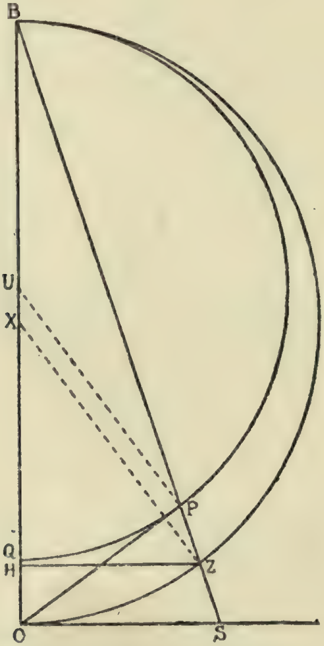

Fig. 130.—Heyland's Circle Diagram.

$$\begin{aligned}\text{maximum power factor } \cos\theta_0 &= \cos \text{POS} = \cos \text{PUO}^{*} \\ &= \frac{\text{UP}}{\text{UO}} = \frac{\text{UQ}}{\text{UQ}+\text{QO}} \\ &= \frac{1}{1+2\sigma}\ \ldots\ldots \qquad (17)\end{aligned}$$

since $\dfrac{\text{QO}}{\text{UQ}} = \dfrac{\sigma\text{QB}}{\text{UQ}} = 2\sigma$, by (15).

Again, the point K (Fig. 129) being now at B (Fig. 130), the perpendicular XY of Fig. 129, which is a measure of the maximum torque, has become equal to the radius of the larger circle. Hence—

$$\frac{\text{maximum torque}}{\text{torque at maximum power factor}} = \frac{\text{XZ}}{\text{HZ}} = \frac{1}{\sin\theta_0} = \frac{1+2\sigma}{2\sqrt{\sigma(1+\sigma)}} \qquad (18)$$

It must be understood that (17) and (18) only hold in the ideal case of a motor whose stator winding is quite negligible.

The construction shown in Fig. 130 was first given by Heyland, and is known as Heyland's Circle Diagram. It only applies, however, to the ideal case of a motor the resistance of whose stator windings is negligible.

§ 112. Effect of Resistance in (1) Stator and (2) Rotor

Let us next suppose that the resistance of the stator winding is steadily increased while L_1, L_2, and M remain unaltered. Then in Fig. 129 the point O will travel away from A, the line OFK will swing round, the point K gradually moving away from B. The most important effect of this change is to *reduce the maximum torque*, for as K moves away from B, XY shortens. And since in every well-designed motor $\sigma = \dfrac{\text{AF}}{\text{FB}}$ is very small, a comparatively small increase in the stator resistance (the length OA) may produce a relatively large reduction in the maximum torque (*i.e.* the overload capacity) of the motor.

In § 110, we obtained an expression for the torque of the form $T = K_6 . ZT$, where $K_6 = \dfrac{K_3 . K_4^2}{K_5}$. Now, of the three constants K_3, K_4, and K_5, one—K_4—does not involve r_2 at all, while K_3 and K_5 both contain r_2 as a factor. Thus K_6 is independent of r_2, that is, the torque scale in our diagram does not depend on the resistance of the rotor windings; so that a given stator current will produce a definite torque, no matter what the resistance of the rotor may be. Increasing the rotor resistance does not, therefore, affect the value of the torque corresponding to a given value of the stator current.

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* $\angle$POS = complement of $\angle$POU = $\angle$PUO.

Again, on referring to equation (12) for the slip, we notice that, corresponding to a given value of OS, and hence also OP (*i.e.* a given value of the stator current), the slip is directly proportional to the rotor resistance. For a given value of the stator current, therefore, an increase in the rotor resistance has the effect of *increasing the slip.*

The introduction of resistance into the motor windings has thus a totally different effect, according as the resistance is introduced into the stator or the rotor windings. For a given value of the stator current, the introduction of resistance into the stator windings reduces the torque and lowers the efficiency; whereas its introduction into the rotor windings leaves the torque unaltered, but increases the slip and lowers the efficiency.*

§ 113. Use of Starting Resistances

Let us next consider the effect of introducing resistance into the rotor windings at the moment of starting, when the slip is unity. By equation (12), § 110, we have, putting $s = 1$—

$$r_2 \times \text{OS} = \text{constant}$$

Hence OS will vary inversely as r_2. Now, in order to secure high efficiency, it is desirable to make the rotor windings of very low resistance. If, therefore, we simply short-circuit the rotor at starting, r_2 being very small, OS will be very large, and hence the point P in our circle diagram (Fig. 129) will lie in the neighbourhood of K. As a result, the length of the perpendicular let fall from Z on OK will be small, *i.e.* the starting torque will be small, in spite of the fact that the current OP taken by the motor is very large. Let us now suppose that r_2 is momentarily increased by the introduction of starting resistances (§ 63) into the rotor windings. Increasing r_2 will cause OS to shorten, the point P travelling downwards from its original position in the neighbourhood of K. This will obviously increase the torque, while at the same time the current is reduced. If necessary, the maximum torque, corresponding to XY, may be obtained at starting by giving to r_2 a value such that the point Z falls on Y.

We thus see that large starting torque can only be obtained by temporarily introducing resistances into the rotor windings, and that incidentally we thereby gain the advantage of a smaller starting current.

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* Since there is a decrease of speed without any change of torque.

In the case of rotors having permanently short-circuited windings, of which the best-known and most widely used type is the squirrel-cage winding (§ 61), the introduction of starting resistances into the rotor windings is, of course, impossible. Such motors are, therefore, incapable of starting against a heavy load. Further, if the output of such a motor is large, the starting current would reach a dangerous value if the full p.d. were applied to the stator windings. Hence it becomes necessary to introduce resistances into the *stator* windings, or to use an auto-transformer (§ 58) in order to limit the current; but this, as we have seen, reduces the torque still further. The squirrel-cage type of winding is, therefore, totally unsuitable for motors which are required to start under load.

§ 114. Effect of Stator Core Loss. Efficiency of Motor

We have hitherto neglected the stator core loss. In the diagram (Fig. 126) of the equivalent transformer, this is represented by the power absorbed by the shunting non-inductive resistance h. The presence of this resistance has the effect of adding to the primary current a component I_h which is in phase with the p.d. The addition of this to the primary current vector in Fig. 129 would displace all points on the primary current locus a distance representing I_h to the right. It is simpler, however, to leave the diagram unaltered, and to measure the primary current not from O, but from a point O′ to the left of O, such that O′O represents, on the scale of primary current, I_h. The primary angle of phase difference will be PO′A instead of POA.

The total *mechanical power* is easily calculated from the known value of the torque and the speed. This latter is easily determined from the slip, as it is equal to $(1 - s) \times$ speed of synchronism. If we express the total mechanical power in watts, then its ratio to the total watts supplied to the primary gives us the *electrical efficiency* of the motor. The *useful* power is obtained by subtracting the rotor friction loss from the total mechanical power, and the ratio of the useful power to the total power supplied to the primary gives us the *commercial efficiency*.

CHAPTER XIII

§ **115**. Experimental data required for construction of circle diagram. Determination of hysteresis loss—§ **116**. Short-circuit test—§ **117**. Construction of torque circle and slip line. Scales for torque and slip—§ **118**. Applications of circle diagram—§ **119**. Brake tests. Soames brake—§ **120**. Air friction dynamometer—§ **121**. Eddy-current brakes—§ **122**. Measurement of slip. Stroboscopic methods—§ **123**. Methods applicable to slip-ring rotors—§ **124**. Measurement of friction loss—§ **125**. Efficiency tests. Methods of Meunier, Sumpner, and Weekes—§ **126**. Alexanderson's method—§ **127**. Heating test—§ **128**. Induction motors with large starting torque.

§ 115. Experimental Data required for Construction of Circle Diagram. Determination of Hysteresis Loss

THE circle diagram furnishes us with an easy method of completely investigating the properties of an induction motor, without requiring any very elaborate tests. In order to be able to construct the diagram, we have to carry out the following measurements.

The motor is supplied at the normal p.d. and frequency, and is allowed to run light. The current I_0 and total power W_0 are carefully measured. If $V_l =$ normal line p.d., then phase p.d. for a three-phase star winding[*] $= \frac{V_l}{\sqrt{3}} = V$ (§ 16). The power per phase is $\frac{1}{3}W_0$, and hence the power factor at no load is—

$$\cos \theta_0 = \frac{W_0}{V_l \cdot I_0\sqrt{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We may now commence the construction of our diagram by drawing a horizontal line O′V (Fig. 131), and from O′ drawing a line $O'P_0 = I_0$, making an angle θ_0 with O′V, θ_0 being given by (1); P_0 will obviously be a point on the circle.

The next step consists in determining the position of the point O in Fig. 129. Since O′O is that part of the power component of the current which corresponds to the stator core loss, we must measure this loss. Now, when the motor is running light, the power taken by it is not all absorbed by the core loss, a certain amount going towards overcoming the frictional losses. If W_h = core loss, and W_f = frictional loss, we have—

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[*] In what follows, we suppose that we are dealing with a three-phase motor; the necessary modifications in the formulæ for a two-phase motor are quite obvious.

$$W_h = W_0 - W_f$$

This equation enables us to find W_h if W_f is known. In order to determine W_f, we take a set of readings connecting the power absorbed by the motor (when running light) with the line p.d. As the p.d. is steadily decreased, the power decreases, on account of the decrease in the hysteresis and eddy-current losses in the stator core. By producing the curve connecting power with p.d. backwards until it intersects the axis of power, we obtain the power corresponding to zero p.d., *i.e.* the power when the core loss is vanishingly small. This power will then simply represent W .

The distance O′O is now given by—

$$O'O = I_h = \frac{W_0 - W_f}{V_l . \sqrt{3}}$$

and on laying this off horizontally from O′ we obtain the point O in our diagram.

We next measure the resistance r_1 of one phase of the primary. The effect due to eddy currents in the stator windings is equivalent to an increase in the resistance of the windings. It is difficult to estimate this equivalent increase of resistance, but as a rough value we may, in order to be on the safe side, take it to amount to 30 per cent. We shall suppose r_1 to be a resistance which is 30 per cent. greater than the resistance of one stator phase as measured by using a continuous current (§ 88). Then $r_1 I_0$ gives us the resistance drop at no load.

Now, on referring to the diagram of Fig. 128, we notice that OB makes an angle α with the vertical such that—

$$\sin \alpha = \frac{r_1}{\sqrt{r_1^2 + p^2 L_1^2}} = \frac{r_1 I_0}{I_0 \sqrt{r_1^2 + p^2 L_1^2}} = \frac{\text{resistance drop at no load}}{\text{primary phase p.d.}}$$

Returning now to Fig. 131, we erect a perpendicular at O, and draw OB making an angle α with the vertical, such that $\sin \alpha = \frac{r_1 I_0}{\frac{V_l}{\sqrt{3}}}$. Along OB lay off a length $OQ = I_0 \sin \theta_0$ equal to the wattless component of I_0, and from Q draw a line QY making an angle α with OB (or 2α with the vertical). On again referring to Fig. 129, we see that the centre of the circle must lie on QY.

§ 116. Short-circuit Test

We now have one point on the circle—P_0—and the line QY, which must contain the centre of the circle. The circle could be drawn if we knew another point on it. Such a point is obtained by determining the magnitude and phase of the *short-circuit current*, *i.e.* the current which would be obtained by clamping the short-circuited rotor and applying the normal p.d. to the stator.

In this form, however, the experiment could only be carried out with extremely small motors, as the currents would be excessive, and a dangerous rise of temperature would result in a very short time. Further, the value so obtained would not be quite correct, as the permeability of the cores would be different from that corresponding to normal working conditions. Hence the plan generally adopted consists in applying to the stator a p.d. of amount V_s which is sufficient to give rise to a short-circuit current I'_s about equal to the normal full-load current of the motor, the power W_s being measured at the same time. If $\cos \theta_s$ denote the corresponding power factor, we have—

$$\cos \theta_s = \frac{W_s}{V_s I'_s \sqrt{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

V standing for the *line* p.d. during the short-circuit test. The short-circuit current I_s, which would have been obtained at the full line p.d. of V volts, is then calculated by means of the equation—

$$I_s = I'_s \frac{V_l}{V_s}$$

the assumption being made that the short-circuit current is directly proportional to the p.d.

As it is found that the value of the short-circuit current depends on the position of the rotor relatively to the stator, it is best to allow the rotor to run *very slowly* during this test.

We can now construct the vector of short-circuit current in our diagram, by laying off $O'P_s = I_s$ (Fig. 131), making an angle θ_s, given by equation (2), with the horizontal. This determines a second point P_s on the circle. The construction is then completed by joining P_0P_s, and at the middle point of P_0P_s erecting a perpendicular to it. The intersection of this perpendicular with QY gives the centre U of the circle, and the circle may now be drawn.

In determining the no-load and short-circuit current vectors, it is best to find a number of points on the curve connecting p.d. with current, and to read off the required value from the curve, rather than rely on an isolated reading. This procedure is in any case necessary, in the no-load test, for the purpose of determining the frictional loss as already explained.

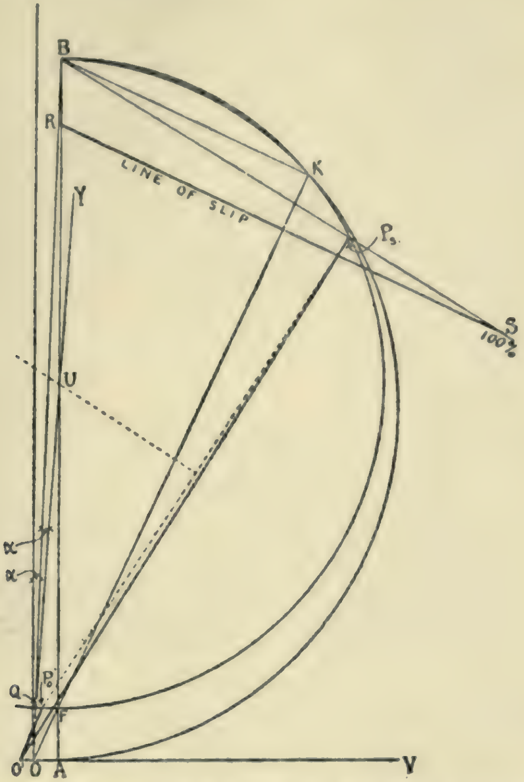

FIG. 131.—Construction of Circle Diagram from Experimental Data.

By drawing a vertical line through U, we are able to determine the dispersion coefficient σ of the motor, given by (§ 110)—

$$\sigma = \frac{AF}{FB}$$

§ 117. Construction of Torque Circle and Slip Line. Scales for Torque and Slip

We next describe the torque circle on OB as diameter, join OF and produce it to meet the circles in K. The torque is, to a certain scale, given by the length of the perpendicular let fall on OK from that point on the torque circle where it is intersected by the line drawn from B to the extremity of the primary current vector (ZT in Fig. 129). If we wish to measure the torque in lb.-feet—*i.e.* in terms of the pull which the motor would exert at the circumference of a pulley one foot in radius—then we may fix the scale of torque very simply, as follows. Assuming, for the moment, that there are no losses at all in the stator, we must have the power transmitted to the rotor equal to that supplied to the stator. Now the former is, in terms of the torque T,* equal to $\frac{p}{P}\cdot\frac{T}{550}\cdot746$ watts, p being $2\pi \times$ frequency and P = number of pairs of poles; while the latter is, for a three-phase motor, $V_l I \cos\theta\sqrt{3}$ (I being primary current and $\cos\theta$ = power factor). Thus—

$$\frac{p}{P}\cdot\frac{T}{550}\cdot746 = V_l I \cos\theta\sqrt{3}$$

or—

$$\begin{aligned} T &= \frac{550}{746}\cdot\frac{P}{p}V_l\sqrt{3}\,.\,I\cos\theta \\ &= kI\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

k being the constant $\frac{550}{746}\cdot\frac{P}{p}\cdot V_l\sqrt{3}$. Now when, as we have momentarily assumed, there are no losses in the stator, the points O′, O, and A in our diagram become coincident, and the maximum torque T_m is represented by $\frac{1}{2}$AB. We may thus write—

$$T_m = \tfrac{1}{2}c\,.\,AB\dagger$$

c being the required constant of proportionality, so that—

$$c = \frac{2T_m}{AB}$$

The value of T_m may be determined by means of (3); in this, k has a known value, and I and $\cos\theta$ may be found by describing a circle on AB as diameter, from its centre drawing a horizontal radius, and joining its extremity to B. The point in which this line intersects the current circle determines I and $\cos\theta$ in equation (3).

The slip might be found as in Fig. 129, but if we wish to consider large values of the slip, the line OS there shown would correspond to an inconveniently large scale for the slip. It is, therefore, more convenient to choose a point R in OB nearer B, and to draw RS parallel to BK. Since P_s corresponds to the short-circuit current when the slip is unity (or 100 per cent.), the point of intersection S of

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* In lb.-feet.

† AB being measured on the scale of *currents*.

BP, produced with RS fixes the slip scale, the length RS representing a slip of 100 per cent. The slip corresponding to any other value of the current is, on the same scale, equal to the distance from R of the point of intersection of the line drawn from B to the extremity of the current vector.

§ 118. Applications of Circle Diagram

Having constructed our diagram, we may use it to find the relation connecting any two out of the various quantities which we have to consider—current, power factor, slip, torque, mechanical power, and electrical efficiency. These relations are conveniently represented in the usual way by curves, and a very large number of different modes of representation may be adopted. Two of the most commonly employed are those in which the electrical power supplied to the motor, and the mechanical power developed by it, are taken as abscissæ, the remaining quantities being plotted as ordinates. An

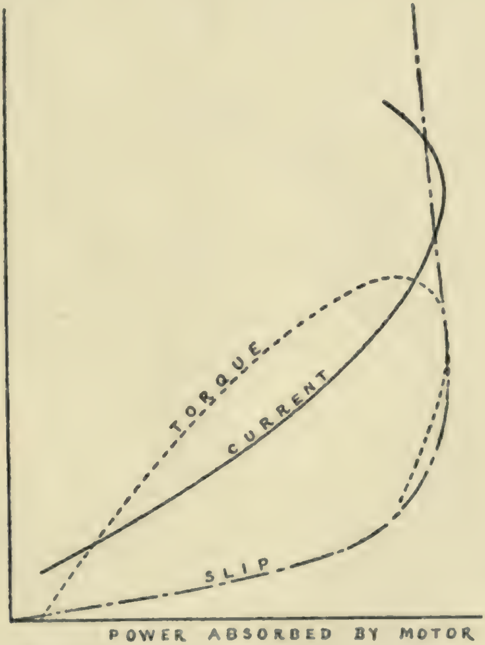

FIG. 132.—Curves derived from Circle Diagram.

example of this mode of representation is shown in Fig. 132, where the current, torque, and slip are plotted against the power supplied to the motor.

§ 119. Brake Tests. Soames' Brake

The experiments required for finding the constants which determine the circle diagram may all be carried out with the expenditure of a comparatively small amount of power, and hence the method of testing which depends on the use of the circle diagram is of particular value in connection with motors of large size, when the

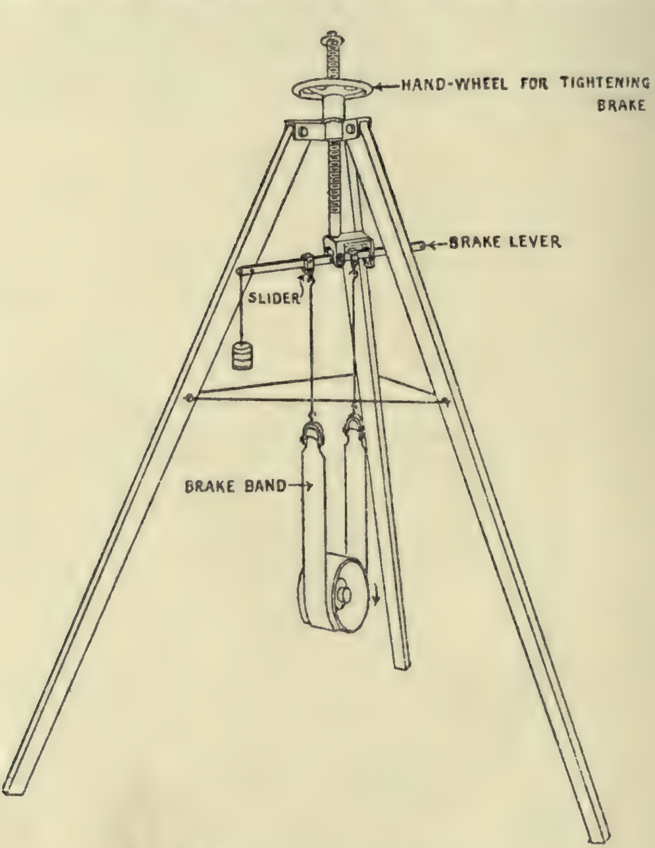

FIG. 133.—Soames' Testing Brake.

power necessary for carrying out brake tests becomes a serious consideration. When dealing with motors of small size, it may be desirable to supplement the no-load and short-circuit tests by brake tests and measurements of the slip, which furnish additional checks on the accuracy of the circle diagram. We shall now consider some of these supplementary tests.

For carrying out direct brake tests, one of the most convenient appliances is Soames' Testing Brake, shown in Fig. 133. This brake may be used up to about 5 h.p. It consists of a rigid tripod stand, at the top of which is a nut fitted with a hand-wheel. Passing through this nut is a screwed shank, which ends in a fork fitted with hardened steel plates on which rest the knife-edges of the brake lever. The middle portion of the lever is graduated, so that the two sliders supporting the brake band may be set at equal distances from the knife-edge, each distance being equal to the radius of the pulley. The tripod is adjusted so as to bring the knife-edge vertically over the centre of the shaft. The brake band passing around the pulley consists of a thin braided band of hemp. A suitable weight—depending on the load required—is hung from one end of the lever, so that when the brake band is tightened the motor tends to lift the weighted end of the lever. The displacement of the lever in either direction is limited by the height of the rectangular opening in the supporting frame through which it passes on one side of the fork. By means of the hand-wheel the friction between the pulley and the brake band is adjusted until the lever is gently swaying in a horizontal position. From the weight W, its known distance d (in feet) from the lever knife-edge, and the speed m in revs. per min., the b.h.p. is easily calculated by means of the formula—

$$\text{brake horse-power} = \frac{2\pi d \mathrm{W} \,.\, m}{33{,}000}$$

but since $\frac{2\pi d}{33{,}000}$ is a constant whose value is supplied by the makers, all that is necessary is to multiply the product Wm by this constant. The *useful torque* * exerted by the motor is, in lb.-ft., Wd.

§ 120. Air-friction Dynamometer

An exceedingly compact, simple, and ingenious form of brake dynamometer is the one shown in Fig. 134. This is the invention of Mr. W. G. Walker, and may be used up to about 20 h.p. It consists simply of two strong square blades which may be bolted at varying distances from the shaft to the cross-arm supporting them. This cross-arm consists of two parts, which may be easily bolted to the shaft of the motor as shown. The power is absorbed entirely by air friction, and it need hardly be pointed out that the apparatus keeps perfectly cool. The b.h.p. is calculated from the speed and the position of the blades, but the relation connecting these three quantities is not a simple one, and the results of calibration tests by the makers have to be utilized.*

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* This torque is not the same as the T in the formulæ given in connection with the circle diagram. The latter is the *total* torque, and includes the torque required to overcome the resistances due to friction and hysteresis in the rotor core.

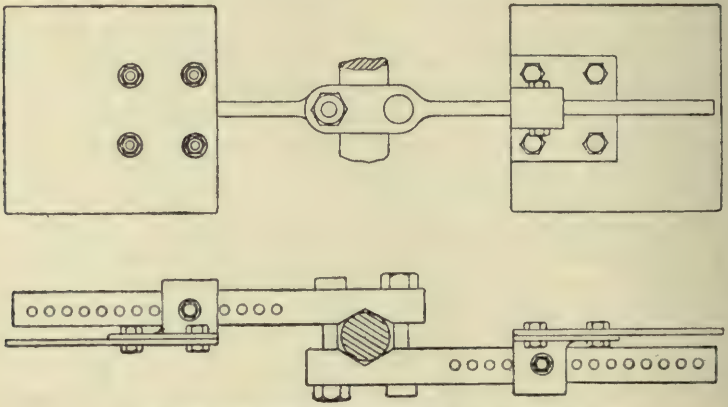
Fig. 134.—Walker's Air-friction Dynamometer.

§ 121. Eddy-current Brakes

Various forms of eddy-current brakes have been devised. These are extremely accurate, though more expensive and not so easy to apply as the simple forms just described. They are particularly valuable in cases where very small amounts of brake-power have to be accurately measured. One form of such a brake, designed by Prof. A. Grau † and suitable for small motors, is shown in Fig. 135. It consists of a copper brake-disc mounted on the motor shaft, and a lever, one end of which carries an electromagnet, between whose poles the copper disc rotates, while the other end, which is graduated, carries a sliding weight. A perspective view of the apparatus is given in Fig. 135 (*a*), while Fig. 135 (*b*) shows a portion of it in plan.

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* Further particulars regarding this brake are obtainable from Mr. W. G. Walker, 47, Victoria Street, Westminster, S.W.

† *Elektrotechnische Zeitschrift*, vol. xxi. p. 265 (1900).

The zero position of the sliding weight W is that in which it just balances the weight of the magnet pole-pieces on the other side of the knife-edge when the motor is at rest. The force F due to eddy currents is given by the equation—

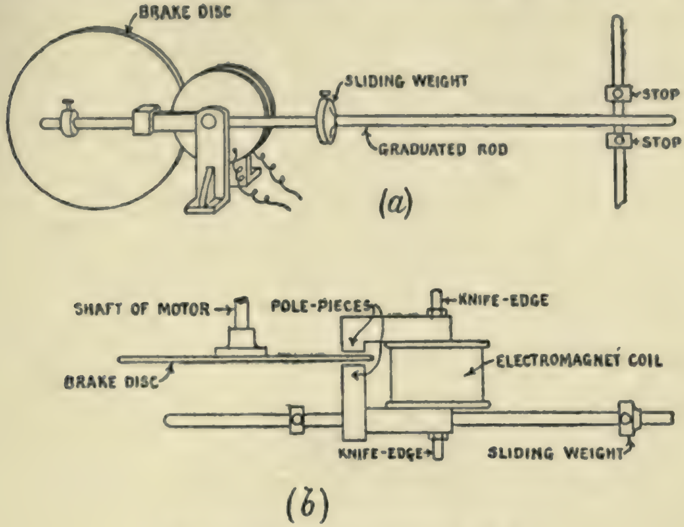

FIG. 135.—Grau's Eddy-current Brake.

$$F = \frac{dW}{a}$$

d being the displacement of W from its zero position, and a the distance of the middle line of the pole-pieces from the knife-edge. The corresponding useful torque of the motor is given by $F . l$, where l is the distance of the middle line of the pole-pieces from the centre of the shaft. By suitably varying the exciting current of the brake magnet, the lever may be kept absolutely steady.

An extremely convenient form of eddy-current brake has recently been described by Messrs. D. K. Morris and G. A. Lister.* The *entire* brake is mounted on the motor shaft, replacing the pulley. The essential parts of the brake are shown in Fig. 136. There are *two* brake rings of copper, each of which is supported by a steel disc and a number of steel arms, riveted to a hub. Each hub is keyed to a bush, which in its turn is keyed to an inner bush, the latter being finally keyed to the motor spindle. The reason for providing an inner bush is that by having several inner bushes of different internal diameters, the brake may be made to fit different sizes of motor spindles. The stationary part consists of a number of electromagnets (8) and two external yoke rings.* The cores of the electromagnets are provided with expanded pole-pieces, which are supported by brass plates bolted to a casting fitted with a gunmetal bush which rides loose on the bush supporting the brake rings. Screwed into the casting, diametrically opposite to each other, are two tubes, a short one to carry a counterpoise, and a long graduated one which forms the brake lever. The external yoke rings are attached to the magnet system by means of aluminium castings, which overarch the brake rings (the mode of attachment is not shown in Fig. 136). The brake rings rotate in the narrow gaps between the magnet poles and the yoke rings. The coil of each magnet is provided with a ventilating duct in the middle.

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* ***The Electrician***, **vol. lv. p. 88 (1905).**

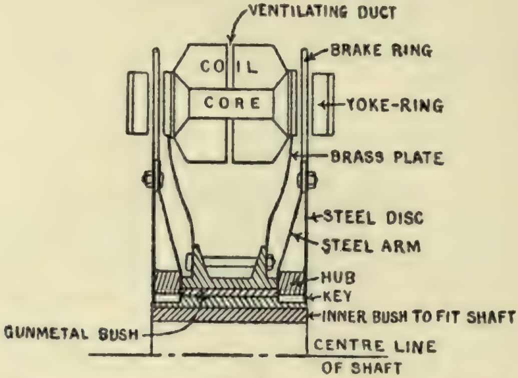

FIG. 136.—Morris and Lister's Eddy-current Brake.

§ 122. Measurement of Slip. Stroboscopic Methods

We shall next consider the measurement of *slip*. For the sake of securing high efficiency, the slip is arranged to be small at all ordinary loads, not exceeding, as a rule, some 5 per cent. at full load even in small motors, and being much less in larger motors (§ 67). Hence its measurement by a direct comparison of the actual speed with the speed of synchronism is incapable of yielding accurate results, as the error involved in the speed measurement is quite comparable with the slip, and may even exceed it considerably at light loads. Special methods have, therefore, been devised for measuring *small* values of the slip.

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* The poles of the electromagnets on the same side of the brake are alternately north and south.

In one of these, the shaft of the motor carries a cardboard disc divided into a number of sectors, alternately black and white, the total number of sectors being equal to twice the number of poles. The disc is illuminated by means of an arc lamp, which derives its current from the same source as the motor. Now, an alternating-current arc is extinguished twice during each complete wave of current, so that the light coming from it will in reality consist of a succession of flashes. If we suppose that the motor is being driven at synchronous speed, then since the time taken by a white sector to move into the position of the white sector next in advance of it is equal to a half-period, it follows that as each flash reaches its maximum brightness the white and black sectors are in the same relative positions, and thus the disc appears to be stationary. If slip is allowed to take place, the positions of the sectors will be retarded relatively to the flashes, and the disc will appear to rotate.* By counting the number of apparent revolutions of the disc during any convenient interval of time, we obtain the number of slip revolutions. If we at the same time determine the number of revolutions of the motor (during the same interval of time, while counting the slip revolutions), then the slip is given by—

$$\frac{\text{slip revolutions}}{\text{motor revolutions} + \text{slip revolutions}}$$

Instead of a cardboard disc divided into white and black sectors, a simple radial chalk line drawn across the web of the pulley is sufficient, provided the pulley is illuminated mainly by the light of the arc. The experiment may be conducted in daylight.

A disadvantage connected with the use of the arc is that as it draws a fairly heavy current from the mains, it is liable to unbalance the three-phase system, so that the phase p.d.'s across the stator windings are not alike, and the motor is not running under entirely normal conditions.

Samojloff† has, however, found that if the rotating disc be in darkness, an incandescent lamp (whose periodic partial extinctions, due to alternate heating and cooling of the filament, are ordinarily quite imperceptible to the eye) may be successfully employed for the same purpose. With an incandescent lamp, which only takes a small current, no serious unbalancing can, of course, take place.

Although by the above means *small* values of the slip are easily determined with a high degree of accuracy, the method fails when the slip is considerable, since it then becomes impossible to count the rapid revolutions of the stroboscopic disc. The range of the stroboscopic method may, however, be greatly extended by various special devices, one of the most successful of which is that embodied in the direct-reading slip indicator recently devised by Dr. C. V. Drysdale,[*] and shown in Fig. 137. The instrument consists of a boxwood cone mounted on a spindle whose end may be applied to the motor shaft, so that the cone will be driven at the speed of the motor. Resting on the cone is a pivoted disc mounted at the end of a light fork, which is pivoted in a slider. The position of the slider may be varied by means of a screw, and may be read off on a scale which gives the corresponding value of the slip directly. Attached to the surface of the disc is a stroboscopic disc of paper, the number of black or white sectors being equal to the number of poles

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* In a direction opposed to the direction of rotation of the motor.

† *Annalen der Physik*, 3. 2. p. 353 (1900).

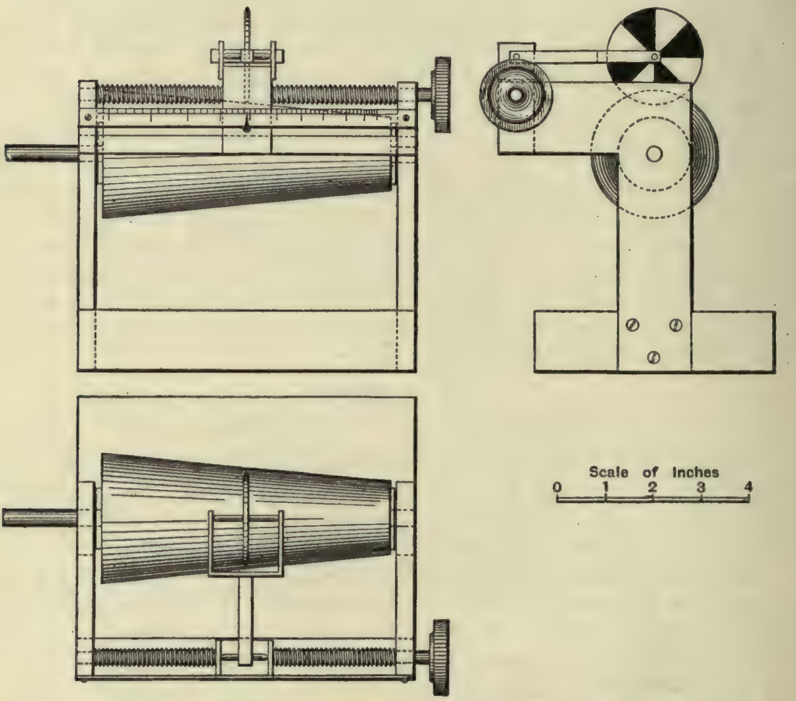

Fig. 137.—Drysdale Direct-reading Slip Indicator.

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[*] *The Electrician*, vol. lv. p. 734 (1905). For the use of Fig. 137 the author is indebted to Dr. Drysdale and *The Electrician* Printing and Publishing Co., who kindly supplied the block for this illustration.

in the motor. The disc is illuminated as usual by a source of light connected to the same mains as those supplying the motor. If the position of the disc is such that its diameter is equal to that of the cone, then disc and cone will revolve at the same speed, and if this speed were that of synchronism, the disc would appear to be stationary. This position gives the zero of the scale. Since the speed of the motor is, however, below that of synchronism, the disc will appear to rotate if placed at the zero of the scale. But its speed may be increased by traversing it towards the thicker end of the cone, until it appears stationary; the method is thus a *zero* method. The value of the slip corresponding to a given displacement of the disc from the zero position is easily calculated from the angle of the cone in constructing the direct-reading scale.

§ 123. Methods applicable to Slip-ring Rotors

The method just described is applicable to motors having either wound rotors provided with slip-rings or permanently short-circuited rotors. With wound rotors, many other methods for measuring small slips are available. Thus, a sensitive dead-beat moving-coil galvanometer or milli-ammeter may be connected as a shunt across one of the connecting leads between the slip-rings and the starting resistance, and the number of throws to the same side of zero determined per second. This number gives the frequency n_2 of the rotor currents, and if n_1 is known, $s = \frac{n_2}{n_1}$. Instead of the milli-ammeter, a telephone receiver may be used; the frequency n_2 is then given by *half* the number of clicks per second.

§ 124. Measurement of Friction Loss

We have already, in considering the circle diagram, explained one method of finding the friction loss in a motor. Another very convenient method, which may be used as a check on the former, is to take one set of wattmeter readings when the motor is running light at the normal p.d. with its rotor short-circuited as usual, and another set when the rotor has been suddenly open-circuited, the second set of readings being obtained as quickly as possible after open-circuiting the rotor. The difference of power in the two cases practically represents the frictional loss.

F. Blanc[*] finds that for well-constructed motors of $\frac{1}{10}$ to 300 h.p. the friction loss may be represented by the formula—

$$\text{friction loss} = C\sqrt{\text{h.p.} \times D}$$

D being the rotor diameter in cms., and C a constant whose value ranges from 20 to 30.

§ 125. Efficiency Tests. Methods of Meunier, Sumpner, and Weekes

A very accurate method of determining the efficiency of two motors of about the same output is the following modification of the well-known Hopkinson test for dynamos, which appears to have been first suggested by M. P. Meunier,[†] and independently by Dr. Sumpner and Mr. R. W. Weekes,[‡] who have tested the method practically with very satisfactory results. The arrangement of connections is shown in Fig. 138. The two motors are belted together, the sizes of the pulleys being different, so that while one of the machines—which has the larger pulley—is running as a *motor*, the other—which has the smaller pulley—is being driven mechanically above the speed of synchronism. It will be shown (§ 129) that when the rotor of an induction motor whose stator is connected across the supply mains is driven *above* the speed of synchronism, the motor will act as a *generator*, supplying power to the mains. Thus one of the machines shown in Fig. 138 will act as a motor, receiving power partly from the mains, partly (the greater amount) from the second machine, which acts as a generator. The stator terminals of the generator are marked G_1, G_2, G_3, those of the motor, M_1, M_2, M_3. The difference in the diameters of the pulleys must be sufficient to give slips corresponding to the maximum load when the rotors are short-circuited. Smaller loads may be obtained by introducing suitable resistances into the rotor circuits.[§] Let w = power supplied from mains (algebraical sum of readings of wattmeters W_1 and W_2 in Fig. 138). This power is required to make up for the total losses. Now, one source of loss is that represented by the power w_b required to bend the belt and drive it against air friction. In order to find w_b, the wattmeters W_1 and W_2 are read (1) when both stators are across the mains, and the rotor of the motor is short-circuited, driving the open-circuited rotor of the generator by belt; and (2) when both machines are running light with the belt off. The difference in the power drawn from the mains in the two cases gives w_b. Thus $w - w_b$ gives us the power actually wasted *in the machines.* Part of this waste will occur in the motor, the remainder in the generator. Now, since the motor is necessarily more heavily loaded than the generator, it will waste a larger amount of power; the power wasted in each machine may be taken to be proportional to the slip. Thus, if s_m, s_g stand for the slips of motor and generator respectively, the power lost is—

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[*] *Elektrotechnische Zeitschrift*, vol. xxi. p. 131 (1900).

[†] *Éclairage Électrique*, vol. xxxiv. p. 228 (1903).

[‡] *Electrical Engineer*, vol. xxxiii. p. 918; also vol. xxxiv. p. 310 (1904).

[§] An allowance can easily be made for the power wasted in the regulating resistances.

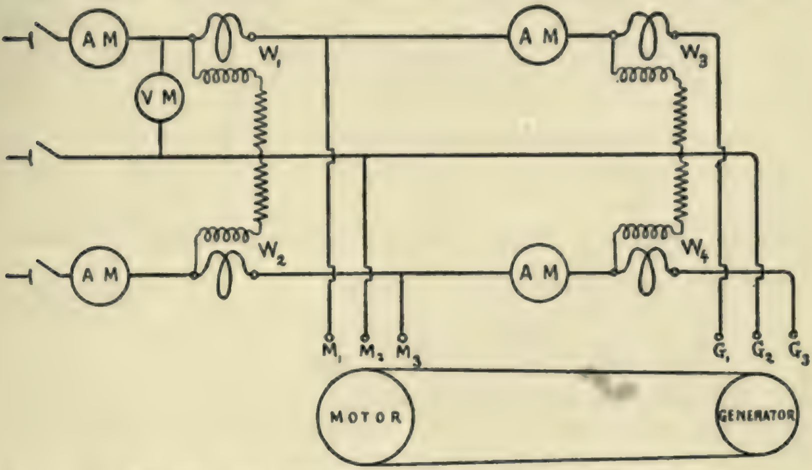

FIG. 138.—Efficiency Test of Two Similar Induction Motors.

$$\frac{w - w_b}{s_m + s_g} \cdot s_m \text{ in the motor}$$

and—

$$\frac{w - w_b}{s_m + s_g} \cdot s_g \text{ in the generator}$$

Now, if w_g = power developed by generator (algebraical sum of readings of W_3 and W_4 in Fig. 138) and transmitted to motor, the total power received by motor is $w_g + w - w_b$. On subtracting from this the power wasted in the motor, we obtain the useful power transmitted to the generator, and on dividing this latter by $w_g + w - w_b$, we obtain the motor efficiency. Similarly, the generator efficiency is obtained by dividing its useful output w_g by the power transmitted to it by the motor.*

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* Having subtracted w_b from the power drawn from the mains, we deal with the machines as if they were connected by means of a perfectly pliable belt which encounters no resistance in its motion through the air.

There is, however, an additional source of loss due to the belt drive which we have so far left out of consideration. This is due to the slipping of the belt on both pulleys. So long as the machines are lightly loaded, the belt slip is inappreciable, and w_b represents the only loss taking place outside the machines. But with a heavy load, the rate of heat production at the pulleys due to slipping of the belt over their surfaces may represent an appreciable fraction of the brake-power of the motor. A correction thus becomes necessary. If d_m = dia. of motor pulley, d_g = dia. of generator pulley, and t = thickness of belt, all measured in terms of the same unit of length, then the effective diameters of the pulleys are $d_m + t$ and $d_g + t$, and if the belt did not slip, the ratio of the speed of the generator to that of the motor would be $\frac{d_m + t}{d_g + t}$. By reason of slipping, however, this is reduced in a certain ratio, say $\frac{1 - b}{1}$, where b may be termed the *belt slip*. Thus—

$$\frac{\text{speed of generator pulley}}{\text{speed of motor pulley}} = \frac{d_m + t}{d_g + t}(1 - b)$$

But this ratio may be measured directly. For if s_m = slip of motor rotor relatively to its stator field, and s_g = slip of generator rotor,* we have—

$$\frac{\text{speed of generator rotor}}{\text{speed of motor rotor}} = \frac{1 + s_g}{1 - s_m}$$

Equating the two expressions for this ratio, we find—

$$1 - b = \frac{1 + s_g}{1 - s_m} \cdot \frac{d_m + t}{d_g + t}$$

Now, if by reason of belt slip the speed of the generator is reduced in the ratio $\frac{1 - b}{1}$, it is obvious that the power transmitted to it by the motor is reduced in the same ratio. Thus of the total power w_m † developed by the motor a fraction b is lost in producing heat at the pulleys. If η_m = motor efficiency, approximately calculated as already explained, then in order to balance the loss due to belt slip we have to draw from the mains an amount of power $\frac{bw_m}{\eta_m}$. We have now to recalculate both efficiencies, assuming that

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* Numerical values of the slip being considered (*algebraically*, the generator slip is *negative*).

† $$w_m = \frac{w_g + w - w_b - \frac{w - w_b}{s_m + s_g} \cdot s_m}{w_g + w - w_b}$$

no losses whatever are occasioned by the belt drive, and that the total waste of power in the machines is—

$$w - w_b - \frac{bw_m}{\eta}$$

Although we have explained in detail how to apply the correction due to belt slip, we may state that in most cases this correction is extremely small, and need only be taken account of at heavy loads and where the highest possible degree of accuracy is required.

§ 126. Alexanderson's Method

The following approximate method of testing induction motors has been devised by Mr. E. Alexanderson,* and is used by the General Electric Co. of America. The principle of the method is based on the fact that for ordinary loads the slip is practically proportional to the torque. The load consists of a separately excited continuous-current generator belted to the motor. The useful output of the generator is measured by means of a voltmeter and ammeter, and is equal to Vi (V being the p.d. and i the current); its total output is $Vi + ri^2$, r being the armature resistance. The slip s of the motor corresponding to this load is measured (by the stroboscopic method of painting a number of white sectors on the pulley equal to the number of poles, and illuminating the pulley by an alternating arc supplied from the same circuit as the motor). Let $w =$ total mechanical power of motor, and $w_1 =$ power required to balance losses due to rotor bearing friction and windage, belt losses, and frictional and core losses of generator; and let T and T_1 be the torques corresponding to the total power w and the losses w_1 respectively. Then obviously—

$$\frac{w}{w_1} = \frac{T}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The ratio $\frac{T}{T_1}$ is determined by switching off the load from the continuous-current generator, but maintaining its excitation, and finding the new value s_1 of the slip—which corresponds to the torque T_1. We then have $\frac{T}{T_1} = \frac{s}{s_1}$ approximately, and (1) becomes $\frac{w}{w_1} = \frac{s}{s_1}$. Subtracting unity from each side, we get $\frac{w - w_1}{w_1} = \frac{s - s_1}{s_1}$,

or—

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* *Electrical World and Engineer*, vol. xliv. p. 212; or *The Electrician*, vol. liii. p. 869 (1904).

$$\frac{w - w_1}{s - s_1} = \frac{w_1}{s_1} = \frac{w}{s}, \text{ so that } w = (w - w_1)\frac{s}{s - s_1}$$

But since $w - w_1$ = total output of generator $= \mathrm{V}i + ri^2$, we have—

$$w = \frac{s}{s - s_1}(\mathrm{V}i + ri^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus w, the *total* mechanical power of the motor, becomes known. In order to find its useful or brake-power, we have to determine the power w_f corresponding to the rotor frictional loss at the speed for which the total mechanical power of the motor is w. Let T_f be the corresponding torque. Then we have $\frac{w}{w} = \frac{\mathrm{T}}{\mathrm{T}_f}$. To find the latter ratio, we remove the belt, allow the motor to run light, and measure its slip s . Then $\frac{\mathrm{T}}{\mathrm{T}_f} = \frac{s}{s_f}$, so that $\frac{w}{w_f} = \frac{s}{s}$. Proceeding as before, we find—

$$\text{brake-power of motor} = w - w_f = (s - s_f)\frac{w_f}{s_f} = (s - s_f)\frac{w}{s}$$

or, using (2)—

$$\text{brake-power} = \frac{s - s_f}{s - s_1}(\mathrm{V}i + ri^2)$$

The brake-power being known, the efficiency is at once obtained by dividing it by the total power supplied to the motor, as measured by the two-wattmeter method.

Now, in rough measurements, it would not be necessary to measure the slip corresponding to any other value of the load in order to find the brake-power. For, using the same method as before, we find—

$$\text{losses of generator with belt} = w_1 - w_f = \frac{s_1 - s_f}{s - s_1}(\mathrm{V}i + ri^2) \ . \ (3)$$

and hence—

brake-power = total output of generator + losses in belt and generator

Thus the brake-power is obtained by simply adding to the total generator output the value of the losses as given by (3), the assumption being made (which is obviously not quite correct) that these losses are constant within the working range of the motor speed.

§ 127. Heating Test

In addition to the special tests described, it is necessary, in an exhaustive test of an induction motor, to carry out the insulation (§ 89) and heating tests, which apply to all classes of electrical machinery. The heating test may be conveniently carried out by clamping and short-circuiting the rotor, and applying a p.d. to the stator windings sufficient to give rise to the full-load currents. The rise of temperature should be determined by resistance measurements (§ 93).

§ 128. Induction Motors with Large Starting Torque

In an induction motor having permanently short-circuited windings, high efficiency (which involves low rotor resistance) is incompatible with large starting torque. Such motors are, therefore, incapable of starting against a heavy load, and it is usual to provide a fast and loose pulley, the motor being first allowed to run up to speed on the loose pulley, and the driving belt being then shifted to the fast pulley. Instead of this arrangement, various forms of friction clutch may be used. In a device recently patented by De Lignières,* the stator as well as the rotor is capable of rotating, and the stator is provided with a suitable brake, by means of which it may be gradually reduced to rest. In starting, the stator is allowed to run up to full speed, and the brake is then gradually applied to it, the torque increasing as the speed decreases, until the rotor is started; the stator being then gradually reduced to rest, and the rotor gaining speed.

M. Déri has devised an extremely ingenious method of securing large starting torque with high efficiency in motors having permanently short-circuited rotors.† This method is used by the Helios Co. in their induction motors. It consists essentially in halving the number of poles at starting, and in using a special form of short-circuited rotor winding, such that while under normal running conditions, with the full number of poles in use, the resistance of the rotor winding is low, its resistance when only half the number of poles is used is considerable, thereby enabling the rotor to develop a powerful starting torque. The change in the number of stator poles is effected by means of a switch. The arrangement of the rotor winding is shown in Fig. 139. The four conductors shown arranged on the rotor periphery form one element of the winding. The zig-zag lines R_1 and R_2 are high resistances bridged across the end-connections of the winding as shown. It will be easily seen that with a rotating two-pole field (starting position of switch) the total e.m.f. induced around the closed circuit formed by the four conductors always vanishes (Fig. 139 (*a*)), and that the only paths open to the currents are those through R_1 and R_2 (which come into play alternately, the currents flowing through them being in quadrature with each other), *i.e.* the circuit of the winding is a high-resistance one, enabling the rotor to exert a large starting torque. But as soon as the switch is thrown over into the running position, corresponding to a four-pole field—Fig. 139 (*b*)—the e.m.f.'s around the circuit formed by the four conductors are added together, the currents flowing around a low-resistance circuit (corresponding to high efficiency) without traversing R_1 and R_2.

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* *Western Electrician*, vol. xxxiii. p. 228 (1903).

† *Zeitschrift für Elektrotechnik*, vol. xvi. p. 285 (1898); also *Elektrotechnische Rundschau*, vol. xxi. p. 24 (1903).

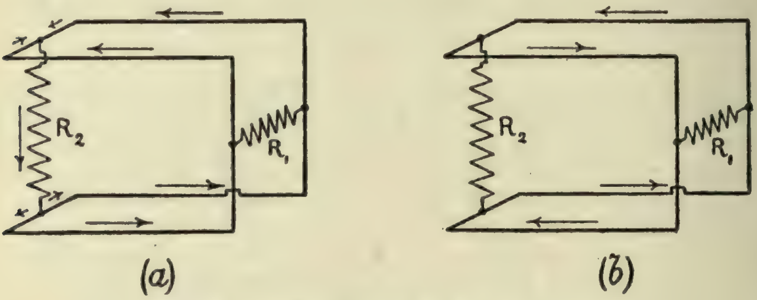

FIG. 139.—Diagram of Déri Winding for Rotors.

The most commonly adopted method for limiting the starting current and securing a large starting torque consists in using wound rotors provided with slip-rings, by means of which starting resistances may be introduced into the rotor circuits. In some cases, these resistances are mounted inside the rotor itself, and an automatic switch, controlled by a centrifugal governor, gradually cuts them out as the motor gains speed. In an arrangement recently patented by H. S. Meyer,* the starting resistance takes the form of iron wires of graded size wound at the bottom of the slots containing the copper winding, so that the wires form part of the magnetic circuit of the rotor.

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* *Western Electrician*, vol. xxxv. p. 287 (1904).

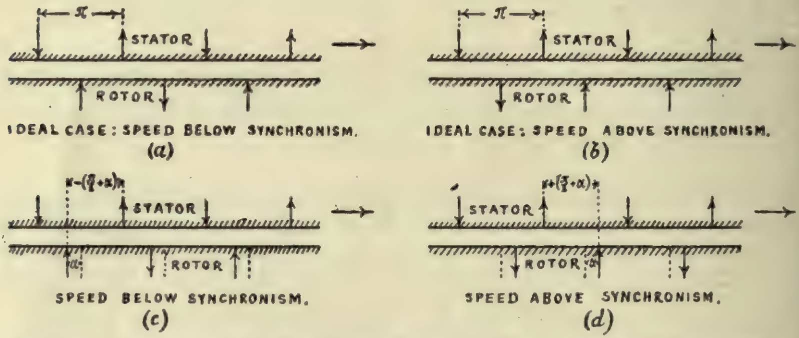
CHAPTER XIV

§ 129. Generator action of induction motor at hypersynchronous speed. Phase relation of stator and rotor currents—§ 130. Vector diagram of e.m.f.'s and its transformation—§ 131. Generalized circle diagram—§ 132. Characteristic features of induction generator—§ 133. Speed control of induction motors. Rheostatic control—§ 134. Speed control by change in number of poles—§ 135. Tandem control. Multiple motor method—§ 136. Single-phase induction motors. Theory of motor at rest—§ 137. Torque exerted by single-phase induction motor when running—§ 138. Effect of varying resistance of rotor circuits—§ 139. Starting of single-phase induction motors.

§ 129. Generator Action of Induction Motor at Hypersynchronous Speed. Phase Relation of Stator and Rotor Currents

A POLYPHASE induction motor in many respects resembles an ordinary shunt-wound, continuous-current motor. Each runs at a speed which decreases but little with increase of load, so that practically we may regard each type as a constant-speed motor. Each develops a torque which increases, for ordinary load conditions, practically in proportion to the decrease of speed (slip) from no-load speed. Further, each, if *driven* mechanically at a sufficient speed above the no-load speed, is capable of acting as a *generator*. We are thus led to consider the generator action of an induction motor when its rotor is mechanically driven above the speed of synchronism. An induction motor when so used forms an *induction generator*.

In order to bring out as clearly as possible the relation connecting the phase differences between the primary and secondary currents in the two cases when the speed is a certain amount below and above that of synchronism, we may consider the four diagrams (*a*), (*b*), (*c*), and (*d*) of Fig. 140. The stator field is supposed to move from left to right in each case. The arrows drawn along the face of the stator core and at right angles to it indicate the polarity of the stator field, and are drawn at the points where the induction due to the stator current *alone* reaches a maximum value. Diagrams (*a*) and (*c*) refer to the case of a motor running with *positive slip*, *i.e.* below the speed of synchronism. The sine wave of magnetic flux due to the stator current in sweeping past the rotor conductors induces e.m.f.'s in them, so that there will be a sine wave of e.m.f. travelling along the rotor conductors, and this sine wave of e.m.f. will give rise to a sine wave of current. Now, if the rotor current were exactly in phase with the hypothetical rotor e.m.f., a simple application of the rule for finding the direction of an induced current would show that the field due to the rotor currents would be as shown by the arrows drawn at intervals along the surface of the rotor core in diagram (*a*). (This would be approximately the case for very small values of the slip, when the frequency of the rotor currents is so low that they lag by a relatively small amount behind the hypothetical e.m.f. producing them.) Owing, however, to the lag of the rotor currents behind their e.m.f.'s, the hypothetical field due to the rotor currents will be as shown in diagram (*c*), where the current is supposed to lag by an angle a behind the e.m.f. Similar reasoning will show that if the motor be driven above the speed of synchronism, so as to have a *negative slip*, in which case the sine wave of the stator flux is reversed in its motion relatively to the conductors, the distribution of the hypothetical rotor field in the ideal case of no lag is as in (*b*), while the actual distribution is as in (*d*). The amount of lag, it is to be noted, depends solely on the slip,* and will have the same numerical value a whether the slip be positive or negative, provided the numerical value of the slip is the same.



IDEAL CASE: SPEED BELOW SYNCHRONISM. (*a*)

IDEAL CASE: SPEED ABOVE SYNCHRONISM. (*b*)

SPEED BELOW SYNCHRONISM. (*c*)

SPEED ABOVE SYNCHRONISM. (*d*)

FIG. 140.—To illustrate Phase Relation of Stator and Rotor Currents in Induction Motor and Generator.

On comparing diagrams (*c*) and (*d*), we notice that in passing from any positive value of the slip to an equal negative value, we change the phase difference between the wave of hypothetical rotor flux and that of hypothetical stator flux by an amount $\pi + 2a$.

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* Which determines the *frequency* of the rotor currents, and hence the angle of lag.

§ 130. Vector Diagram of e.m.f.'s and its Transformation

We may next, in order to reduce the stator and rotor currents to the same frequency (§ 106), imagine the negative slip increased, and at the same time resistances introduced into the rotor circuits so as to maintain the current and phase difference unaltered, until the slip becomes numerically equal to unity, *i.e.* until the speed is double that of synchronism. If the original slip was numerically equal to s, then, r_2 denoting the original resistance of one phase of the rotor circuit—*i.e.* the resistance of the short-circuited phase — and R_2 the required equivalent resistance for a negative slip of unity, we must have $R_2 = \frac{r_2}{s}$. For, by increasing the slip in the ratio $1 : s$, we have increased the hypothetical rotor e.m.f. and the rotor reactance in this ratio, so that in order to have the current and its phase relatively to the e.m.f. unaltered, the total resistance must also be increased in the same ratio. If, as before, r denote the added external resistance to each rotor phase, $r = R_2 - r_2 = \frac{1 - s}{s} r_2$ (*cf.* § 106).

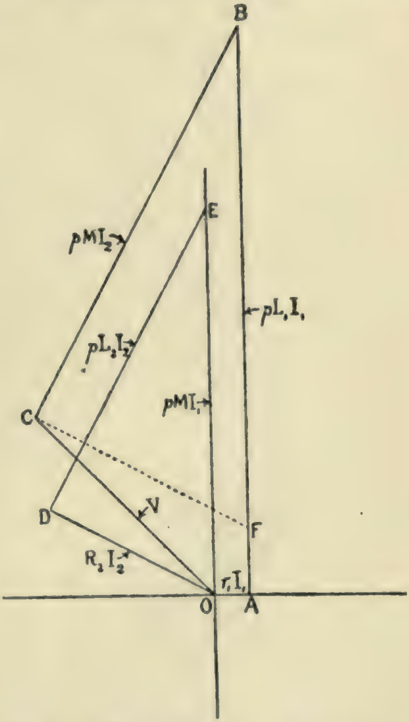

Fig. 141.—Vector Diagram of Induction Generator.

The currents in the two windings having by this artifice been reduced to the same frequency, we may proceed to construct the vector diagram of e.m.f.'s similar to the motor diagram shown in Fig. 127. The lines $OA = r_1I_1$ and $AB = pL_1I_1$ remain unaltered in phase. But the vector r_2I_2 of the secondary drop must, in accordance with diagrams (*c*) and (*d*) of Fig. 140, be shifted forward (*i.e.* in the counter-clockwise direction) by an amount $\pi + 2a$, so as to occupy the position OD shown in Fig. 141, the angle DOE being equal to

$a = \theta_2 = \tan^{-1}\frac{pL_2}{R_2}$. We may now complete the primary vector diagram by drawing $BC = pMI_2$ in a direction at right angles to OD. The vector OC then corresponds to the constant stator phase p.d. V. The triangle of secondary e.m.f.'s, ODE, may now be completed.*

Having thus obtained our vector diagram, we proceed to transform it just as we did in dealing with a motor (§ 109), and so establish the *circle diagram* for an *induction generator.*

§ 131. Generalized Circle Diagram

The circle representing the locus of the extremity of the primary current vector is identical with the circle obtained for the machine

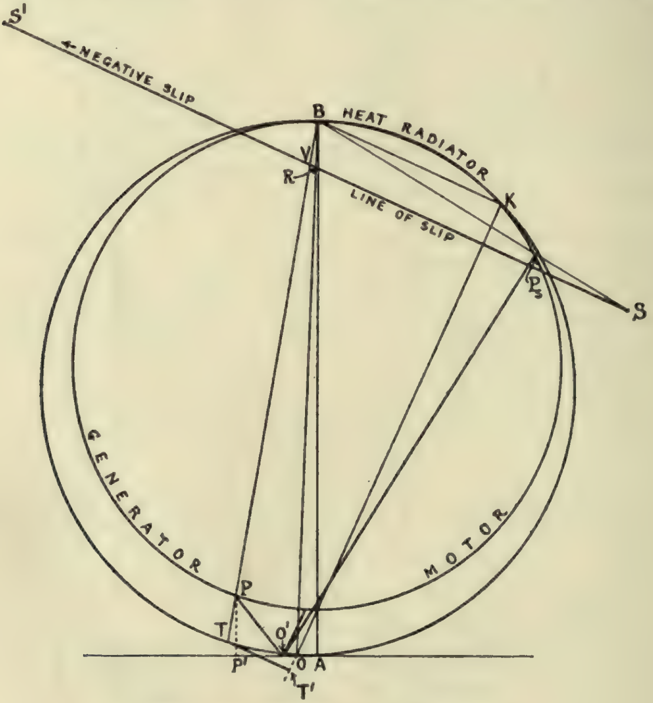

Fig. 142.—Generalized Circle Diagram.

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* If we wish to regard the rotating triangle ODE as giving us, by the projection of its sides on a fixed straight line, the instantaneous values of the three e.m.f. components, then we must suppose the rotation to be *clockwise*, that of OABC being *counter-clockwise.* This is due to the fact that the motion of the stator field relatively to the stator conductors is opposed to its motion relatively to the rotor conductors.

when used as a motor; and the constructions for electrical power, slip, and torque remain unaltered (§ 110).

Such a complete circle diagram, embracing every possible condition of operation of the machine, whether as generator or motor, is shown in Fig. 142.* Thus, O′P represents a certain value of the stator current. Its projection O′P′ on the line of phase p.d. is proportional to the *useful* output of the generator outside its terminals.† If we draw BT, the intercept RV on the line of slip represents the negative slip of the generator to the same scale as that to which RS represents a slip of unity for the motor. Again, the perpendicular TT′ gives the value of the torque which is actually instrumental in generating electrical energy, *excluding* the additional torque which must be provided to compensate for rotor friction. The power factor is given by the cosine of the angle P′O′P.

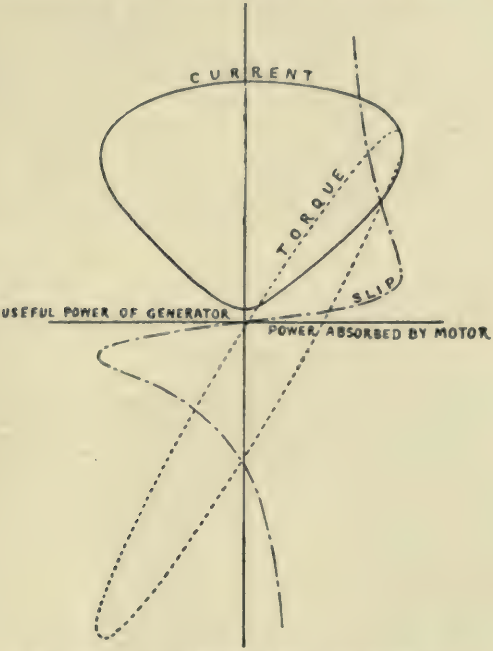

Fig. 143.—Curves Derived from Circle Diagram.

If, using such a complete circle diagram, we determine the relations connecting power supplied to terminals of motor, or power delivered at terminals of generator, with stator current, slip, and torque, we obtain the elegant graphical representation shown in Fig. 143. The ordinates of the three curves drawn correspond to stator current, slip, and torque respectively, the abscissæ representing power absorbed from, or supplied to, the mains.

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* The experimental data required for the construction of the diagram have already been fully considered in Chapter XIII.

† Total useful output = O′P′ × phase p.d. × number of stator phases.

Between the regions of generator and motor action—the generator action extending, in the circle diagram of Fig. 142, to a point a little to the left of B (vertically over O′), and the motor action to P_s—we have a region, corresponding roughly to the arc P_sB, but extending slightly beyond B, within which the machine is giving out neither electrical nor mechanical energy, but is absorbing both, acting as a sink of energy pure and simple. This region, it need hardly be pointed out, is of purely theoretical interest, not only because for obvious reasons it would never be used in practice, but because, even if we did attempt to reach it,* the insulation of the coils would be rapidly destroyed by the excessive currents. This portion of the diagram is marked "heat radiator," as the only function then performed by the machine is to absorb energy and convert it into heat.

§ 132. Characteristic Features of Induction Generator

It must be carefully noted that the generator action of an induction motor depends wholly on the fact of its stator terminals being in connection with a polyphase generator which supplies the necessary exciting current required to maintain the rotating field. An induction generator is thus not a self-exciting machine; it can only be run in parallel with a polyphase generator of ordinary construction. A remarkable feature of the induction generator is the fact that its frequency is independent of the speed, and that for small values of the slip its output is roughly proportional to the slip—as is immediately evident from the "slip" curve of Fig. 143.

If we suppose the induction generator to be working on a non-inductive load, then the current supplied to the load will be represented by the component O′P′ in the diagram of Fig. 142. The wattless component of the current, obtained by taking the projection of O′P on the vertical, will be considerable, amounting, even in the neighbourhood of maximum power factor, to some 30 per cent. of O′P′. An induction generator thus requires a very large exciting current; and a plant consisting of induction generators would have to include a supplementary plant of synchronous machines, of capacity equal to some 30 per cent. of that of the induction generators for providing the necessary exciting or wattless current. It is mainly for this reason that induction generators have not come into use, in spite of the fact that a great deal of ingenuity has been devoted to special types of this machine.*

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* The point K is, in any case, inaccessible for purely mechanical reasons, corresponding, as it does, to an infinite speed.

Since an induction generator does not run at synchronous speed, it is frequently termed an *asynchronous generator.*

The fact that an induction motor when driven above synchronous speed is capable of acting as a generator is turned to useful account in connection with railways employing such motors, the motors in descending a steep gradient being made to return power to the line by their generator action; the power is thereby saved, instead of being dissipated by the use of ordinary brakes.

§ 133. Speed Control of Induction Motors. Rheostatic Control

The induction motor is, as we have seen, practically a *constant speed* motor, its slip at full load amounting to not more than 5 per cent. even in the case of small motors, and to much less in larger motors. Now, for many practical purposes—such as traction or crane work—it is essential to have a motor whose speed may be varied within much wider limits. We shall now consider some of the methods of speed control which are used in practice.

The method most commonly employed consists in introducing resistances into the rotor windings. We have already seen (§ 106) that the introduction of non-inductive resistances into the rotor circuits has the effect of reducing the speed while leaving the torque, the currents, and their phase relations unaltered. The total power drawn from the mains is obviously also unaltered, and the loss of mechanical power due to speed reduction (at constant torque) is accounted for by the power dissipated in the external resistance. This method, although giving any desired range of speed variation, is obviously very wasteful, especially at the lower speeds. Hence it can only be tolerated when the periods of variable speed are comparatively short, and when during the greater portion of the time the motor is running at its normal speed. It is, however, even in other cases used in combination with the methods described below.

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* Leblanc has devised an interesting form of exciter for use with an induction generator (*Éclairage Électrique*, vol. xviii. pp. 161, 376 (1899)). McAllister has found that the wattless current may be supplied by condensers (*Electrical World and Engineer*, vol. xli. p. 109). An induction generator requires no synchronizing before being thrown into parallel with other machines, and is entirely free from the trouble of "hunting" or "phase-swinging" to which synchronous generators are occasionally liable. The special forms of induction generators in which commutators are used are dealt with in Chapter XVI.

§ 134. Speed Control by Change in Number of Poles

The speed of synchronism of an induction motor being given, in revs. per sec., by $\frac{n}{P}$, n being the frequency of the stator currents and P the number of pairs of poles, it is evident that doubling the number of poles will halve the speed. By suitably arranging the stator winding,* and providing a pole-changing switch, it is possible to run at two different speeds, the lower of which is about half the higher. This method of speed control has been used by the Oerlikon Co. since 1893. In some motors, two independent windings, corresponding normally to different numbers of poles, each of which may be halved by a pole-changing switch, are provided, so that four different speeds are available. Thus, a motor wound for 12 and 6 poles, and 8 and 4 poles, will give (synchronous) speeds, at a frequency of 50, of 500, 1000, 750, and 375 revs. per min.

§ 135. Tandem Control. Multiple Motor Method

Another method which has been used is that known as the *cascade* or *tandem* or *concatenation* method of coupling induction motors. In its simplest form, it consists in using two motors mechanically coupled (the coupling may be direct, as when the motors are mounted on the same shaft; or indirect, as in the case of the two motors of a tramcar or railway carriage; or some form of mechanical gearing having any desired speed ratio may be used) so as to run at the same speed, the stator of the first motor, which we may call motor I, being connected to the mains, and the rotor of this motor being connected to the stator of the second motor (motor II), whose rotor circuits are closed, either by being short-circuited or through suitable resistances, as at starting. Let us suppose that rotor II has been short-circuited, and that stator I is switched on. The motors start, their torques at first increasing as they gain speed. But as the speed increases, the frequency of the currents in stator II

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* With a squirrel-cage rotor, no special device is required in the rotor when the number of stator poles is changed. But *wound* rotors must be of special construction—such as that illustrated in Fig. 139, in order to enable them to run when the number of poles is altered.

(which is the same as that of the currents in rotor I) steadily decreases, and if we suppose that the speed has nearly reached half the speed of synchronism (for motor I) then motor II will be running at nearly synchronous speed. Hence at this speed the torque exerted by it will be very small, the current being also small and nearly wattless. The current in stator II will, therefore, also be small and nearly wattless, and so will the current in rotor I. The torque exerted by motor I will thus also be small. We therefore find that as half the speed of synchronism is approached the combined torque decreases, so that two motors mounted on the same shaft and coupled in cascade will run at approximately *half the speed of synchronism.*[*] If both motors are connected in parallel across the mains, they will run at nearly synchronous speed, with the result that the speed will be doubled. Thus by changing from the parallel to the cascade grouping we are able to halve the speed.

It may be pointed out that in the cascade arrangement motor I performs two functions; it acts as a motor, and also as a *frequency transformer*, practically reducing the frequency of supply to half its value. The rotor winding of motor I must, of course, be wound for the voltage of the supply mains, if the change from the tandem to the parallel grouping is to be feasible.

Various modifications of this arrangement may be used. Thus, motor II may have a different number of poles from motor I, so as to give a speed greater or less than half the speed of synchronism; and a third speed will be obtainable by fitting motor II with a pole-changing switch.[†]

Closely allied to the cascade method is what we may describe as the multiple motor method.[‡] This is used by Messrs. C. Wüst & Co., of Switzerland, and consists in mounting several rotors (in practice, from 2 to 4) on the same shaft, each rotor being surrounded by a corresponding stator. By choosing suitable numbers of poles for the different stators, as many different speeds as there are motors may be obtained by using each motor singly. Other speeds are obtainable by using two or more motors simultaneously, and the speeds will be different according as the torques developed by the motors all act in the same direction or oppose each other.

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[*] So far as motor II is concerned, this speed is, of course, *its* speed of synchronism.

[†] A large motor of this type, designed by Mr. E. Danielson, has recently been installed at the iron-works at Sandviken, Sweden (*Elektrotechnische Zeitschrift*, vol. xxv. p. 43 (1904)).

[‡] *Elektrotechnische Zeitschrift*, vol. xxiv. p. 694 (1903).

§ 136. Single-phase Induction Motors. Theory of Motor at Rest

Induction motors are also used on single-phase circuits. The stator winding in such motors consists of a single-phase winding, while the rotor has a winding of precisely the same type as in polyphase motors. A two-phase induction motor may be run as a single-phase motor by using only one of the phases in the stator winding.

We shall suppose the single-phase stator winding to be so arranged as to give rise to a sine wave of magnetic induction in the air-gap. The stationary or simple alternating sine wave of induction so produced may be written in the form (§ 18)—

$$b = \mathrm{B} \sin pt \,.\, \sin \frac{\pi}{\tau} x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where b is the magnetic induction at time t at a point distant x from the origin, the latter being taken at a point where the induction is always zero; τ stands for the pole-pitch or half the wave-length (*cf.* § 18).

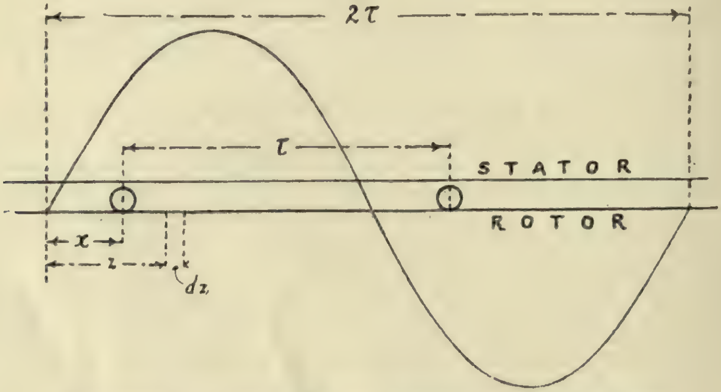

Fig. 144.—To illustrate Theory of Single-phase Induction Motor.

In order to simplify the treatment as much as possible, we shall suppose the rotor winding to consist of isolated coils, each short-circuited on itself, the width of a coil being equal to the pole-pitch τ. One such coil is shown in Fig. 144, the left-hand side of the coil being at a distance x from the origin. The sine curve shows the distribution of the induction at time t, so that the maximum value of the induction at this instant is B sin pt. At a distance z from the origin, the induction has the value $\mathrm{B}\sin pt\,.\sin\frac{\pi}{\tau}z$, so that if l = length of coil (in cms.) measured parallel to the shaft, the flux through a narrow strip of width dz is $\mathrm{B}l\sin pt\,.\sin\frac{\pi}{\tau}z\,.\,dz$, and hence the total flux through the coil in the position shown is—

$$f = \int_{x}^{x+\tau} \mathrm{B}l\sin pt\,.\sin\frac{\pi}{\tau}z\,.\,dz$$

$$= \mathrm{B}l\sin pt\int_{x}^{x+\tau}\sin\frac{\pi}{\tau}z\,.\,dz$$

$$= \frac{2\tau l}{\pi}\mathrm{B}\sin pt\,.\cos\frac{\pi}{\tau}x$$

Assuming the rotor to be at rest, and hence x constant, we may write for the e.m.f. induced in the coil shown in Fig. 144—

$$e = -\frac{df}{dt}^{*} = -\frac{2p\tau l}{\pi}\mathrm{B}\cos\frac{\pi}{\tau}x\,.\cos pt = -\mathrm{E}\cos\frac{\pi}{\tau}x\,.\cos pt$$

where $\mathrm{E} = \frac{2p\tau l}{\pi}\mathrm{B}$. Hence if r, λ stand for the resistance and *leakage* self-inductance respectively of the coil, the current is—

$$i = -\frac{\mathrm{E}}{\sqrt{r^2 + p^2\lambda^2}}\cos\frac{\pi}{\tau}x\,.\cos(pt - \theta)\,\dagger$$

where $\tan\theta = \frac{p\lambda}{r}$.

This shows (a result which is also otherwise obvious from an inspection of Fig. 144) that, so long as the rotor is at rest (*i.e.* so long as x is not a function of the time), there are different currents circulating in different portions of the rotor winding; the greatest current circulating in the coil which faces a pole-piece, and for which $x = 0$, and $\cos\frac{\pi}{\tau}x = 1$, and zero current in the coil whose sides are in line with the centre lines of the pole-pieces ($x = \frac{1}{2}\tau$, $\cos\frac{\pi}{\tau}x = 0$).

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[*] The negative sign being taken in accordance with Lenz's Law (§ 4). It is to be particularly noted that f here stands for the *resultant actually existing* oscillating flux *common* to the stator and rotor, and due to the combined action of their windings; it is *not* the hypothetical flux due to the stator winding *alone*.

[†] The current being expressed, for the sake of simplicity, in C.G.S. units.

Thus, when the rotor is at rest, and the stator supplied with current, excessive heating will take place in some of the rotor coils, while others will remain quite cool. This behaviour of a single-phase motor at rest is in striking contrast to that of a polyphase motor, in whose rotor coils uniform heating would take place.

It will be noticed that the coil in which the induced current has a maximum value has its sides placed in a field of zero intensity, and is thus unable to exert any torque, while the coil whose sides are in the region of maximum field intensity conveys no current, and is thus also unable to develop any torque. Any coil in an intermediate position, such as the coil shown in Fig. 144, will, however, be subjected to a definite tangential pull, which will obviously be equal to twice the tangential force on one side of it;* this latter force is given by the product of length of conductor, current, and field intensity. We thus get for the total tangential pull T on the coil in the position shown in Fig. 144—

$$T = 2lib$$

or, using the values for b and i obtained above—

$$T = -T_0 . \sin\frac{2\pi}{\tau}x . 2\sin pt . \cos(pt - \theta)$$

where—

$$T_0 = \frac{lEB}{2\sqrt{r^2 + p^2\lambda^2}}$$

Since $2 \sin pt . \cos(pt - \theta) = \sin(2pt - \theta) + \sin\theta$, the above may be written—

$$T = -T_0 \sin\frac{2\pi}{\tau}x\{\sin(2pt - \theta) + \sin\theta\}$$

Now, since the mean value of $\sin(2pt - \theta)$ over any complete number of periods vanishes, we see that the mean value of the tangential pull on the coil is given by—

$$T_m = -T_0 \sin\theta . \sin\frac{2\pi}{\tau}x$$

In order to find out the direction of the pull, let us imagine a *positive* current sent round the coil in Fig. 144, *i.e.* a current tending to produce a magnetic field in the same (upward) direction as the given field. Since both field and current are positive, their product will be positive; but an application of Dr. Fleming's rule easily shows us that in this case the coil will tend to move to the *left.* Thus a negative value of T_m corresponds to a force acting from left to right, or in the positive direction.

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* We are assuming the coils to be placed on the surface of the rotor core, so that the pull is exerted on the coils; in reality, the coils would be embedded in the core, and the pull would come mainly on the *core*. Since, however, the total pull would remain unaltered, it is immaterial, so far as calculation goes, whether we assume the coils to be placed on the surface or in slots.

From this it will be at once seen that any coil for which $x < \frac{1}{2}\tau$ experiences a pull in the *positive* direction (from left to right), while any coil for which $\tau > x > \frac{1}{2}\tau$ experiences a pull in the *negative* direction (from right to left). And since corresponding to every coil at a distance x from the origin there is another at a distance $x + \frac{1}{2}\tau$, it follows that the tangential pulls acting on the various coils will balance each other so that *the resultant torque on the rotor of a single-phase motor which is at rest vanishes.* A single-phase induction motor is, therefore, not self-starting.

§ 137. Torque Exerted by Single-phase Induction Motor when Running

Let us next investigate the relations which obtain when the motor is running. Considering any one coil, we must now regard x as variable, and we may write $x = vt$, v being the peripheral velocity of the rotor. The positive direction of x in Fig. 144 being from left to right, this would correspond to a displacement of the coil from left to right. The magnetic flux f at any time t is now given by (§ 136)—

$$\begin{aligned} f &= \frac{2\tau l}{\pi}\mathrm{B}\sin pt \cos\frac{\pi v}{\tau}t \\ &= \frac{\tau l\mathrm{B}}{\pi}\{\sin(p+\omega)t + \sin(p-\omega)t\} \end{aligned}$$

ω standing for $\frac{\pi v}{\tau}$. The e.m.f. induced by this flux is—

$$e = -\frac{df}{dt} = -\frac{\tau l\mathrm{B}}{\pi}\{(p+\omega)\cos(p+\omega)t + (p-\omega)\cos(p-\omega)t\}$$

while the current is given by—

$$\begin{aligned} i = -\frac{\tau l\mathrm{B}}{\pi}\Big\{&\frac{p+\omega}{\sqrt{r^2+(p+\omega)^2\lambda^2}}\cos[(p+\omega)t-\theta_1] \\ &+ \frac{p-\omega}{\sqrt{r^2+(p-\omega)^2\lambda^2}}\cdot\cos[(p-\omega)t-\theta_2]\Big\} \end{aligned}$$

θ_1 and θ_2 being given by the equations—

$$\tan\theta_1 = \frac{(p+\omega)\lambda}{r};\ \tan\theta_2 = \frac{(p-\omega)\lambda}{r}. \quad . \quad . \quad . \quad (2)$$

Since $\sin\theta_1 = \frac{(p+\omega)\lambda}{\sqrt{r^2+(p+\omega)^2\lambda^2}}$, and $\sin\theta_2 = \frac{(p-\omega)\lambda}{\sqrt{r^2+(p-\omega)^2\lambda^2}}$, the expression for the current may be written in the form—

$$i = -\frac{\tau l\mathrm{B}}{\pi\lambda}\{\sin\theta_1 . \cos[(p+\omega)t-\theta_1] + \sin\theta_2 \cos[(p-\omega)t-\theta_2]\}$$

In order to find the value of the total tangential pull T on the coil, we have, as before, to form the product $\mathrm{T} = 2lib$. Now, b may be written in the form —equation (1) of § 136—

$$b = \tfrac{1}{2}\mathrm{B}\{\cos(p-\omega)t - \cos(p+\omega)t\}$$

In determining the mean value T_m of T, we need only consider the products of terms of the same frequency, since the mean value of the product of two terms of different frequency vanishes.* Now, since—

$$\cos[(p+\omega)t-\theta_1]\cos(p+\omega)t = \tfrac{1}{2}\{\cos[2(p+\omega)t-\theta_1] + \cos\theta_1\}$$

the mean value of which is $\cos\theta_1$, and since similarly the mean value of $\cos[(p-\omega)t-\theta_2]\cos(p-\omega)t$ is $\cos\theta_2$, it follows that—

$$\mathrm{T}_m = -\frac{\tau l^2\mathrm{B}^2}{\pi\lambda}(-\sin\theta_1 . \cos\theta_1 + \sin\theta_2 . \cos\theta_2)$$
$$= -\frac{\tau l^2\mathrm{B}^2}{4\pi\lambda}(\sin 2\theta_2 - \sin 2\theta_1)$$

§ 138. Effect of Varying Resistance of Rotor Circuits

Remembering (§ 136) that when T_m is negative the pull is from left to right, *i.e.* in the direction of motion, we see that the motor will exert a *driving* torque if $\sin 2\theta_2 > \sin 2\theta_1$, and an opposing torque if $\sin 2\theta_2 < \sin 2\theta_1$. The angles θ_1 and θ_2 are given by the equations (2). At standstill, $\theta_1 = \theta_2$, and the torque vanishes—which confirms our previous result. If, as is always the case, $\frac{p\lambda}{r}$ is a large quantity, then for a small value of ω, *i.e.* at a low speed, both θ_1 and θ_2 will be very large angles, and $2\theta_1$, $2\theta_2$ will both exceed 90°, θ_1 being the greater of the two, so that $\sin 2\theta_2 > \sin 2\theta_1$, as is evident from an inspection of Fig. 145; the motor will, therefore, exert a driving torque. By plotting $\sin 2\theta_2 - \sin 2\theta_1$ as a function of the speed, we obtain a curve which shows the variation of the torque with speed on the supposition that B^2 is constant.* The exact shape of this curve will depend on

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* Thus, $\cos[(p+\omega)t-\theta_1]\cos(p-\omega)t = \frac{1}{2}[\cos(2pt-\theta_1) + \cos(2\omega t-\theta_1)]$, the mean value of which is zero.

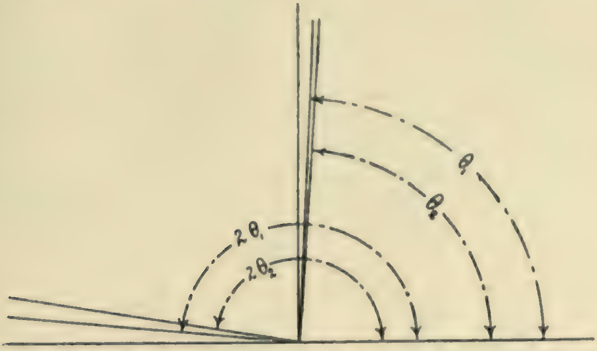

FIG. 145.—To illustrate Theory of Single-phase Induction Motor.

the value of $\dfrac{p\lambda}{r} = \dfrac{\text{leakage reactance at synchronism}}{\text{resistance}}$. In Fig. 146 are plotted the curves connecting $\sin 2\theta_2 - \sin 2\theta_1$ with the speed, expressed as a fraction of the synchronous speed, for four different values of $\dfrac{p\lambda}{r}$. The curves marked A, B, C, and D correspond to values of $\dfrac{p\lambda}{r}$ equal to 20, 5, 2, and 1 respectively. Curves A and B may be regarded as fairly typical of ordinary single-phase induction motors. Curves C and D correspond to effects which might be produced by introducing additional resistances into the rotor windings. It will be seen that the introduction of resistance has an effect markedly different from that obtained with a polyphase motor. Not only is the maximum value of the torque rapidly reduced, but when $\dfrac{p\lambda}{r}$ is made equal to unity, the motor becomes quite incapable of exerting a driving torque at any speed whatsoever (curve D). Hence the introduction of resistances into the rotor for purposes of speed regulation is attended with the serious disadvantage of reducing the overload capacity of the motor, and cannot be used as freely and advantageously as with polyphase motors.†

It may be shown that for a single-phase induction motor—as for a polyphase one—there exists a circle which represents the locus of the extremity of the primary current vector. But since the simple and elegant graphical representation of slip, torque, etc., is in this case no longer possible as with the polyphase motor, the circle diagram for a single-phase induction motor possesses but little practical interest.

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* This condition is, of course, not strictly fulfilled, on account of the existence of the primary or stator leakage flux; as the latter increases, B decreases.

† Resistances are introduced at starting to limit the starting current.

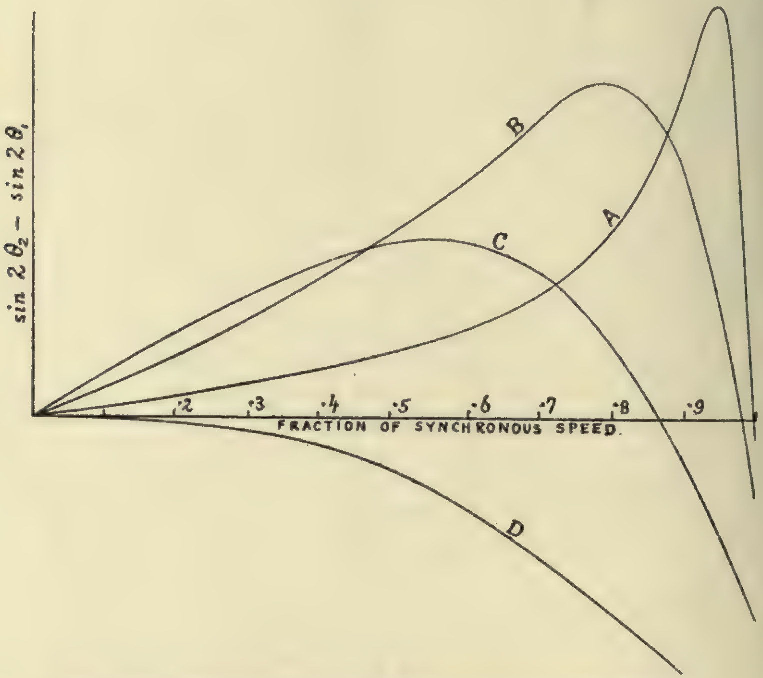

FIG. 146.—Torque-speed Curves of Single-phase Induction Motor.

§ 139. Starting of Single-phase Induction Motors

The single-phase induction motor not being self-starting, special starting devices have to be provided for it. The arrangement most commonly in use is one consisting of an auxiliary starting winding, which occupies the same position relatively to the running winding as the two windings of a two-phase induction motor do relatively to each other. The motor will start if there is a sufficient phase difference between the currents in the two windings. The necessary phase difference may be obtained (1) by connecting the starting winding in series with an external reactance coil or condenser (both methods having been used in practice); (2) by arranging the starting winding so that the magnetic leakage corresponding to it is much greater than that corresponding to the running winding—a method very successfully employed by Heyland, and not involving the use of any reactance coils or condensers external to the motor; (3) by using a rotor provided with a winding resembling that of an ordinary continuous-current machine, and having a commutator and brushes, by means of which the motor may be started as a "repulsion" motor; commutator motors of this type are considered in Chapter XVII.

CHAPTER XV

§ **140**. Rotary converters and their uses—§ **141**. Voltage ratio in converters—§ **142**. Ratio of currents—§ **143**. Relative outputs of armature when generating continuous, single-, two-, and three-phase currents—§ **144**. Heating of single-phase converter—§ **145**. Heating of converter having N slip-rings—§ **146**. Effect of number of slip-rings on output of converter—§ **147**. Six-phase converter supplied from three-phase mains—§ **148**. Twelve-phase converter—§ **149**. Hunting of rotary converters—§ **150**. Voltage regulation of rotary converters—§ **151**. Starting of rotary converters—§ **152**. Racing of inverted rotaries—§ **153**. Converters *v.* motor-generators.

§ 140. Rotary Converters and their Uses.

A ROTARY converter is a machine by means of which may be effected the transformation of alternating (single- or polyphase) currents into continuous current, or the transformation of continuous into alternating current. In most cases, the machine is employed in connection with the first-mentioned purpose, and its chief practical importance is due to the fact that while the transmission of power over long distances is most readily and economically accomplished by means of alternating currents, its actual distribution to the motors of an electric railway or tramway system has hitherto been effected chiefly by means of continuous current. The rotary converter supplies one of the necessary links between the high-voltage alternating-current transmission system, and the low-voltage (500 to 600 volts) continuous-current distribution system of an electric railway or tramway. Were it not for its importance in electric railway work, the rotary converter would probably have received much less attention than has actually been bestowed on it.

In some few cases, converters have been used for effecting the opposite kind of transformation, viz. from continuous to alternating current. When so used, they have sometimes been termed *inverted rotaries.* Cases of this kind may occasionally arise in connection with a large continuous-current generating station, when it is desired to transmit a limited amount of power to a greater distance than could economically be accomplished by the use of continuous current. Continuous current is in such cases supplied to an inverted rotary at the generating station, the polyphase currents from the rotary being raised to a high voltage by means of a step-up transformer, and at the far end transformed down and fed into another rotary converter.

The general principles underlying the construction of a rotary converter have already been briefly considered in § 39. We shall now study it somewhat more in detail, and shall, in the first place, investigate the voltage relations on the alternating- and continuous-current sides.

§ 141. Voltage Ratio in Converters

If we assume the space distribution of the magnetic flux to follow the sine law, then the e.m.f. induced in each coil of the armature winding will be a sine function of the time. The sine wave e.m.f.'s induced in the consecutive coils will differ in phase by a constant amount, represented by the phase angle $\frac{\pi}{c}$, c being the number of coils per pole-pitch. These e.m.f.'s may, in a vector diagram, be represented, as in Fig. 147, by a series of radial vectors of equal length and spaced at equal angular intervals apart, each angular interval amounting to $\frac{\pi}{c}$. In order to find the resultant alternating e.m.f. between any two points of the winding, we have to add vectorially the e.m.f.'s of all the coils included between those two points, using the construction known as the polygon of vectors. Thus, taking two points which correspond to a span equal to the pole-pitch, and thus embrace all the coils whose e.m.f. vectors in Fig. 147 are included between A and B, we get the open polygon OQP of Fig. 148, the closing side OP of which is the resultant e.m.f. of the group of coils considered. If each of the vectors be taken to represent the maximum value of the alternating e.m.f., then the vector OP in Fig. 148 will represent the maximum value of the e.m.f. in a group of coils included within a pole-pitch; but this is evidently equal to the continuous e.m.f. between two brushes. For any smaller group of coils, such as that represented by the vectors included between A and C in Fig. 147, we get for the maximum value of the e.m.f. a vector OQ (Fig. 148).

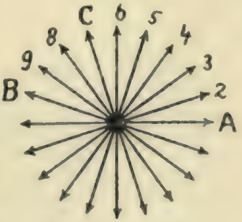

FIG. 147.—Vector Diagram of e.m.f.'s in Consecutive Coils.

Now if—as is generally the case—the number of coils between two brushes is considerable, the regular polygon (OQP in Fig. 148), which results from the vectorial addition of the e.m.f.'s of consecutive coils, becomes indistinguishable from a circular arc. Hence, in order to find the voltage relations on the continuous- and alternating-current sides, we may adopt the following construction. On a line OP (Fig. 149) as diameter—the length of OP representing, to a suitable scale, the voltage between two consecutive brushes—describe the semicircle OSQP. If the distance between the points of connection of two given slip-rings is $\frac{1}{d}$th of the pole-pitch, then, in order to find the maximum value of the alternating voltage between the slip-rings, lay off from the centre R of the semicircle a line RQ such that the angle ORQ $= \frac{\pi}{d}$ in circular measure, or $\frac{180}{d}$ in degree measure. Join OQ; then OQ gives the required maximum value of the alternating voltage. The r.m.s. value of this voltage is given by $\frac{OQ}{\sqrt{2}}$, but

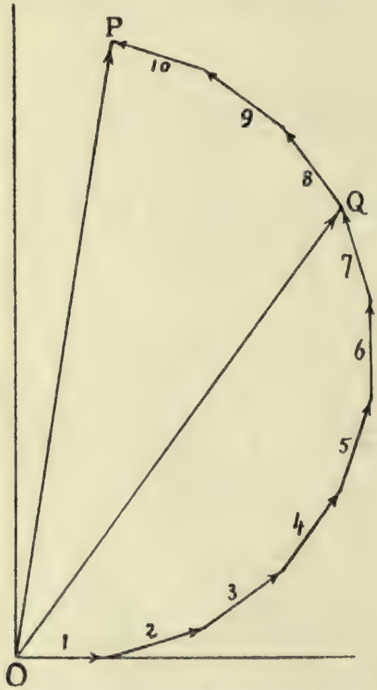

Fig. 148.—Polygon Construction for Resultant e.m.f.

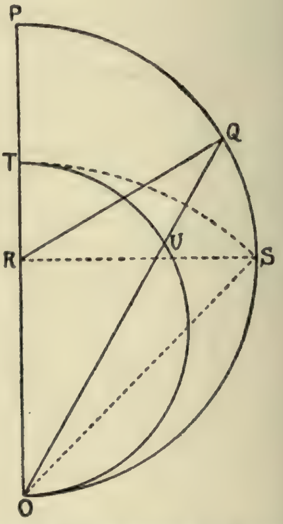

Fig. 149.—Construction for Voltage Ratio in Rotary Converter.

may be immediately obtained by an additional very simple graphical construction, which is as follows. At R erect the perpendicular RS, and join OS. With O as centre and radius OS describe the arc ST, intersecting OP at T, and on OT as diameter describe the semicircle OUT. Then, since $\mathrm{OT} = \mathrm{OS} = \sqrt{2}\ \mathrm{OR}$, we have $\frac{\mathrm{OT}}{\mathrm{OP}} = \frac{\sqrt{2}\ \mathrm{OR}}{2\mathrm{OR}} = \frac{1}{\sqrt{2}}$, or $\mathrm{OT} = \frac{\mathrm{OP}}{\sqrt{2}}$. But the semicircles being similar and similarly situated with respect to O, it follows that the ratio of any two secants drawn from O and making the same angle with OP will also be $\frac{1}{\sqrt{2}}$. Thus $\mathrm{OU} = \frac{\mathrm{OQ}}{\sqrt{2}}$, *i.e.* OU gives the r.m.s. value of the alternating voltage between the slip-rings.*

A simple formula, immediately evident by a reference to the diagram, may also be used for calculating the voltage ratio. For we have—

$$\text{r.m.s. value of voltage} = \mathrm{OU} = \frac{\mathrm{OQ}}{\sqrt{2}} = \frac{\mathrm{OP}\cos\mathrm{POQ}}{\sqrt{2}} = \frac{\mathrm{OP}}{\sqrt{2}}\sin\tfrac{1}{2}\mathrm{ORQ}$$

and since ORQ in degree measure is equal to $\frac{180}{d}$,

$$\text{r.m.s. value of slip-ring voltage} = \frac{\text{continuous voltage}}{\sqrt{2}} . \sin\frac{90^\circ}{d} \quad . \quad (1)$$

the value of d being given by—

$$\text{distance between points of connection of slip-rings} = \frac{\text{pole-pitch}}{d}$$

Using this formula, or the graphical construction just explained, we have, calling the continuous voltage 100, the following values of the alternating (r.m.s.) voltage for a single-, two-, and three-phase converter $\left(\frac{1}{d} = 1, 1 \text{ and } \frac{2}{3} \text{ respectively}\right)$—

Continuous voltage.	Single-phase.	Two-phase.	Three-phase.
100	70·7	70·7	61·2

These values, it must be remembered, are only *approximate*, not only because we have taken no account of the resistance drop in the converter, but because the assumed sine law of the magnetic flux distribution around the armature periphery is more or less departed from in practice.

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* This elegant construction is due to Mr. O. J. Ferguson (*Electrical World and Engineer*, vol. xliv. p. 733 (1904)).

§ 142. Ratio of Currents

We shall next consider the ratio of the continuous current to the line currents on the alternating-current side. An approximate value for this ratio may be easily obtained by assuming that the power factor is unity, and that the losses in the converter are negligible; the latter supposition requiring equality of power on the two sides of the converter.

Let V, I stand for the voltage and current respectively on the continuous-current side; and V_1, I_1; V_2, I_2; V_3, I_3 for the voltages and currents on the alternating-current side of a single-, two-, and three-phase converter respectively. The power factor being unity, and the losses being neglected, we must have—

$$VI = V_1I_1 = 2V_2I_2 = \sqrt{3} \cdot V_3I_3.^*$$

Now since, as we have just seen, $V_1 = 0{\cdot}707V$, $V_2 = 0{\cdot}707V$, and $V_3 = 0{\cdot}612V$, we get—

$$I_1 = 1{\cdot}414I;\ I_2 = 0{\cdot}707I;\ I_3 = 0{\cdot}943I$$

Exhibited in tabular form, these results are—

Continuous current.	Alternating Line Currents.		
	Single-phase.	Two-phase.	Three-phase.
100	141·4	70·7	94·3

§ 143. Relative Outputs of Armature when generating Continuous, Single-, Two-, and Three-phase Currents

One of the most important problems in connection with converters is the question of *heating*. Before proceeding with this problem, however, we shall investigate the relative output of a continuous-current armature when used to generate continuous, single-, two-, and three-phase currents, the limit of output being determined by the greatest permissible rise of temperature. As a basis of our comparison we therefore adopt the same temperature rise in each case, assuming also that the speed and the effective flux per pole are maintained constant.

It is evident that to obtain the same temperature rise the current traversing the armature winding must be the same in each case. Let I, I', I'', and I''' be the line currents corresponding to maximum safe temperature rise of the armature when generating continuous, single-, two-, and three-phase currents respectively. If V, V_1, V_2, and V_3 denote the corresponding line voltages, then the relative outputs are—

---

* The three-phase power $= 3V_3I_3'$, where I_3' is the phase current; now $I_3' = \frac{1}{\sqrt{3}} I_3$ (§ 16), so that the power $= \sqrt{3} . V_3I_3$.

Continuous current.	Single-phase.	Two-phase.	Three-phase.
VI	$V_1I'=0{\cdot}707VI$	$2V_2I''=1{\cdot}414VI$	$V_3I'''\sqrt{3}=0{\cdot}612VI'''\sqrt{3}$

For the sake of simplicity, we may assume the machine to be a two-pole one. This will not in any way affect the generality of our conclusions, for an ordinary lap-wound multipolar armature is equivalent to a number of simple two-pole armatures connected in parallel, and any result deduced for a two-pole machine will apply to a section of the multipolar winding under cover of two neighbouring pole-pieces, and hence will also apply to the entire multipolar armature.

Assuming, then, a two-pole armature, we have, for the currents in the armature windings, in the four cases under consideration—

$$\tfrac{1}{2}I \qquad \tfrac{1}{2}I' \qquad \frac{\sqrt{2}}{2}I'' \qquad \frac{1}{\sqrt{3}}I'''$$

For equal heating, these currents must be equal. Hence—

$$I' = I;\ I'' = \frac{1}{\sqrt{2}}I;\ I''' = \frac{\sqrt{3}}{2}I$$

Substituting these values in the expressions for the output in the various cases, we obtain—

Continuous current.	Single-phase.	Two-phase.	Three-phase.
VI	$0{\cdot}707VI$	VI	$0{\cdot}918VI$

§ 144. Heating of Single-phase Converter

The heating of a converter armature is a very complicated problem, due mainly to the fact that the rate of heat generation is not uniform throughout the armature winding (as it was in the cases just considered), but varies according to the position of a coil relatively to the point of connection to a slip-ring, the greatest rate of heat production occurring in the coils close to the slip-ring connections. For the sake of simplicity, we shall, as before, assume the armature to be a two-pole one, the power factor to be unity, and shall determine the relative outputs, for the same mean temperature rise, when the armature is used as a converter from single-, two-, and three-phase currents respectively to continuous current.

The points of connection to the slip-rings being fixed relatively to the armature winding, while the brushes are constantly changing their position with respect to it, it is evident that the distribution of currents in the armature at any instant will depend (among other things) on the position of the brushes relatively to the slip-ring attachments. Since the effect is one depending on relative position, we shall find it convenient to suppose that the armature is stationary, while the brushes are carried round by the revolving field.

In Fig. 150, the circle is intended to represent the closed winding of a stationary two-pole armature, R and S being the fixed points in the winding to which the two slip-rings of the single-phase converter are attached, while the brushes B_1, B_2 are carried round the winding with an angular velocity p, such that $p = 2\pi \times$ frequency of the single-phase currents. Let the instantaneous value of the single-phase current be given by—

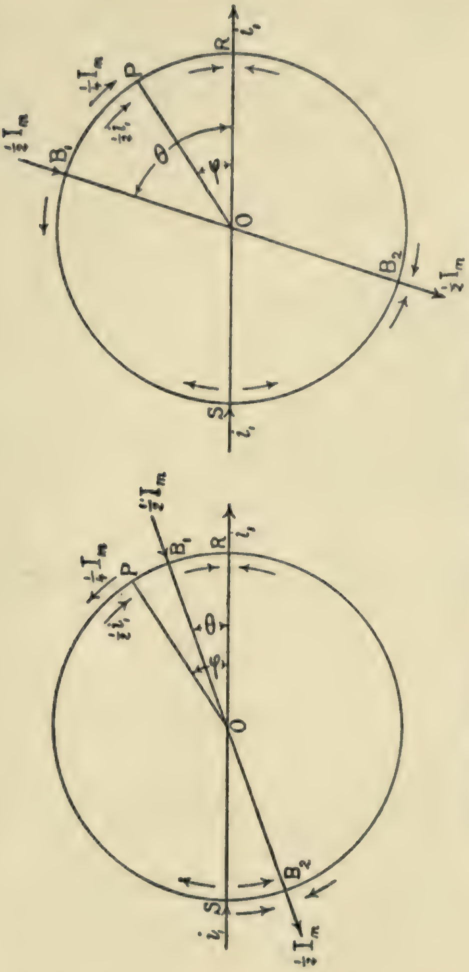
$$i_1 = I_m \cos pt = I_m \cos \theta$$

where we write, for the sake of brevity, θ for pt. The current reaches its maximum positive value at $t = 0$, *i.e.* $\theta = 0$. In the figure, we assume the positive direction of the current to be from left to right, as shown by the arrows. Now, since by supposition the power factor is unity, the maximum value of the current must occur when the p.d. reaches its maximum value, *i.e.* when B_1B_2 is horizontal. The value of the continuous current of the converter is, by § 3, $\frac{1}{\sqrt{2}} \times$ r.m.s. value of alternating current $= \frac{1}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = \frac{1}{2}I_m$, so that the continuous current is half the maximum alternating current. When $t = 0$, therefore, the brushes are horizontal, the alternating current has the value I_m, and of this $\frac{1}{2}I_m$ passes into the external circuit, while the remaining $\frac{1}{2}I_m$ flows into the armature, driving it as a motor.

Consider any point P in the winding at an angular distance $POR = \phi$ from R. It is evident from considerations of symmetry that the cycle of changes in the numerical value of the current in the winding at P during the first half-revolution of B_1B_2 will be repeated during the next half-revolution. So far as the heating at P is concerned, therefore, it will be sufficient to study the changes in the current at P during one half-revolution of B_1B_2, *i.e.* while B_1 moves, in a counter-clockwise direction, from R to S,

Now, it is evident that as the brush B_1 sweeps past the point

P, the continuous-current component of the current in the coil at P undergoes a sudden reversal. Hence we divide the entire cycle of current changes at P during half a period into two stages: (1) while



Figs. 150 and 151.—To illustrate Heating of Single-phase Converter.

$\theta < \phi$, and the continuous current is flowing through P in a counter-clockwise direction, as shown in Fig. 150; and (2) while $\theta > \phi$, and the continuous current is flowing past P in a clockwise direction, as shown in Fig. 151.

During the first stage (Fig. 150) the total instantaneous current at P is given by—

$$i = \tfrac{1}{2}i_1 - \tfrac{1}{4}I_m = \tfrac{1}{2}I_m(\cos\theta - \tfrac{1}{2})$$

while during the second stage (Fig. 151) it is given by—

$$i' = \tfrac{1}{2}i_1 + \tfrac{1}{4}I_m = \tfrac{1}{2}I_m(\cos\theta + \tfrac{1}{2})$$

The *squares* of the currents during the two stages are given by—

$$i^2 = \tfrac{1}{4}I_m^2(\cos^2\theta + \tfrac{1}{4} - \cos\theta) = \tfrac{1}{4}I_m^2(\tfrac{3}{4} + \tfrac{1}{2}\cos 2\theta - \cos\theta)\,*$$

and—

$$i'^2 = \tfrac{1}{4}I_m^2(\tfrac{3}{4} + \tfrac{1}{2}\cos 2\theta + \cos\theta)$$

respectively.

In order to find the mean value of the square of the current—which determines the heating effect at P—we may suppose the square of the total current at P plotted as a function of θ. By determining the area of the curve, and dividing this by the base (π), we obtain the mean value of the ordinate, *i.e.* the mean square of the current at P. The determination of the area is most easily accomplished by a simple integration. Taking first the portion of the curve corresponding to the first stage, during which $\phi > \theta > 0$, we have, $i^2d\theta$ representing a narrow strip of the area—

$$\text{area corresponding to first stage} = \int_0^\phi i^2 d\theta = \tfrac{1}{4}I_m^2\int_0^\phi(\tfrac{3}{4} + \tfrac{1}{2}\cos 2\theta - \cos\theta)d\theta$$

$$= \tfrac{1}{4}I_m^2\Big[\tfrac{3}{4}\theta + \tfrac{1}{4}\sin 2\theta - \sin\theta\Big]_0^\phi$$

$$= \tfrac{1}{4}I_m^2(\tfrac{3}{4}\phi + \tfrac{1}{4}\sin 2\phi - \sin\phi)$$

Similarly, we find—

$$\text{area corresponding to second stage} = \int_\phi^\pi i'^2 d\theta = \tfrac{1}{4}I_m^2\Big[\tfrac{3}{4}\theta + \tfrac{1}{4}\sin 2\theta + \sin\theta\Big]_\phi^\pi$$

$$= \tfrac{1}{4}I_m^2(\tfrac{3}{4}\pi - \tfrac{3}{4}\phi - \tfrac{1}{4}\sin 2\phi - \sin\phi)$$

The total area is thus—

$$\tfrac{1}{4}I_m^2(\tfrac{3}{4}\pi - 2\sin\phi)$$

and the mean ordinate, *i.e.* the mean square of the current at P, is—

$$\tfrac{1}{4}I_m^2\left(\frac{3}{4} - \frac{2}{\pi}\sin\phi\right)$$

* Since $\cos^2 \quad = \tfrac{1}{2}(1 + \cos 2\theta)$

This result shows that the heating effect at P depends on ϕ, *i.e.* on the position of P relatively to the slip-ring attachments. The heating is *least* when $\phi = \frac{\pi}{2}$, or when P is halfway between the slip-rings, and it is *greatest* when $\phi = 0$, *i.e.* when P is in direct connection with a slip-ring. In the first case, the mean square of the current at P is $\frac{1}{4}I_m^2\left(\frac{3}{4} - \frac{2}{\pi}\right) = \frac{1}{4}I_m^2 \times 0{\cdot}113$, and in the second, $\frac{1}{4}I_m^2 \times 0{\cdot}75$, the ratio of the latter to the former being $\frac{0{\cdot}75}{0{\cdot}113} = 6{\cdot}6$. Thus, in a single-phase converter, heat is generated in a coil in direct connection with a slip-ring at 6·6 times the rate at which it is generated in a coil halfway between the slip-rings. There is thus great *local* heating of certain coils.

In Fig. 153 is plotted a curve, marked $N = 2$, whose abscissæ are different values of ϕ, and whose ordinates represent the mean square of the corresponding current—$\frac{1}{4}I_m^2(\frac{3}{4} - \frac{2}{\pi}\sin\phi)$. In order to find the average rate of heating of the coils, we must determine the area of this curve. This may be done either graphically or by integration. Adopting the latter method, we get for the area—

$$\frac{1}{4}I_m^2\int_0^\pi\left(\frac{3}{4} - \frac{2}{\pi}\sin\phi\right)d\phi = \frac{1}{4}I_m^2\left[\frac{3}{4}\phi + \frac{2}{\pi}\cos\phi\right]_0^\pi$$

$$= \frac{1}{4}I_m^2\left(\frac{3}{4}\pi - \frac{4}{\pi}\right)$$

and for its mean ordinate (the mean value of the mean square of the current in the various coils)—

$$\frac{1}{4}I_m^2\left(\frac{3}{4} - \frac{4}{\pi^2}\right) = \frac{1}{4}I_m^2 \times 0{\cdot}3448$$

So far, therefore, as the *average* rise of temperature of the armature coils is concerned, the effect is the same as if they were traversed by a continuous current, in each half of the winding, of amount $\frac{1}{2}I_m\sqrt{0{\cdot}3448} = 0{\cdot}2929I_m$, or by a *total* continuous current (which divides equally between the two parallel paths of the winding) of amount $0{\cdot}5858I_m$.

For the same average rise of temperature, therefore, as that which occurs in the single-phase converter, the armature could be loaded with a continuous current of amount $0{\cdot}5858I_m$, and if V = continuous voltage at the given speed, the output of the armature as a simple continuous-current generator would be $0{\cdot}5858\mathrm{VI}_m$. Its output as a single-phase converter is, however, only $\dfrac{\mathrm{V}}{\sqrt{2}}\cdot\dfrac{\mathrm{I}_m}{\sqrt{2}} = 0{\cdot}5\mathrm{VI}_m$. Thus the single-phase converter output of a continuous-current armature is, for a given average temperature rise, only $\dfrac{0{\cdot}5}{0{\cdot}5858}$, or 85·4 per cent. of its continuous-current output.

§ 145. Heating of Converter having N Slip-rings

A similar mode of investigation might be applied to a two-phase converter. We shall, however, find it convenient to develop a perfectly general method applicable to any number of slip-rings. Let there be N slip-rings, dividing the armature winding into N phases. The angle ROS (Fig. 152) subtended by each phase at O in the case of a two-pole armature is $\dfrac{2\pi}{\mathrm{N}}$. If V = continuous voltage between brushes, then, by applying the method explained in § 141 for determining the slip-ring voltage, we have, putting $d = \dfrac{\mathrm{N}}{2}$, and using formula (1) of § 141—

$$\text{maximum voltage of each phase} = \mathrm{V}\sin\frac{\pi}{\mathrm{N}}$$

Hence if $\mathrm{I_N}$ = maximum current in each phase, then, assuming a power factor of unity, the power supplied to each phase of the converter is $\frac{1}{2}\mathrm{VI_N}\sin\dfrac{\pi}{\mathrm{N}}$, and the total power = number of phases × power per phase $= \dfrac{\mathrm{N}}{2}\mathrm{VI_N}\sin\dfrac{\pi}{\mathrm{N}}$. Let I be the current on the continuous-current side, so that VI = output on continuous-current side. Neglecting losses in the converter, and equating the input to the output, we get $\dfrac{\mathrm{N}}{2}\mathrm{VI_N}\sin\dfrac{\pi}{\mathrm{N}} = \mathrm{VI}$, or—

$$\mathrm{I_N} = \frac{2\mathrm{I}}{\mathrm{N}\sin\dfrac{\pi}{\mathrm{N}}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

In Fig. 152, R and S are the points of connection of two neighbouring slip-rings, and $\mathrm{B_1B_2}$ is the line of brushes, making, at time t, an angle $pt = \theta$ with OR. If i_N = instantaneous alternating current supplied to the phase RS (through the slip-rings at R and S), we may write—

$$i_\mathrm{N} = \mathrm{I_N}\sin(\theta + \alpha)$$

where the angle α is as yet unknown. In order to determine α, we notice that the phase voltage and current reach their maxima values when T, the middle point of RS, is in line with the middle of the pole-piece, *i.e.* when the line of brushes is at right angles (as shown by dotted line) to OT. We must then have $i_N = I_N$, or $\sin(\theta + \alpha) = 1$, *i.e.* $\theta + \alpha = \frac{\pi}{2}$; but since, referring to the figure, we at this instant have $\theta = \frac{\pi}{N} + \frac{\pi}{2}$, it follows that $\alpha = -\frac{\pi}{N}$. We thus have—

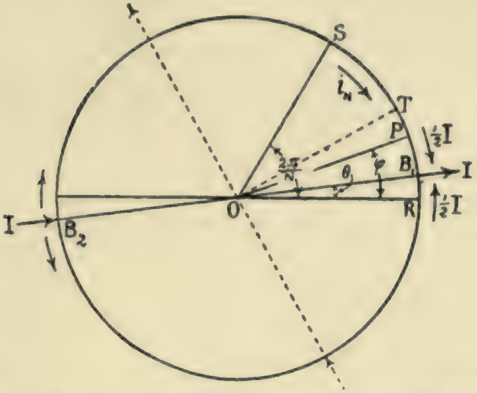


FIG. 152.—To illustrate Heating of Converter with N Slip-rings.

$$i_N = I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

Consider now any portion of the winding between R and S, such as that at P, and let ROP $= \phi$. Then, so long as $\theta < \phi$, the current at P is given by—

$$i_N + \tfrac{1}{2}I = \tfrac{1}{2}I + I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

while for $\theta > \phi$ the current at P is—

$$i_N - \tfrac{1}{2}I = -\tfrac{1}{2}I + I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

The square of the current is given by—

$$i^2 = \tfrac{1}{4}I^2 + I_N^2 \sin^2\left(\theta - \frac{\pi}{N}\right) \pm II_N \sin\left(\theta - \frac{\pi}{N}\right)$$

or—

$$i^2 = \tfrac{1}{4}I^2 + \tfrac{1}{2}I_N^2 - \tfrac{1}{2}I_N^2 \cos\left(2\theta - \frac{2\pi}{N}\right) \pm II_N \sin\left(\theta - \frac{\pi}{N}\right) \quad . \quad (3)$$

since $\sin^2\left(\theta - \frac{\pi}{N}\right) = \frac{1}{2}\left[1 - \cos 2\left(\theta - \frac{\pi}{N}\right)\right]$. The upper sign in the last term corresponds to $\theta < \phi$, and the lower sign to $\theta > \phi$.

To obtain the mean rate of heat production at P, we must find the mean value of i^2 during a half-period, that is, between $\theta = 0$ and $\theta = \pi$. Now, we notice that the first two terms in (3), $\frac{1}{4}I^2 + \frac{1}{2}I_N^2$, are constant; their mean value is thus $\frac{1}{4}I^2 + \frac{1}{2}I_N^2$. The third term contains the factor $\cos\left(2\theta - \frac{2\pi}{N}\right)$, which is of frequency double that of the supply current; thus half a period of the supply current —from $\theta = 0$ to $\theta = \pi$—will correspond to a whole period of the term $\cos\left(2\theta - \frac{2\pi}{N}\right)$, and since the mean value of the cosine over a whole period vanishes, this term drops out in the mean value of i^2. Lastly, we have the term $\pm II_N \sin\left(\theta - \frac{\pi}{N}\right)$, the *plus* sign to be taken so long as $\theta < \phi$, and the *minus* sign when $\theta > \phi$. If we plot this term as a function of θ, the area enclosed by its graph is—

$$II_N\left\{\int_0^{\phi} \sin\left(\theta - \frac{\pi}{N}\right)d\theta - \int_{\phi}^{\pi} \sin\left(\theta - \frac{\pi}{N}\right)d\theta\right\} = -2II_N \cos\left(\phi - \frac{\pi}{N}\right)$$

and the mean ordinate of the curve, obtained by dividing the area by the base, is—

$$-\frac{2}{\pi}II_N \cos\left(\phi - \frac{\pi}{N}\right)$$

We thus obtain for the mean value of the square of the current at P—

$$\tfrac{1}{4}I^2 + \tfrac{1}{2}I_N^2 - \frac{2}{\pi}II_N \cos\left(\phi - \frac{\pi}{N}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This expression shows that the heating is greatest at R and S, the points of slip-ring connection, and least at T, the point halfway between them.

If, using (2), we plot the expression (4) as a function of ϕ for $N = 2, 3, 4, 6$, and 12, the extreme values of ϕ being always 0 and $\frac{2\pi}{N}$, so as to correspond to the distance between two neighbouring slip-rings, we obtain the curves of Fig. 153. The value of I—the continuous current—has been assumed to be the same for all the curves. The vertical scale is the same for all, but the horizontal scale is different, having been so chosen that the length of the base line in each case represents $\frac{2\pi}{N}$, *i.e.* the base line in each case is proportional to the distance between two neighbouring slip-rings.

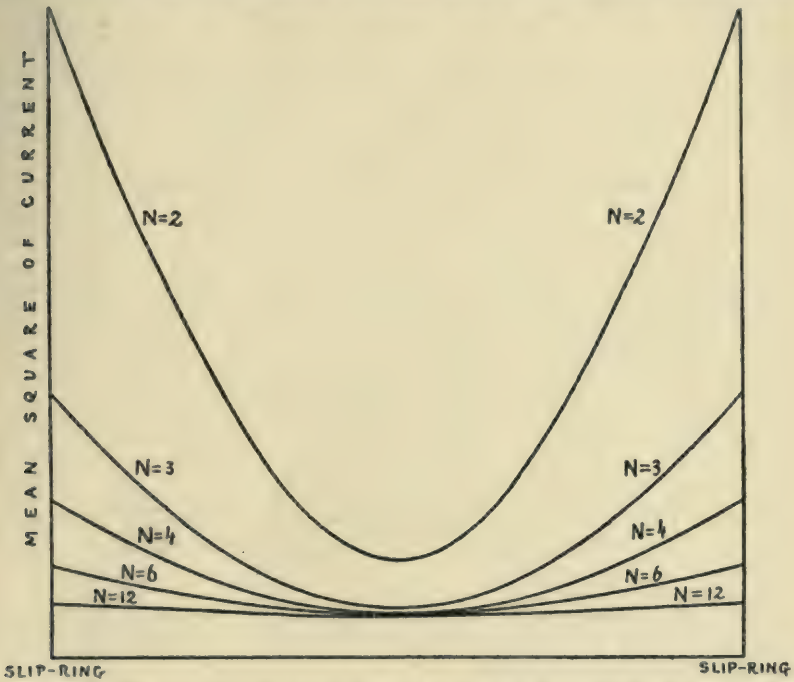


Fig. 153.—Showing Relative Rate of Heat Production in Converter with Varying Number of Slip-rings.

The rapid decrease in the total rate of heat production, and the increase in the uniformity of heat generation in the various coils, as the number of slip-rings is increased, are exhibited in a striking manner by these curves.

§ 146. Effect of Number of Slip-rings on Output of Converter

In order to find the mean rate of heat generation in the various coils lying between two slip-rings, we have to find the mean ordinate of the corresponding curve of Fig. 153. A simple integration enables us to obtain an expression for the mean value of the mean square of the current in the coils included between two slip-rings. Returning to expression (4), we notice that the first two terms are constant, while the mean value of the last term is given by—

$$-\frac{2}{\pi}\mathrm{II}_{\mathrm{N}} \times \text{mean value of } \cos\left(\phi - \frac{\pi}{\mathrm{N}}\right) \text{ between } \phi = 0 \text{ and } \phi = \frac{2\pi}{\mathrm{N}}$$

$$= -\frac{2}{\pi}\mathrm{II}_{\mathrm{N}} \times \frac{\mathrm{N}}{2\pi}\int_0^{\frac{2\pi}{\mathrm{N}}} \cos\left(\phi - \frac{\pi}{\mathrm{N}}\right)d\phi$$

$$= -\frac{2}{\pi^2}\mathrm{II}_{\mathrm{N}}\mathrm{N}\sin\frac{\pi}{\mathrm{N}}$$

Hence the mean value of the mean square of the current in a group of coils between two slip-rings is—

$$\tfrac{1}{4}\mathrm{I}^2 + \tfrac{1}{2}\mathrm{I}_{\mathrm{N}}^2 - \frac{2}{\pi^2}\mathrm{II}_{\mathrm{N}} \,.\, \mathrm{N}\sin\frac{\pi}{\mathrm{N}}$$

or, using (2)—

$$\left(\frac{1}{4} + \frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - \frac{4}{\pi^2}\right)\mathrm{I}^2 = \left(\frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - 0{\cdot}1553\right)\mathrm{I}^2$$

Hence the mean rate of heat production in the armature is the same as if each of its coils were traversed by a continuous current of amount $\sqrt{\mathrm{K}}\,.\,\mathrm{I}$, where—

$$\mathrm{K} = \frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - 0{\cdot}1553 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or as if the armature were generating a continuous current of amount $2\sqrt{\mathrm{K}}\,.\,\mathrm{I}$.

If we assume, as before, that the limit of output is fixed by the permissible rise of temperature, and if VI = maximum output of armature when acting as a converter, then its output as a continuous-current generator is equal to $2\sqrt{\mathrm{K}}\,.\,\mathrm{VI}$, or—

$$\text{converter output} = \frac{1}{2\sqrt{\mathrm{K}}} \times \text{continuous-current generator output} \quad (6)$$

Using (5) and (6) we get the following relative outputs, that of a continuous-current generator being taken as 100:—

N ... ... =	2	3	4	6	12
Relative output =	85·2	133	162	193	219·5

The output, it will be seen, increases with the number of slip-rings, rapidly at first, then more slowly, tending ultimately to the limit 230 as the number of slip-rings is increased indefinitely.

As a matter of fact, the output would increase even more rapidly with the number of slip-rings than would appear from the above figures. For the practical limit of output should be determined, not by the *average* temperature rise of the coils, but by the highest temperature to which any one coil is raised. Now, although, owing to the partial equalization brought about by the conduction of heat along the coils and core, and the more powerful radiation of heat from the hotter parts of the armature, the curves of temperature rise in the various cases will not be quite so steep as the curves of Fig. 153 (the end portions becoming depressed, and the middle portion raised, by the combined effect of conduction and radiation), yet—especially in the case of the 2-ring (single-phase) converter—the coils close to the slip-rings will attain a considerably higher temperature than those midway between the slip-rings. It might be thought that a simple method of surmounting this difficulty of excessive local temperature rise would be to increase the cross-section of the conductors close to the slip-rings; but this would introduce a fresh difficulty—excessive sparking in certain positions of the armature, due to the destruction of the symmetry of the winding.

Single-phase converters are of no practical interest; on account of their relatively small output, the enormous concentration of the heating on the coils close to the slip-rings, and the further difficulty due to their tendency to spark by reason of the fluctuating armature reaction, they are never employed in practice, motor-generators being invariably used instead. Most of the large power transmission plants making use of either two-phase or three-phase transmission lines, both two-phase (4-ring) and three-phase (3-ring) converters are largely used, and it might be thought that these two types are the only ones practically available. Such, however, is by no means the case, for on account of the important advantages presented by six-phase and twelve-phase converters, the former has been coming very largely into use * with three-phase transmission lines, while quite recently an arrangement has been patented for using a twelve-phase converter in connection with a three-phase transmission line.

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* The four largest rotary converters hitherto constructed (by the General Electric Co., for the New York Edison Co.), each capable of giving an output of 2500 k.w. continuously, are operated six-phase (*Electrical World and Engineer*, vol. xliii. p. 1207 (1904)).

§ 147. Six-phase Converter supplied from Three-phase Mains

The arrangement of connections for supplying a six-phase converter (*i.e.* one having six slip-rings) will be understood by reference to Fig. 154. Let the circle in Fig. 154 (*a*) represent diagrammatically the winding of a 2-pole armature, fitted with six slip-rings connected to the points A, B, C, A′, B′, and C′ of the winding. Remembering the rule (§ 141) for finding the magnitude and phase of the e.m.f. in any section of the winding, we see that the e.m.f.'s between the slip-rings A, B, C form a three-phase system, as also do those between the slip-rings A′, B′, and C′. We thus get a double Δ of voltages; and since AB is in phase with A′B′, BC in phase with B′C′, and CA in phase with C′A′, it is evident that both Δ's may be obtained from the same set of three-phase transformers. For this purpose, the transformers are wound as shown in Fig. 154 (*b*), a mesh connection for both primaries and secondaries being used by preference.† P_1P_2, P_2P_3, and P_3P_1 are the high-voltage primaries. Each transformer is provided with two equal secondaries, which are connected to form the Δ's, ABC, and A′B′C′. The junctions A, B, C; A′, B′, and C′ of the two sets of secondary windings are connected to the points A, B, C; A′, B′, C′ of the armature winding respectively.

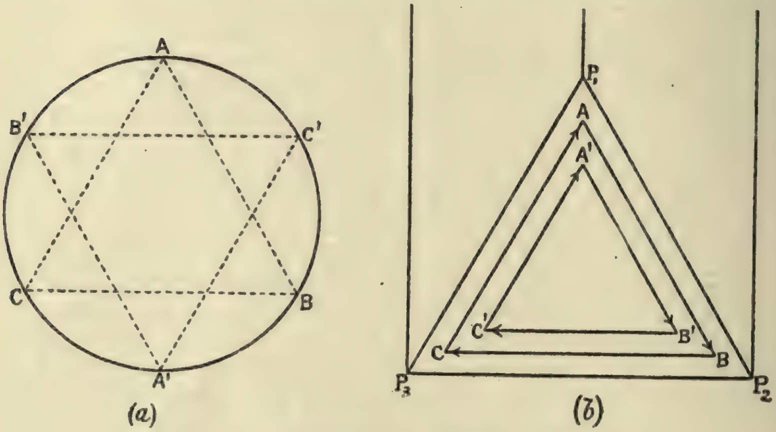

FIG. 154.—Connections of Six-phase Converter.

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† With a mesh connection of transformers, continuity of supply will be maintained even if one of the transformers should break down (§ 57).

§ 148. Twelve-phase Converter

For the purpose of operating a twelve-phase converter from three-phase transformers, the arrangement recently patented by A. D. Lunt, of Schenectady, N.Y., may be used.* This is shown diagrammatically in Figs. 155 (*a*) and (*b*). The armature winding is, as before, represented by a circle, and the points of connection to the twelve slip-rings are marked A, D, C′, etc. It will be noticed that the voltages corresponding to the nine dotted lines in the diagram fall into three groups, such that those corresponding to any one group (*e.g.* AB, FK, A′B′, which are all represented by parallel lines) are all derivable from secondaries of the same transformer; hence all the nine voltages may be supplied by means of only three transformers, each transformer having three distinct secondary windings, as shown in Fig. 155 (*b*). The two groups ABC and A′B′C′ of secondary windings are arranged precisely as for a six-phase converter, and are mesh-connected. Each of these groups contains the same number of turns. The third set of windings, DG, FK, and HE, Fig. 155 (*b*), consists of coils having a larger number of turns than the coils of the other two groups, in the ratio of $\frac{DG}{AC}$,† and these

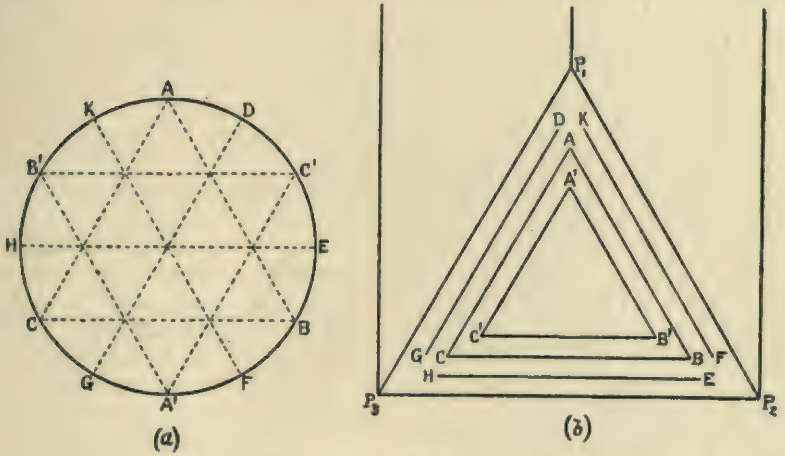

Fig. 155.—Connections of Twelve-phase Converter.

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* *Western Electrician*, vol. xxxv. p. 306 (1904).

† For a sine wave of magnetic flux distribution, $\frac{DG}{AC} = \frac{2}{\sqrt{3}} = 1{\cdot}155$.

coils are not directly connected, each being joined to two diametrically opposite points in the armature winding—D and G, F and K, and H and E respectively in Fig. 155 (*a*).

§ 149. Hunting of Rotary Converters

The diminished heating of the armature of a converter as compared with that of a continuous-current generator supplying an equal current is due to the partial neutralization of the continuous by the alternating current in the armature windings, and a further result of this neutralization is reduced field distortion. For this reason, a converter is less liable to spark, and is capable of standing a much heavier momentary overload without sparking than a continuous-current generator, *provided no hunting takes place.* The hunting or phase-swinging trouble has in some cases assumed a very acute form, and has at times rendered the running of converters impossible. The polyphase currents circulating in the armature give rise to a rotating field, which, owing to the synchronous rotation of the armature, is stationary in space, *i.e.* stationary with respect to the field-magnets. If hunting takes place, however, this field begins to oscillate, and its oscillations may cause vicious sparking at the brushes. A converter which is likely to hunt is thus peculiarly liable to the sparking trouble.

Difficulties due to hunting have been experienced mainly in cases where the angular velocity of the generators was not sufficiently uniform, or where the converters were supplied through a very long line of comparatively high resistance. Hunting may be to a large extent prevented by the use of copper damping bridges placed so as to connect neighbouring poles (the equivalent of a squirrel-cage being thereby obtained), the poles themselves being of solid metal, not laminated. Any oscillation of the field is rapidly damped out by this equivalent of a squirrel-cage winding. One such arrangement, as used by the Union E. G. of Berlin, is shown in Fig. 156.

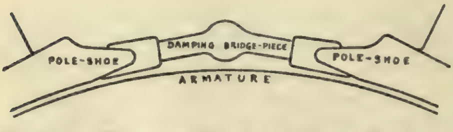

Fig. 156.—Damping Circuit of Rotary Converter.

Practical experience has shown that where a number of converters have to be run in parallel, it is not advisable to run them from a common bank of transformers; each rotary should be supplied from a separate transformer or bank of transformers.

§ 150. Voltage Regulation of Rotary Converters

A very important problem in connection with converters is that of voltage regulation. The voltage ratio is approximately constant, and is but little affected by altering the exciting current. Hence the voltage of a rotary converter cannot be controlled (except within extremely narrow limits) by the method commonly in use with continuous-current generators—that of varying the field excitation. A marked change in the voltage on the continuous-current side can only be obtained by changing the alternating voltage. The two methods commonly in use in practice for effecting this change are (1) the hand regulation method, involving the use of an *induction regulator*, and (2) the automatic method, depending on the use of choking coils and a compound winding on the converter field.

The induction regulator method is mostly in use where the converter supplies a lighting load, in which case very perfect voltage control is desirable. An induction regulator is essentially a polyphase transformer with movable primary. In construction, it is identical with a polyphase induction motor whose wound rotor represents the primary, the stator carrying the secondary winding. The rotor is normally held fast, and can only be slowly rotated in either direction by means of worm gearing driven by a small auxiliary induction motor mounted on the top of the regulator case. In one of the extreme positions of the primary, the rotating magnetic fields due to primary and secondary are in exact coincidence of phase, while in the other extreme position they are in direct opposition of phase.

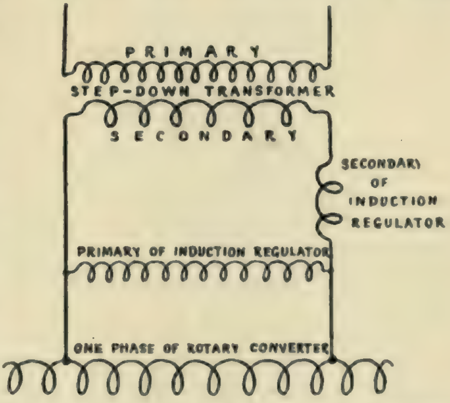
PRIMARY
STEP-DOWN TRANSFORMER
SECONDARY
SECONDARY OF INDUCTION REGULATOR
PRIMARY OF INDUCTION REGULATOR
ONE PHASE OF ROTARY CONVERTER

Fig. 157.—Connections of Induction Regulator.

The diagram of connections corresponding to one phase is given in Fig. 157. The primary of the induction regulator is connected across the corresponding phase of the converter armature, while the secondary is in the main circuit. Let us suppose that with a certain current passing into the converter the primary of the induction regulator is rotated in such a direction as to increase the separation of phase between the primary and secondary hypothetical fields. Now, so long as the secondary current remains unaltered, and the voltage across the primary is maintained approximately constant, it is evident that both the secondary hypothetical field and the resultant or actually existing field must remain approximately constant. Hence, if the separation of the phases of the two hypothetical fields is increased, the primary field must necessarily increase in order to maintain the resultant constant, and as a consequence the resultant will swing round so as to be nearer (as regards phase) the more powerful primary component. The e.m.f. induced in the secondary, whose magnitude and phase depend on those of the resultant field, will thus remain approximately unaltered in magnitude, but will be brought more nearly into coincidence of phase with the secondary voltage of the step-down transformer, so that their vectorial resultant will be increased. The two extreme positions of the induction regulator are those in which the regulator secondary voltage is arithmetically subtracted from, and added to, the voltage of the step-down transformer.

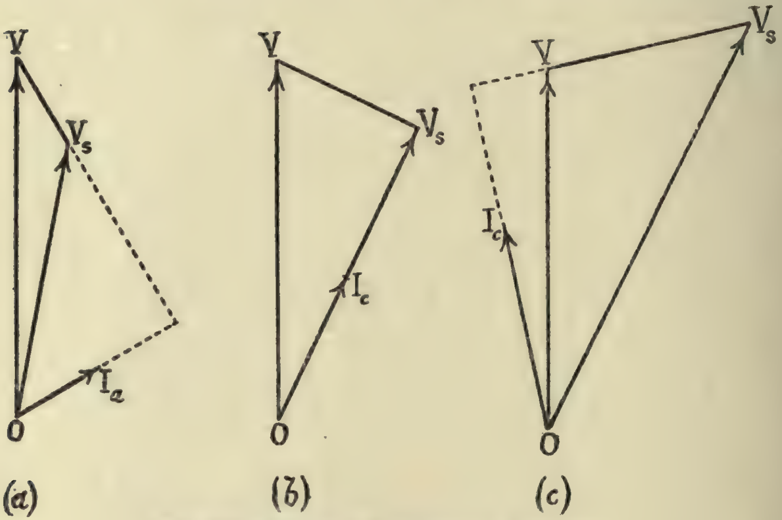

Fig. 158.—Automatic Regulation of Converter.

In the automatic method of regulation, the field of the converter is compound wound, so that with increasing load the field is automatically strengthened. Now, a mere strengthening of the field would, as already pointed out, produce no appreciable rise of voltage on the continuous-current side unless there is a rise on the alternating side. In order to effect this latter rise, suitable choking or reactance coils are inserted in series with each phase of the converter. Let us assume that the voltage at which the step-down transformer is supplied is constant, and let its secondary voltage, also approximately constant, be denoted by V. The excitation due to the shunt is such that at light loads the converter takes a lagging current, represented by I_a in Fig. 158 (*a*). In order to obtain the slip-ring voltage V_s, we have to subtract (vectorially) from V the drop due to the reactance coil—VV_s in Fig. 158 (*a*).* As the load increases, the field is strengthened, and the current comes more nearly into phase with V_s (§ 72), so that when, as in Fig. 158 (*b*), coincidence of phase is reached, V_s may, in spite of the increase in VV_s, remain the same or even increase in value. This constancy or increase is maintained when, as in Fig. 158 (*c*), the current has become a leading one. By this method, an amount of over-compounding of from 10 to 15 per cent. may be obtained without difficulty.

§ 151. Starting of Rotary Converters

There are various methods of starting rotary converters. Where possible they are run up to synchronous speed from the continuous-current side, synchronized, and switched on to the alternating-current mains like ordinary alternators. But in some cases a supply of continuous current may not be available. The best method is, then, to provide a small auxiliary induction motor coupled direct to the converter shaft, and having a somewhat smaller number of poles, so as to enable the converter to be run up to a speed slightly above that of synchronism. The induction motor is then switched off, and as the speed of the converter slowly diminishes it is synchronized and thrown into circuit.†

§ 152. Racing of Inverted Rotaries

When a converter is used as an "inverted rotary," to transform continuous into alternating currents, difficulties may arise on account of the excessive weakening of the field by lagging currents on the alternating side (which is now the generator side), and the dangerously high speed to which the converter armature may run up. In order to prevent this "running away" of an inverted rotary, it is provided with a separate exciter coupled direct to the converter shaft, and designed so that under normal running conditions its field is comparatively weak, and hence very sensitive to an increase in the exciting current. Any tendency on the part of the converter armature to race produces a very rapid increase in the exciting current, which immediately checks any further increase of speed. This arrangement is used by the Westinghouse Co.

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* VV_s being, of course, perpendicular to the current vector.

† A third method, which does away with the auxiliary starting motor, consists in opening the field circuit—preferably in several places—and switching on the alternating current to the armature, through suitable resistances or reactance coils. By the action of eddy currents and hysteresis, the armature will run up to practically synchronous speed, and the field winding is then closed (it may, however, be closed at the wrong moment, so that in order to prevent this a polarized instrument must be provided on the continuous-current side). This method is not to be recommended, as it throws a severe strain on the insulation of the field coils, in which very high e.m.f.'s are induced at the instant of switching on.

§ 153. Converters *v.* Motor-generators

The relative merits and demerits of rotary converters as compared with motor-generator sets—that is, alternating-current motors coupled to continuous-current generators—have formed the subject of much controversy. Considered by itself, a rotary converter is considerably cheaper and more efficient than a motor generator. But while the latter is self-contained and requires no auxiliary apparatus (such as step-down transformers, for the alternating-current motor may be wound for the high voltage of the line—unless this is exceptionally high), the rotary converter requires various accessories, which have the double effect of increasing the total cost of the transforming plant and reducing its efficiency. When the additional cost of the step-down transformers, the induction regulator or the reactance coils and compound winding, are taken into account, the difference in first cost between a rotary converter and a motor-generator plant is very slight—not exceeding some 5 per cent.—the rotary converter plant being the cheaper of the two. Similarly, as regards efficiency, there appears to be but little to choose between them, the rotary converter plant being again slightly more efficient. On the other hand, the motor-generator plant allows of a simpler mode of voltage control over a wider range. A good deal depends on the frequency, and on low-frequency (say, 25-cycle) circuits rotary converters would probably be preferred.

Two types of motor-generators are in use, depending on the type of motor—the synchronous and the asynchronous or induction motor type. For capacities up to 300 k.w., the induction motor-generator set would appear to be slightly cheaper, while for outputs exceeding 500 k.w., it would appear to be slightly more costly than the synchronous set. The induction set is self-starting; on the other hand, the losses with the synchronous set may be made somewhat smaller by reason of the higher power factor.

CHAPTER XVI

§ **154**. Compensated induction motor—§ **155**. Equivalent of compensated motor—§ **156**. Effect of varying number of rotor conductors. Speed control—§ **157**. Effect of brush displacement—§ **158**. Case of synchronous speed—§ **159**. Latour alternator—§ **160**. Heyland alternator.

§ 154. Compensated Induction Motor

WHILE the induction motor possesses many advantages—among which the extreme simplicity of its construction (especially when a squirrel-cage rotor is used) and the small amount of attention (due to the absence of a commutator) which it requires are the most important—it also suffers from a somewhat serious disadvantage—a comparatively low power factor at light loads (*cf.* § 67). Not only does a low power factor increase the losses in the generators, supply mains, and motor itself, but it renders the voltage regulation of the system much more difficult. By adopting for the rotor an ordinary continuous-current armature having a commutator, Heyland and Latour have succeeded in constructing induction motors whose power factor closely approaches unity at all loads, and which also possess certain other remarkable peculiarities. Such induction motors are known as *compensated induction motors*. The advantages which they possess over ordinary induction motors have, however, been only gained by a sacrifice of simplicity of construction.

Although the researches of Heyland and Latour are comparatively recent—dating back to 1900 only—yet the most prominent feature of their compensated induction motors—a rotor having a commutator and resembling in every respect (except as regards position and number of brushes) the armature of a continuous-current machine—is a device of much older date, having been patented as far back as 1888 by Wilson in England, and in 1891 by Görges in Germany. Although Görges gave a sketch of the theory of such motors in 1891, there is no doubt that the possibilities presented by the use of a commutator rotor were by no means fully realized at that time, and it was only by the later labours of Heyland and Latour that this type of motor was once more brought into prominence.

Imagine a stator wound for 2 P poles, and into this stator let there be introduced a lap-wound continuous-current armature, wound for a 2 P-pole field. Instead of the usual two sets of brushes per pole-pair which would be used in connection with a continuous-current machine, let there be provided three sets of brushes per pole-pair, spaced $\frac{2}{3}$ of the pole-pitch ($= \frac{1}{3}$ period) apart, the brush sets occupying similar positions relatively to the stator field being connected together. Further, let the brushes be placed in such positions relatively to the stator windings that when three-phase currents are fed into the rotor through the brushes, the rotating field due to them is at every instant coincident in position with the stator field. Lastly, let the three groups of rotor brushes be connected across the mains, so that the stator and rotor windings are in parallel with each other, and that their ampere-turns are at every instant added arithmetically when the windings are both traversed by currents coming from the mains.

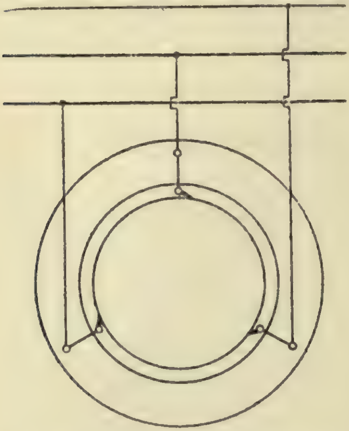

FIG. 159.—Connections of Compensated Induction Motor.

The arrangement of connections will be easily understood by reference to Fig. 159, where, for the sake of simplicity, a two-pole field is shown. The windings are not indicated, but only the three points of connection to the stator windings, and the three rotor brushes, spaced 120° apart. The directions of the two windings are supposed to be such that if each of them is supplied with currents from the mains, the fields to which such currents give rise are at every instant coincident as regards both position and direction.*

§ 155. Equivalent of Compensated Motor

A moment's consideration will show that, in so far as the action of the motor is concerned, the arrangement of Fig. 159 is practically identical with that of Fig. 160, where the rotor is shown disconnected from the mains, and short-circuited through negligible resistances R_1, R_2, R_3, which, however, contain alternating e.m.f.'s. The supply mains are, in fact, bridged by the low-resistance generator coils at the station, and by numerous lamps, motors, etc., throughout the entire area of supply, so that the *resistance* across any two mains is extremely low. If, then, the combined resistance of the entire network were the only point to be considered, the rotor might just as well be simply short-circuited by connecting together its three sets of brushes, and it would then be equivalent to an ordinary short-circuited rotor.* The mains, however, in addition to offering paths of negligible resistance, *also contain e.m.f.'s*—furnished by the generators at the station. Hence, in order to render the arrangement of Fig. 160 equivalent to that of Fig. 159, we must suppose that into the short-circuiting resistances of the rotor there are injected e.m.f.'s, equal in magnitude and phase to the p.d.'s across the mains.

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* So that if in Fig. 159 we assume the stator winding to be an ordinary *right-handed* Gramme or ring winding, the rotor ring winding would have to be a *left-handed* one if the brushes are in the positions shown in the figure. With both windings either right- or left-handed, the brushes would have to be rotated through 180° (or, generally, through a pole-pitch or a distance corresponding to $\frac{1}{2}$ period in a multipolar field). Otherwise, the condition may be expressed by saying that two conductors lying along the same radius on opposite sides of the air-gap should convey currents identical in phase.

It is the injection of these e.m.f.'s—due to the connection of the brushes across the mains instead of across simple short-circuiting resistances—that so profoundly modifies the behaviour of the motor.

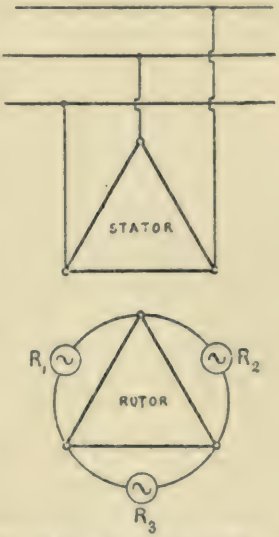

Fig. 160.—Equivalent of Compensated Induction Motor.

For the sake of simplicity, we shall neglect magnetic leakage, and shall, further, assume that the stator and rotor windings are of the same type, so that the e.m.f. induced in each winding by the rotating field is simply proportional to the product of the number of conductors into the relative velocity of the conductors and the rotating field. Let C_s = number of stator conductors, and C_r = number of rotor conductors. We shall consider the effect on the behaviour of the motor due to a variation in the number of rotor conductors.

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* The only difference being that in an ordinary rotor the points of short-circuit are three fixed points in the winding, whereas in a commutator rotor whose brushes are short-circuited the position of the points varies during the rotation. This, however, does not introduce any material difference: in the first case, the frequency of the currents in the short-circuiting connections is the same as that of the rotor currents; in the second, it is equal to the frequency of the stator currents.

§ 156. Effect of Varying Number of Rotor Conductors. Speed Control

First, let $C_r < C_s$. Assume the rotor to be originally at rest and unloaded, and consider the *resultant* e.m.f. acting in each of the three closed circuits formed by the short-circuited rotor windings (Fig. 160). This resultant e.m.f. is the arithmetical difference of the e.m.f. induced in the rotor windings and the p.d. of the mains. Now, if V = p.d. across mains, and E_s, E_r be the e.m.f.'s induced in the stator and rotor windings respectively, we may write, neglecting the stator resistance drop—

$$V = E_s = kC_s, \text{ and } E_r = k\,C_r$$

k being a constant. Hence—

$$\text{resultant rotor e.m.f.} = E_r - V = k(C_r - C_s) . \quad . \quad . \quad (1)$$

But by supposition $C_r < C_s$, so that the resultant rotor e.m.f. is *negative, i.e.* the rotor current flows in the direction of the p.d., and *against* the induced e.m.f. of the rotor. Now we know that if the rotor current were allowed to flow *in* the direction of the induced e.m.f. (as happens in an ordinary induction motor), such a current would develop a torque in the direction of the rotating field. A reversal of the rotor current (or a change of π as regards phase) will reverse the torque. We thus see that our commutator rotor will experience a torque *opposed* to the direction of rotation of the field. The rotor will accordingly begin to run in a direction opposed to that corresponding to an ordinary induction motor, and as its speed increases, E_r will increase and the rotor current decrease, until it is just sufficient to provide the small torque required to make up for the losses due to friction, etc.

If a load be now applied to the rotor, its speed will slightly decrease, and the torque and current will increase. To balance the increased rotor ampere-turns due to the load, an equivalent number of ampere-turns *having the opposite direction* must be provided in the stator.* Hence, the stator current will flow *against* the p.d. of the mains, *i.e.* the stator will act as a *generator*, supplying power to the mains. The stator of the motor now acts, in fact, as if it were the secondary of a transformer of which the rotor is the primary.

Let us next suppose that $C_r = C_s$, the rotor being as before originally at rest and unloaded. A reference to equation (1) shows that in this case E_r is practically balanced by V, so that there is no current in the rotor except the very small magnetizing current (V being slightly greater than E_r). The rotor will remain at rest, as there is no torque.

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* The supply p.d. being constant, and magnetic leakage assumed to be negligible, it follows that the magnetic flux, which is common to stator and rotor, must also remain constant—thus involving a constancy of the resultant ampere-turns.

Lastly, consider the case in which $C_r > C_s$, and which is of more practical interest than the others. We now have the resultant rotor e.m.f. *positive*, according to equation (1), so that a current flows through the rotor windings, as in an ordinary induction motor, in the direction of the induced rotor e.m.f. Hence the rotor experiences a driving torque in the direction of the rotating field, and it will run up to a speed at which the induced e.m.f. is nearly balanced by the p.d. If a load be applied to the rotor, the speed will decrease slightly, the current and torque increasing to the amounts required to deal with the resisting torque corresponding to the load. The increase in the rotor ampere-turns due to the load current must be balanced by an equivalent number of *opposing* ampere-turns in the stator. Now, since the rotor load current flows in the direction of the induced e.m.f., the stator load current must flow in the direction of the p.d., *i.e.* the stator will *absorb* power from the mains, and the rotor *return* power to them.

As in the case of an ordinary induction motor, there will be a very small percentage change of speed from no load to full load.* This is due to the low resistance of the rotor windings. The motor will, therefore, run at nearly constant speed. The slip s of the rotor may easily be found approximately, since, if m denotes synchronous speed, we must, neglecting resistance drops and magnetic leakage, have the relations—

$$E_s = KmC_s = V = E_r = KsmC_r$$

K being a constant. If we denote the rotor speed $(1 - s)m$ by m_r, the above equation gives—

$$mC_s = smC_r = (m - m_r)C_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The speed of the motor may, therefore, be arranged to have any desired value between zero and near synchronism by suitably choosing C_r,† and may even be made negative by making $C_r < C_s$.

This suggests a simple means of varying the speed of such a motor without any sacrifice of efficiency such as is involved in the rheostatic method of control sometimes employed with induction motors of ordinary construction. We cannot, of course, alter at will the number of conductors on the rotor, but we can easily effect what is an equivalent change—viz. alter the p.d. impressed on the brushes. This may be done by means of a transformer interposed between the mains and the rotor, and having a variable number of turns in its secondary. A decrease of p.d. in the ratio of $r:1$ is clearly equivalent to an increase of C_r in the ratio of 1 to r.

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* By introducing non-inductive resistances between the mains and the rotor brushes, the speed could be controlled as in an ordinary induction motor.

† In this respect, a commutator induction motor closely resembles an ordinary shunt-wound continuous-current motor, which is also a nearly constant-speed motor, and whose speed, for a given p.d. of supply, may be made to have any desired value by suitably choosing the number of armature conductors.

§ 157. Effect of Brush Displacement

We have so far neglected magnetic leakage, and if this were really negligible, the power factor of our motor would be very high—as high as that of a good transformer working on a non-inductive load. Unfortunately, magnetic leakage is by no means negligible, and its effect is to lower the power factor. Now, in the ordinary type of induction motor we have no means of altering the power factor. It is in this connection that one of the most important advantages of the commutator motor comes to light, for by suitably displacing the brushes *we can adjust the phase of the rotor and hence also that of the stator current so as to bring the latter into practical coincidence of phase with the p.d.*, and thereby secure a power factor approaching unity.

Originally, we assumed the brushes to be placed in such a position that the induced rotor e.m.f. was in exact opposition of phase to the p.d. Now, by displacing the brushes we can introduce a phase difference between the p.d. and e.m.f., thus altering the phase of the resultant e.m.f. and of the rotor and stator currents. The *immediate* effect of shifting the brushes, one way or the other, from the position originally assumed, is to increase the resultant e.m.f., and hence increase the current. But although the current is increased, the torque may be either increased or decreased, depending on the direction in which the brushes are displaced. Let us suppose the displacement to have been such as to increase the torque; acceleration will occur, the induced e.m.f., and with it the current, decreasing until equilibrium is once more established. If the brushes be now further displaced in the same direction, a further increase of torque, accompanied by further acceleration and reduction of the induced e.m.f., will take place. It will thus be seen that by continuing to displace the brushes in the proper direction, the speed may ultimately be brought up to synchronism, and *the induced e.m.f. made to disappear;* the rotor winding now behaving as a simple non-inductive resistance, since there is no relative motion between it and the rotating field. By displacing the brushes still further, speeds above synchronism may be realized.

We thus have a twofold method of altering the speed—in one, we alter the p.d. impressed on the brushes; in the other, we alter the position of the brushes.

§ 158. Case of Synchronous Speed

The case of synchronous speed is of particular importance from a practical point of view. At this speed, the rotor winding contains no induced e.m.f., and therefore *behaves as if it were a simple non-inductive resistance.* The rotor current is equal simply to the quotient of the p.d. by the resistance, and is in phase with the p.d. The winding must, therefore, have a sufficiently high resistance to withstand the application of the full p.d. without damage.* The rotor currents—apart from minor fluctuations—are continuous currents (*i.e.* of zero frequency), and the rotor behaves as if continuous current were supplied to it through three slip-rings, instead of alternating current through brushes bearing on a commutator. It is, further, clear that at synchronous speed the motor entirely ceases to act as an induction motor—it becomes a synchronous motor pure and simple. At a speed slightly above or below that of synchronism, it may be regarded as a combination of a synchronous with an induction motor.

The importance of maintaining nearly synchronous speed is due to the fact that at this speed sparking troubles are reduced to a minimum. In Latour's compensated induction motors, no special devices are employed to suppress sparking; in Heyland's motors, the commutator segments are shunted by non-inductive resistances.

Although compensated motors of this type may be made to have a higher power factor than ordinary induction motors, they have not come into use to any large extent, mainly on account of the additional complications involved. Their designers have, however, been led by a study of their peculiarities to the construction of self-exciting and compounded *alternators.*

§ 159. Latour Alternator

The armature winding of a polyphase generator does not in any way differ from the stator winding of an induction motor. Now, we have just seen that if a rotor of the commutator type be supplied with three-phase currents, when running at nearly synchronous speed it becomes equivalent to a rotating magnet excited by means of continuous currents. Hence by substituting for an ordinary field-magnet of the revolving type a commutator rotor, and supplying this latter with three-phase currents from a transformer whose primary is connected across the terminals of the armature, we obtain a *self-exciting polyphase generator*. This is the arrangement used by Latour. Such a generator, it may be noted, need not run at absolutely synchronous speed.

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* Since this would necessitate the use of very fine wire and many turns on the rotor if the brushes were directly across the mains, in motors of this type as actually constructed a transformer for reducing the stator p.d. is used. The stator winding itself is made to act as the primary of this transformer, the secondary consisting of a few turns embedded in the stator slots and having the rotor winding for its load.

Since at synchronous speed there is no relative motion of the field and the rotor core, it is evident that there is no need to laminate the core: it may be constructed of solid steel.

§ 160. Heyland Alternator

In Heyland's self-excited and compounded *synchronous* alternator, the field-magnet is of the ordinary type of construction employed in separately excited generators, consisting of a flywheel whose rim carries a number of polar projections wound with exciting coils. These field coils are connected in a certain manner—explained below—to a commutator, and by means of three brushes spaced a distance apart corresponding to $\frac{1}{3}$ period, exciting currents are fed into the field winding from the secondaries of two transformers, one of which supplies a nearly constant current, sufficient to produce the required generator p.d. on open circuit, while the other supplies a current which increases in simple proportion to the *wattless* component of the load current, and so maintains the generator p.d. approximately constant.

In the latest form of this type of generator, the commutator contains eighteen segments per magnetic circuit, *i.e.* per pole-pair. The first six segments of each group of eighteen are active segments (*i.e.* segments connected to the field winding); the next three are dummy segments, which are not in connection with anything; then comes another set of six active segments, followed by a set of three dummy segments. Corresponding active segments of each group are interconnected.* The width of each brush is slightly in excess of the width of three segments.

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* Thus, if we suppose the segments to be numbered consecutively, we should have the following twelve groups of interconnected segments:—

Group I: 1—19—37—55, etc.
„ II: 2—20—38—56, etc.
„ III: 3—21—39—57, etc.
„ IV: 4—22—40—58, etc.
„ V: 5—23—41—59, etc.
„ VI: 6—24—42—60, etc.

Group VII: 10—28—46—64, etc.
„ VIII: 11—29—47—65, etc.
„ IX: 12—30—48—66, etc.
„ X: 13—31—49—67, etc.
„ XI: 14—32—50—68, etc.
„ XII: 15—33—51—69, etc.

There is, of course, no occasion to connect the idle or dummy segments 7, 8, 9; 16, 17, 18, etc.

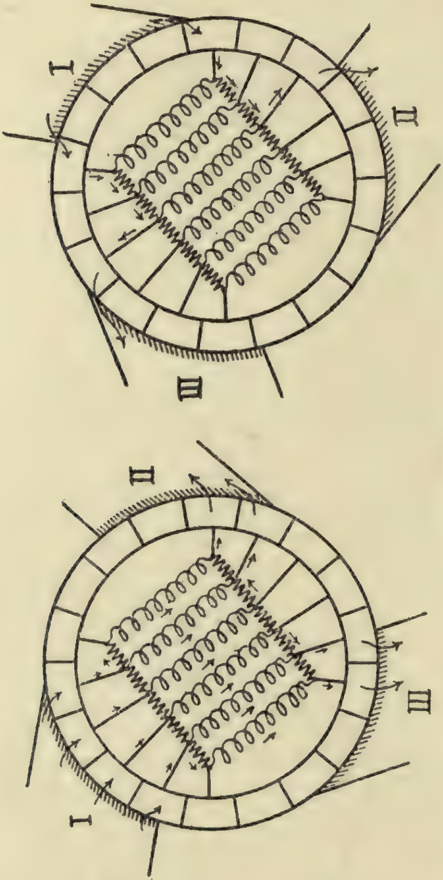
It is evident that we may replace the actual commutator of a multipolar field of 2 P poles by a commutator consisting of eighteen segments only (which would correspond to a two-pole construction), but running at P times the speed of the field-magnet, and having its brushes spaced 120° apart. Such an equivalent two-pole commutator (running at a correspondingly higher speed) is shown in Fig. 161, and on account of its greater simplicity it is more convenient to consider than the actual multipolar commutator.

The entire field winding is divided into six groups of coils. The field coils may be wound in the ordinary way if the number of pole-pairs is a multiple of six; otherwise, each pole would be provided with two or more windings—the coil being wound with two or more wires laid side by side (in a two-pole magnet, six wires would be wound side by side)—so as, in each case, to subdivide the entire field winding into six groups of equal resistance. These six groups are then connected to the twelve active sets of segments as shown in Fig. 161, and across the ends of the groups are connected non-inductive resistances as shown.

Let us suppose that the brushes are in connection with the secondary of a transformer whose primary is across the generator terminals. By shifting the brushes the commutator may be arranged to occupy any desired position relatively to a brush at the instant when the current through that brush is at its maximum value. Let the brushes be so adjusted that when any brush is receiving maximum current it rests in the position of symmetry on the active segments—being in contact with the four middle active segments, as shown in Fig. 161. The field winding is then in the best position for receiving current, while most of the non-inductive resistances are not traversed by currents at all. In order to make this point quite clear, we may imagine the brushes displaced through 90° ($\frac{1}{4}$ period), so that the relative position of brushes and commutator at the particular instant considered is as shown in Fig. 162. It is at once evident that now the current flows mainly through the non-inductive resistances.

Assuming the brushes to have been placed in the most favourable position, corresponding to Fig. 161, as the current through brush I is decreasing, the active segments gradually recede from brush I and approach brush II. After one-third of a revolution, which corresponds to $\frac{1}{3}$ period, the active segments will be under cover of brush II, which is now receiving maximum current and delivering it to the field winding in the same direction as was done by brush I. After another one-third of a revolution, the active segments will come under cover of brush III, which in its turn is now receiving maximum current, and so on. Thus, although the current through the field winding will vary slightly (the period of its fluctuation being clearly one-third of that of the generator period), the variations are kept within extremely small limits by the large self-inductance of the winding, and for practical purposes the current may be regarded as constant.

In order to compound the generator for a highly inductive load, an additional current must be injected into the field coils. Let us suppose that a second set of three brushes, spaced 120° apart, is at our disposal, and that these brushes are connected to the secondary of a transformer whose primary is in series with the armature winding. Let the brushes be so adjusted that when a brush is receiving maximum wattless current, it is in the position of brush I, Fig. 161, relatively to the commutator. The field excitation will clearly be



Figs. 161 and 162.—Commutator Connections of Heyland Alternator.

considerably increased, and the additional exciting current will be proportional to the wattless component of the generator current. On the other hand, when the load component reaches its maximum value

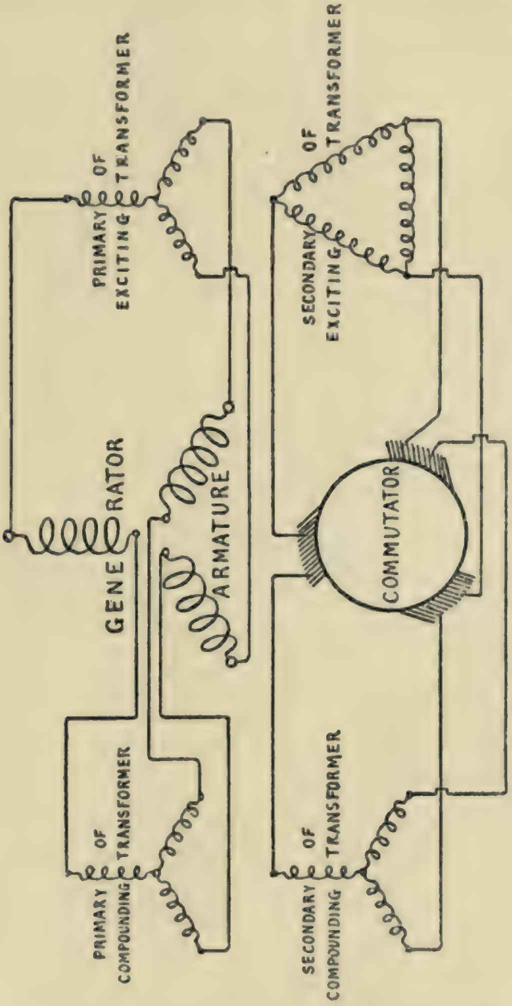

PRIMARY OF EXCITING TRANSFORMER
SECONDARY OF EXCITING TRANSFORMER
GENERATOR
ARMATURE
COMMUTATOR
PRIMARY OF COMPOUNDING TRANSFORMER
SECONDARY OF COMPOUNDING TRANSFORMER

Fig. 163.—Connections of Heyland Compounded Alternator.

at any brush, the relative position of brush and commutator will be that corresponding to Fig. 162, and the current will flow mainly through the non-inductive resistances, and thus contribute practically nothing towards the excitation.

Let us assume, for a moment, that all the windings of the two transformers employed for supplying current to the field coils are star-connected. Then it follows (since a wattless current lags 90° behind the p.d.) that the second set of brushes (which supply the compounding current) must be displaced 90° ahead of the first set. Now, if we replace the star connection of the secondaries of the first transformer by a mesh connection, we accelerate the phase of the secondary p.d., and hence also that of the current supplied by it, by 30°.* In order to maintain the first set of brushes in their proper position relatively to the commutator, we therefore have to shift them backwards through 30°. But this will make the angle between the two sets $90° + 30° = 120°$—which means that the two sets of brushes become coincident.

Thus, by adopting a star connection for the secondaries of one transformer, and a mesh connection for those of the other, we do away with the necessity of using two sets of brushes, a single set being sufficient. The complete arrangement of connections is shown in Fig. 163. The three armature circuits, it will be noticed, are connected to a neutral point not directly, but through the primaries of the compounding transformer. Exact compounding is obtained by varying the number of turns in the compounding transformer, and adjustable resistances are introduced into the circuits of the exciting transformer.

The non-inductive resistances bridged across the ends of the field coils perform three important functions: (1) they provide a path for the load component of the generator current; (2) they serve to distribute the current more or less equally among the six parallel circuits; and (3) they prevent sparking.

The Heyland self-excited and compounded alternator has already achieved a considerable amount of success, and some large machines of this type have been constructed. In addition to its satisfactory self-regulating qualities, it is characterized by its small size as compared with machines of ordinary construction; this is due to the possibility of allowing armature reaction to exceed very considerably the limits admissible in alternators of the ordinary type.

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* Since there is a phase difference of 30° between the phase p.d. and the line p.d. (see § 28).

CHAPTER XVII

§ 161. Single-phase commutator motors—**§ 162.** Series motor. Circle diagram—**§ 163.** Conditions determining power factor of series motor—**§ 164.** Compensated series motor—**§ 165.** Methods of obtaining sparkless running—**§ 166.** Repulsion motor—**§ 167.** Compensated repulsion motors—**§ 168.** Atkinson's repulsion motors—**§ 169.** Mixed-action commutator motors.

§ 161. Single-phase Commutator Motors

WITH the rapid growth of electric railway and tramway systems, the necessity of high-voltage transmission on economical grounds soon became apparent. It was realized that beyond a certain distance the usual 500- or 600-volt continuous-current systems could no longer be successfully employed, as the cost of the conductors required to secure reasonable regulation and losses became a very heavy item in the total cost of the system. This led to the development of the rotary converter system, in which we have three-phase generation and transmission to converter sub-stations at a high voltage, combined with low-voltage (500 volts) continuous-current distribution. But the cost of rotary converter sub-stations is still very heavy. Attempts were then made, in the case of very long lines, to use the three-phase system pure and simple, the current being generated and transmitted at a very high voltage, and then transformed to a lower voltage before being supplied to the three-phase induction motors employed for driving the cars.

Apart from the complication and increased cost of overhead construction necessitated by the use of two overhead conductors, the three-phase system of electric traction suffers from another defect—the unsuitability of induction motors for variable speed work. The induction motor is, as we have seen, essentially a constant-speed motor, and is extremely wasteful during the period of acceleration. Such motors can only be satisfactorily employed for traction work on lines where stoppages are infrequent, and where long runs at constant speed are the rule.

In view of the disadvantages of the three-phase system of traction, and the heavy cost of rotary converter sub-stations, the single-phase system of traction, in which both transmission and distribution are effected by the use of single-phase current, has attracted a great deal of attention within recent years, and has led to the rapid development of single-phase *commutator* motors. In the present chapter we shall give an account of the theory and construction of such motors.

All single-phase commutator motors * may be referred to one or other of three types, viz. (1) the series-wound motor, with or without a compensating winding; (2) the repulsion motor; (3) the compensated repulsion motor (Latour-Winter-Eichberg).

§ 162. Series Motor. Circle Diagram

The general arrangement of the plain series-wound single-phase motor closely resembles that of an ordinary continuous-current motor; in fact, such motors may, and in practice actually are, used on both single-phase and continuous-current circuits. The differences between the ordinary continuous-current and the single-phase types are partly constructional, partly differences of design. It is obvious that, in order to prevent excessive eddy-current loss, and to enable the field-magnet to develop its full magnetic flux,† the core of the field-magnet must be *laminated throughout.*

In Fig. 164 is given the diagram of connections of a plain series-wound single-phase motor. A is the armature, FW being the field winding. T_1 and T_2 are the motor terminals. The dotted arrow indicates the direction of the field flux, which, as in a continuous-current motor, is at right angles to the line of the brushes.

Since in this type of motor the armature is connected in series with the field, it is obvious that the armature and field currents must necessarily be in phase with each other, so that the armature current reaches its maximum value at the instant of maximum field intensity. So long as the magnet is well below saturation—*i.e.* within the range of approximately constant permeability—the torque at any instant is proportional to the square of the current. From this it follows that the motor will exert equal torques when supplied, in the one case, with continuous current of amount i, and in the other, with

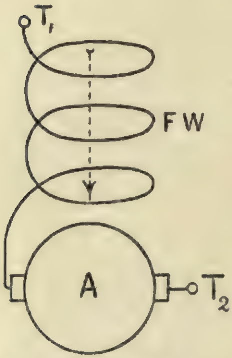

Fig. 164.—Diagram of Connections of Series Motor.

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* The single-phase induction motor is still less suitable for traction than the three-phase induction motor, and its use for this purpose could not be seriously considered.

† Eddy currents exert a demagnetizing or screening effect on the core, so that with a massive core the central portions would not be appreciably magnetized.

single-phase current whose r.m.s. value is i. In the first case, however, the torque is a steady one; in the second, a rapidly fluctuating or pulsating one.

As will be seen presently, it is advisable to have a relatively small number of turns in the field winding of such a motor; this is one of the important differences between an ordinary continuous-current and a single-phase series motor.

The performance of such a motor is best studied by the aid of a *circle diagram.* This we now proceed to establish, neglecting, for the sake of simplicity, the iron or core losses in the motor.

Let in Fig. 165 the horizontal direction be that of the current vector. The p.d. impressed on the motor terminals may be regarded as made up of the following components: (1) the component OC = ri, r being the total resistance of the motor, and i the current; this component is clearly in phase with the current; (2) the component OB = CA, in quadrature with the current, required to balance the induced e.m.f. due to the self-inductance of the field and armature windings; (3) the component AV, in phase with the current, required to balance the e.m.f. induced by the *rotation of the armature in the field.* The resultant OV of these three components gives the impressed p.d.

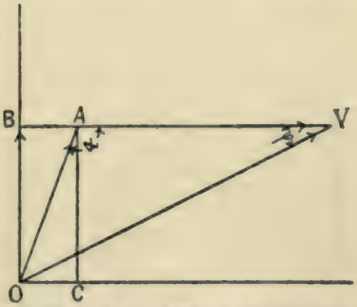

FIG. 165.—Vector Diagram of Series Motor.

Let us now suppose that the p.d. is maintained at a constant value. The vector OV thus remains of constant length. Further, since OA = current × impedance of windings, and since the impedance on the assumption of constant permeability remains constant, it follows that OA is proportional to the current, and may, therefore, be taken to represent the current to a suitable scale. Again, since the angle BAO = angle AOC = $\tan^{-1}\dfrac{\text{reactance}}{\text{resistance}}$, this angle will be constant, and so will also the angle OAV = α, which is its supplement. It is now evident that as the current (OA) changes, OV remaining constant, A must move along the arc of a circle, in order that the angle OAV may, as required, retain a constant value. Thus the locus of the extremity of the current vector is a circle.

The angle BVO = ϕ = angle VOC = angle of lag of current behind p.d., so that $\cos\phi$ = power factor.

We may now construct the diagram shown in Fig. 166, in which the triangle OAV corresponds to the similarly lettered triangle of Fig. 165. Lay off a line OV to represent, to any convenient scale,

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the p.d. Draw RP, making an angle with OV such that angle ROV $= \alpha = \pi - \tan^{-1}\frac{\text{reactance}}{\text{resistance}}$.* At O erect a perpendicular OC to RP, and on OV describe an arc having its centre on OC. Then it is evident from the construction that the angle contained in the segment is α. Again, angle POA $= \pi - \alpha - \angle VOA$; and since we must have $\angle VOA + \alpha + \phi = \pi$, it follows that $\angle POA = \phi$. Hence $\angle POA$ is the angle of lag of current behind p.d., and PO = OA $\cos\phi$ is proportional to the power absorbed by the motor.

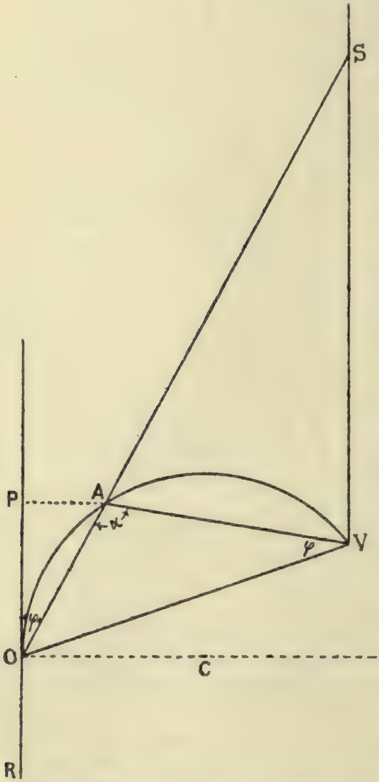

Fig. 166.—Circle Diagram of Series Motor.

The torque of the motor is proportional to OA^2.

A very simple construction for the speed may be obtained as follows. AV is proportional to speed × field (since AV represents the e.m.f. due to the rotation of the armature), *i.e.*—

$$AV \propto \text{speed} \times OA$$

or—

$$\text{speed} \propto \frac{AV}{OA}$$

Draw VS ∥ OP, and produce OA to intersect VS at S. Then the triangles OAV and OVS are similar, so that $\frac{AV}{OA} = \frac{SV}{OV}$, and since OV is constant, we see that—

$$\text{speed} \propto VS$$

and VS will, to a certain scale, represent the speed of the motor.

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* The value of $\tan\phi = \frac{\text{reactance}}{\text{resistance}}$ is easily obtained from $\cos\phi$, the latter value being measured by means of a wattmeter, an ammeter, and a voltmeter when the motor armature is held fast.

§ 163. Conditions Determining Power Factor of Series Motor

A reference to Fig. 165 shows that the power factor (cos ϕ) increases with increase of the ratio $\frac{AV}{OA}$. It is therefore desirable, in order to secure a high power factor, to make this ratio as large as possible. Now, the reactance-voltage CA may be reduced by reducing the number of turns on the field—which will reduce that part of the reactance voltage depending on the field winding—and by designing the magnetic circuit so as to interpose a large magnetic reluctance along the path of the flux due to the armature current, and thereby also reduce the part of the reactance voltage which corresponds to armature self-inductance.

The first method of reducing the reactance is very generally adopted in practice, and leads to the design of motors in which the armature ampere-turns bear a ratio of from 2 to 5 to the field ampere-turns. It must be noted that, beyond a certain point, an increase of power factor by a reduction of the field ampere-turns can only be obtained at a sacrifice of efficiency; for in order to secure a given torque with a very weak field, a larger current will be required (corresponding to a heavier copper loss) than would otherwise be necessary.* It is thus not advisable to reduce the field-turns below a certain limit.

In order to throttle the armature flux as much as possible, it is usual to employ stators or field-magnets having well-defined or salient poles, such as are typical of continuous-current motors. The reluctance to the armature flux may be further increased by making the width of the pole small in comparison with the length of the inter-polar arc, and by working the poles at a high induction so as to reduce their permeability. A further improvement consists in slotting the poles down the central line, an additional air-gap being thereby interposed in the path of the armature flux. Fig. 167 represents a form of construction due to Dr. Finzi, the field core being built up of stampings shaped as shown; there is a wide gap down the middle of each pole-piece, and a further throttling effect is obtained by chamfering the edges of the pole-pieces, instead of providing them with expanded pole-shoes, as would be done in a continuous-current motor.

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* This is on the assumption that the field-turns are reduced without any change in the armature turns. The same result as regards increased armature loss is, however, also arrived at by supposing that the current remains unaltered, and that the original value of the torque is maintained by increasing the number of armature conductors in the same ratio as that in which the field has been weakened. The current remaining unaltered, and there now being more conductors on the armature, *i.e.* a larger armature resistance, it is evident that the copper loss in the armature will have been increased. The increase in the armature copper loss is only to a very feeble extent counterbalanced by the decrease in the field copper loss.

Let the speed at which the motor is required to run when fully loaded be specified. The speed being given, the full-load torque is also known. Now, for a given gap induction, and a given number of ampere-wires per cm. length of armature circumference at full load, the torque is independent of the number of poles. The self-inductance of the field is also independent of this number so long as the gap induction and the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ remain constant.* We may, therefore, vary the number of poles without altering the reactance of the field. If we assume the number of poles to be doubled, then for the same speed, with a lap-wound armature, we shall require the same number of conductors per pole-pitch, but each conductor will now carry only half as much current. The polar arc having been reduced to half its original length, there will be only half the original number of ampere-turns producing the cross-field, and only half the flux linked with the conductors lying between two neighbouring sets of brushes. The armature reactance e.m.f. will thus be reduced to half its original value, and the power factor will be increased.

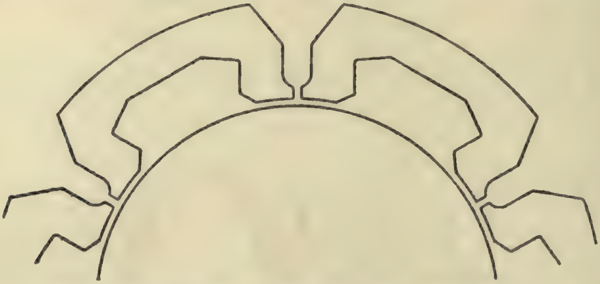

FIG. 167.—Finzi's Design for Magnet of Series Motor.

This shows the advantage gained by increasing the number of poles. There is another way of expressing the same result. Since the speed of synchronism, for a given frequency, varies inversely as the number of poles, increasing this number (for a given speed of the motor) is equivalent to making the ratio $\frac{\text{speed of motor}}{\text{speed of synchronism}}$ larger. The result just established is, therefore, frequently stated in the form that it is best to design the motor so that it will run at a high speed as compared with the speed of synchronism. The normal full-load speed is generally about double that of synchronism.

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* If, *e.g.*, we double the number of poles, the flux per field-turn will be halved, and the number of field-turns doubled, the total flux linked with the field winding thereby remaining unaltered.

It must be clearly understood, however, that the improvement in the power factor is due, not to any effect of increased relative speed *per se*, but to a decrease in the reactance of the motor consequent on increasing the ratio $\dfrac{\text{motor speed}}{\text{speed of synchronism}}$.

§ 164. Compensated Series Motor

Besides the methods already described for improving the power factor, another method is frequently employed. This involves the use of a supplementary field winding, known as a *compensating winding*. This winding is displaced relatively to the main field winding by half a pole-pitch, so as to counteract the magnetic effect of the armature current, and practically suppress the self-inductance of the armature.

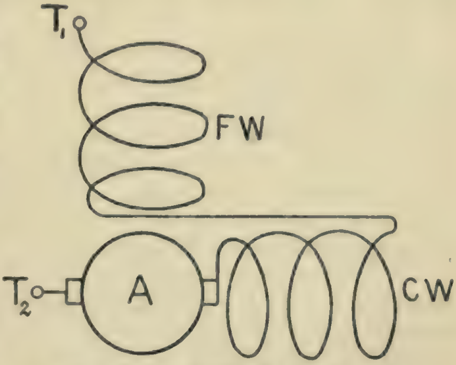

FIG. 168.—Compensated Series Motor.

A series-wound motor fitted with such a compensating winding is known as a *compensated* series-wound motor. There are two methods of connecting the compensating winding. In one, shown in Fig. 168, it is included in the main circuit. The compensating winding CW supplies a number of ampere-turns roughly equal to those on the armature. In the second method, shown in Fig. 169, the compensating winding CW is short-circuited on itself, and acts as the secondary of a transformer, of which the primary is represented by the armature winding. The armature and compensating winding constitute, in fact, an arrangement corresponding to a somewhat leaky short-circuited transformer, and the equivalent

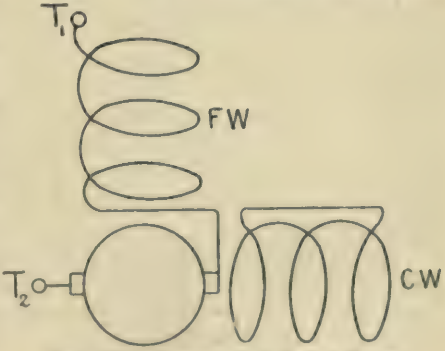

FIG. 169.—Compensated Series Motor.

reactance (due to leakage) is very much less than that of an armature without a compensating winding.*

Since a properly designed compensating winding will destroy the greater portion of the armature reactance, it follows that very little additional advantage will be gained by the use of further devices having the same object in view. There is, for example, no longer the same need to use salient poles, and the stator may be similar to that of an ordinary induction motor. There is, further, very little advantage gained by increasing the number of poles, *i.e.* by running considerably above the speed of synchronism.

§ 165. Methods of obtaining Sparkless Running

A difficulty which proved very serious with the earlier types of series-wound motor was that of *sparking*. The flux through the armature core being an alternating one, a coil short-circuited by a brush is subject to the inductive action of this flux, and may become the seat of excessive induced currents. The difficulty is greatest when the armature is at rest or running at a low speed, since at lower speeds the alternating flux is stronger,† and the coil remains longer short-circuited under its inductive action.

In modern series-wound motors, the difficulty has been overcome by (1) using as many commutator segments as possible; (2) using narrow carbon brushes, so that only one coil is short-circuited at a time; (3) introducing high-resistance connections (which may be placed at the bottom of the slots) between the winding and the commutator; (4) providing special reversing pole-pieces.

Since the width of carbon brush and commutator segment cannot be reduced below a certain limit for purely mechanical reasons, it follows that if the subdivision of the winding is carried as far as possible—*i.e.* with only one turn per coil—the number of armature conductors will be limited by the least possible width of segment. Hence it will be impossible, for a given speed, to construct a motor beyond a certain voltage without overstepping the limitations imposed by considerations of sparkless running. This accounts for the fact that such motors have generally been constructed for voltages not exceeding 200.

As regards both power factor and commutation, better results will be obtained at a lower frequency. Motors of the type under consideration have, for this reason, been generally used on circuits of frequency not exceeding 25.

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* By some writers, the term damping coil is applied to this winding.

† Corresponding to the larger torque, and hence larger current.

If a series-wound motor is intended to operate on both continuous- and alternating-current circuits, then since, when using continuous current, it is desirable to work with a stronger field, the field winding may be divided into two sections, which are connected in series with each other for continuous-current operation, and in parallel for single-phase current.

§ 166. Repulsion Motor

Another type of single-phase commutator motor which has undergone considerable development of late is the *repulsion motor*. This was originally invented by Professor Elihu Thomson in 1887. In its simplest form, it is shown in Fig. 170. The brushes, it will be noticed, are entirely disconnected from the field winding, and are short-circuited by a short length of flexible conductor. They are displaced from the neutral zone through a certain angle. The general principle of action of such a motor is not difficult to understand. The flux impressed on the armature by the field, and represented in Fig. 170 by the vector OF, may be resolved into two components : one—OA—being along the line of brushes, and the other—OB—at right angles to this line. The first component will obviously give rise to an e.m.f. which will produce currents in the armature winding flowing through the short-circuited brushes. The armature winding behaves towards this component of the flux like the short-circuited secondary of an ordinary transformer. The e.m.f.'s induced in the armature winding by the alternations of the second component OB will at every instant balance each other, so that there will be no current due to the two equal and opposite e.m.f.'s.* But since the magnetic axis of the armature is along the line of brushes, while the component OB is at right angles to this line, a torque will be exerted on the armature. When the armature is running, there will, in addition to the two e.m.f.'s in its windings already considered—one of which, that due to the alternations of the flux component OB, is inoperative, and may therefore be left out of account—be two other e.m.f.'s, due to the *rotation* of the armature in the field components OA and OB. The former e.m.f. is easily seen to be inoperative, while the latter is compounded with the e.m.f.

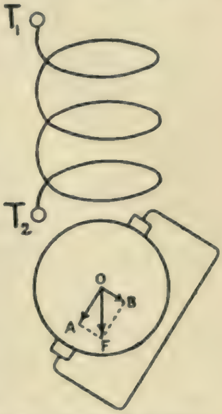

Fig. 170.—Repulsion Motor.

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* These e.m.f.'s would give rise to a current if a pair of brushes were provided at right angles to the first pair.

due to the alternations of the field component OA to produce a certain resultant e.m.f. which gives rise to the actually existing armature current.

As regards the field winding, which is connected across the mains, the impressed p.d. may be regarded as made up of the following components: (1) the component required to balance the effective resistance drop in the field; (2) that required to balance the effective reactance drop; (3) the component required to balance the e.m.f. induced in the field winding by the alternating flux due to that portion of the armature current which results from the rotation of the armature in the field.

By considering these three components, a circle diagram similar to that deduced above for the series-wound motor may be established. The two diagrams will not, however, be identical in every detail. The torque, for example, in the case of the repulsion motor is not proportional to the square of the stator current.

The torque will clearly depend on the angle a which the line of brushes makes with the neutral line, and it is not difficult to see that there are two extreme positions of the brushes for which the starting torque vanishes. The condition, in fact, necessary for the development of a starting torque is the *simultaneous* existence of the two field components OA and OB. If, then, $a = 0$, *i.e.* if the brushes be placed along the neutral line, the component OA vanishes; while if $a = 90°$, the component OB vanishes. Thus in either of these two positions there will be no starting torque. For some intermediate position, the torque will reach a maximum value; this is found to correspond to $a = 60°$ to $70°$.

The maximum voltage of the series-wound motor is, as we have seen, limited by considerations of sparking to about 200 volts. Hence, if such a motor be used for traction purposes in connection with a high-voltage overhead trolley line, a transformer must necessarily be employed to reduce the high voltage. In the repulsion motor, however, the armature is entirely disconnected from the field, so that there is nothing to prevent the latter from being wound for a very high voltage. A transformer may thus be dispensed with. The possibility of connecting a repulsion motor directly across a high-voltage line is its chief advantage over the series-wound motor.

The speed regulation of the repulsion motor may be effected by the simple device of altering the position of the brushes. Since, however, this motor is very sensitive to comparatively small changes in the brush position, this method of speed regulation may prove somewhat troublesome, a very small brush displacement giving rise to a considerable change of speed. The difficulty is partly remedied by using a special arrangement of brushes, due to Déri,* and shown in Fig. 171. OF denotes the direction of the flux impressed on the rotor by the stator. Two sets of brushes are used, B_0B_m and $B_0'B_m'$, which are independently short-circuited. The brushes B_0 and B_0' are always maintained in fixed positions—along the diameter whose direction coincides with that of the impressed field OF. The remaining brushes B_m and B_m' are arranged to be movable. Now, since the magnetic axis of the rotor is given by the bisector of the angle between two brushes not belonging to the same pair (B_0 and B_m', or B_0' and B_m), it follows that a displacement of the brushes B_m, B_m' through a given angle will shift the bisector of the angle between B_0 and B_m' (or B_0' and B_m)—*i.e.* the magnetic axis of the rotor—through *half* the angle of displacement of the brushes. In the simple arrangement of Fig. 170, on the other hand, the magnetic axis of the rotor is displaced through an angle equal to the angle of brush displacement. From this it immediately follows that with the arrangement of Fig. 171 a given displacement of the brushes will have less effect in altering the speed, and speed regulation will be easier.

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* *Elektrotechnische Zeitschrift*, vol. xxvi. p. 72 (1905).

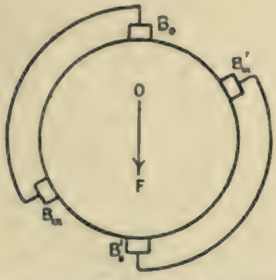

FIG. 171.—Déri's Arrangement of Brushes for Repulsion Motor.

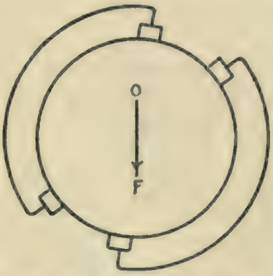

FIG. 172.—Latour's Arrangement of Brushes for Repulsion Motor.

Another modification in the arrangement of the brushes, due to Latour, is shown in Fig. 172.* There are, as in Déri's arrangement, two sets of brushes. All the brushes are, however, arranged to move simultaneously. The difference between this and the simple pair of brushes in Fig. 170 is that only a portion of the winding is short-circuited instead of the entire winding. The advantage gained by adopting the double short-circuit of Fig. 172 is a decreased tendency to spark. In order to explain how the improvement is brought about, we may refer to Fig. 170. The component of the field which gives rise to the sparking difficulty is OB. The short-circuited coils, with the arrangement of Fig. 170, are subject to the full inductive action of this component. By adopting the double short-circuit of

---

* *Elektrotechnische Zeitschrift*, vol. xxv. p. 952 (1904).

Fig. 172, the maximum value of the alternating flux traversing a short-circuited coil is clearly reduced; since now the portion of the flux between the magnetic axis of the rotor and the brush position does not become linked with the short-circuited coils, as it passes into and out of the rotor core outside the arcs spanned by the two sets of short-circuited brushes.

§ 167. Compensated Repulsion Motors

The compensated repulsion motor was invented simultaneously and independently by Latour in France and by Eichberg and Winter in Germany. In its simplest form it is shown in Fig. 173. At first sight, it would appear as if the motor differed but little from the simple series-wound motor of Fig. 164. In fact, the removal of the brushes B_1B_2 would transform it into such a simple series motor. But the presence of these brushes considerably modifies the action of the motor. One effect is to wipe out the self-inductance of the field winding to a very large extent, since the current flowing between B_1B_2 causes the armature winding to act as the short-circuited secondary of a transformer, of which the field winding is the primary. The field winding, therefore, simply acts as a transformer winding, and it no longer (as in the series-wound motor) supplies the entire magnetic field necessary for the production of a torque. This latter field is mainly supplied by the armature current which traverses the armature through the brushes B_3B_4. For this reason, the armature current flowing between B_3 and B_4 is spoken of as the *exciting current*,* while that flowing between B_1 and B_2 is termed the *short-circuit* current. The compensated repulsion motor is characterized by a very high power factor for all speeds above synchronism, and by the absence of sparking troubles.

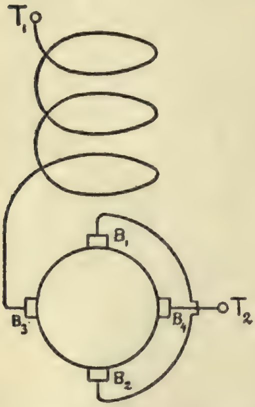

Fig. 173.—Diagram of Compensated Repulsion Motor.

The double short-circuit arrangement of brushes devised by Latour for the simple repulsion motor, and shown in Fig. 172, has also been adapted by him to the compensated repulsion motor. The connections are shown in Fig. 174. The short-circuit currents flow in the portions of the winding spanned by the short-circuited brushes B_1B_2 and B_3B_4

---

* By some writers it is termed the *compensating* current.

respectively, while the exciting currents flow through the remainder of the winding (B_1B_3 and B_2B_4).

In the forms of compensated repulsion motor hitherto considered, the stator could not be wound for a high voltage, as it is connected in series with the armature. By a simple modification, however, indicated in Fig. 175, the use of a high voltage stator winding becomes possible. This is the arrangement actually adopted in the Winter-Eichberg motor. The exciting current, instead of being fed directly into the armature, is supplied to it by the secondary of a transformer whose primary is in series with the stator winding. The secondary of this transformer is, as shown in the sketch, arranged to have a variable number of turns.

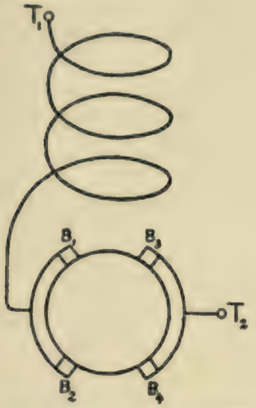

Fig. 174.—Latour's Arrangement of Brushes as applied to Compensated Repulsion Motor.

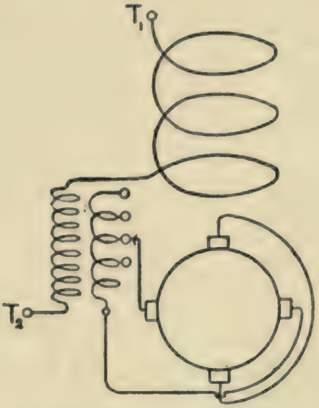

Fig. 175.—Connections of Winter-Eichberg Motor.

§ 168. Atkinson's Repulsion Motors

A form of repulsion motor, due to Atkinson, is represented in Fig. 176. So far as the principle of its action is concerned, this motor is identical with the simple form shown in Fig. 170, the only difference being that whereas in the simple form the two components OA and OB of the impressed field (Fig. 170) are produced by a single winding, in Atkinson's motor they are produced by two independent windings—the transformer winding TW and the exciting winding (or field winding proper) EW. The behaviour of this motor is in every way identical with that of the simple type of Fig. 170; it, however, possesses the advantage that the direction of rotation is easily reversed, by a simple reversal of the connections of one of the stator windings. In order to reverse the simple form of repulsion motor, it becomes necessary to rock the brushes to the other side of the line OF of the impressed field (Fig. 170).

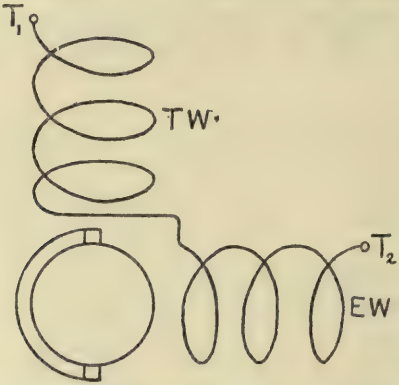

Fig. 176.—Connections of Atkinson's Motor.

A variety of Atkinson's motor is one in which only the transformer winding TW is connected across the mains, the exciting winding EW being across the brushes. The exciting winding then forms a load for the armature winding, regarded as the secondary of a transformer, of which TW is the primary.

§ 169. Mixed-action Commutator Motors

The single-phase commutator motors which we have so far considered are chiefly of interest in connection with railway work; they are variable-speed traction motors. For stationary work requiring approximately constant speed, the simple single-phase induction motor is capable of giving good results. The inherent weakness of this motor is its poor starting torque. Now, the repulsion motor is capable of exerting a powerful torque. If the good starting qualities of the repulsion type of motor could be combined with the satisfactory running qualities of the induction type, a motor would be obtained satisfactory from every point of view. Several forms of such compound or mixed-action commutator motors have been devised.

In the Wagner motor,* the rotor is provided with an ordinary continuous-current winding having a commutator, and at starting the

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* *The Electrician*, vol. li. p. 743 (1903).

brushes short-circuit the winding as in a repulsion motor. A powerful starting torque is thereby obtained. When the speed exceeds a certain limit, a centrifugal governor mounted on the motor shaft causes a collar to slide against the brush rocker ring, lifting the brushes clear of the commutator;* while at the same time a short-circuiting ring is forced into contact with the inner surfaces of the commutator segments, completely short-circuiting the individual armature coils, and thereby converting the motor into a single-phase, constant-speed induction motor.

The Schüler motor (Fig. 177) is provided with an ordinary star-connected stator winding, of which only two phases (in series with each other) are in use at a time. By interchanging one of the active phases and the idle phase (by means of the switch shown in the diagram) the motor may be reversed. The rotor has a continuous-current winding, fitted with a commutator on one side and three slip-rings on the other. At starting, the slip-ring circuits are open, and the motor starts as a simple repulsion motor. By closing the slip-ring circuits through starting resistances such as are ordinarily used in connection with three-phase induction motors, and gradually short-circuiting the resistances, the motor is transformed from the repulsion to the induction type.

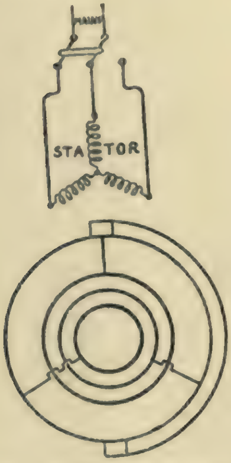

Fig. 177.—Diagram of Schüler Motor.

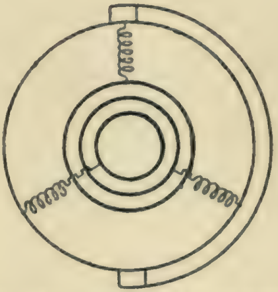

Fig. 178.—Diagram of Fynn Motor.

Another interesting and successful type of repulsion-induction motor is the Fynn motor. The stator of this motor has an ordinary three-phase winding, like that of the Schüler motor, only two phases being in use at a time. The rotor has a *double* winding: an ordinary continuous-current winding with a commutator and short-circuited brushes, and a three-phase winding connected to three slip-rings. The two windings are interconnected; the ends of the three-phase winding, instead of being connected star fashion to a neutral point, as they are in an ordinary induction motor, are joined to three points in the continuous-current winding 120° apart (in a two-pole motor), as shown in Fig. 178.

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* The commutator bearing surface is vertical, forming an annular ring; a displacement of the brush rocker parallel to the shaft will, therefore, move the brushes away from the commutator.

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